IYGB - SYNOPTIC PAPER N - QUESTION 1

a) STARTING WITH THE GRADIENT

$$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{9-4}{3-1} = \frac{5}{2}$$

USING THE POINT $A(1,4)$

$$y - y_0 = m(x-x_0)$$
$$y - 4 = \frac{5}{2}(x-1)$$
$$2y - 8 = 5x - 5$$
$$2y - 5x - 3 = 0$$
$$5x - 2y + 3 = 0$$

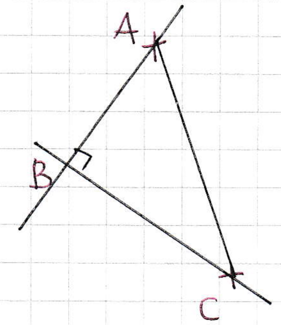
b) L_2 IS PERPENDICULAR TO L_1

$$m_{L_2} = -\frac{2}{5}$$

$B(3,9) \quad C(13,k)$

$$\frac{k-9}{13-3} = -\frac{2}{5}$$
$$\frac{k-9}{10} = -\frac{2}{5}$$
$$k - 9 = -4$$
$$k = 5$$

c) DRAWING A DIAGRAM, $A(1,4)$, $B(3,9)$, $C(13,5)$



$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$|AB| = \sqrt{(9-4)^2 + (3-1)^2} = \sqrt{25+4} = \sqrt{29}$$
$$|BC| = \sqrt{(5-9)^2 + (13-3)^2} = \sqrt{16+100} = \sqrt{116}$$

$$\text{AREA} = \frac{1}{2}|AB||BC| = \frac{1}{2}\sqrt{29}\sqrt{116}$$
$$= \frac{1}{2}\sqrt{29} \times \sqrt{4 \times 29}$$
$$= \frac{1}{2}\sqrt{29} \times 2 \times \sqrt{29}$$
$$= 29$$

IYGB - SYNOPTIC PAPER N - QUESTION 2

SETTING UP AN EQUATION AS FOLLOWS

$$\text{"SHARE BEFORE"} = \frac{840}{n}$$

$$\text{"SHARE AFTER"} = \frac{840}{n-6}$$

HENCE WE NOW HAVE

DIFFERENCE IN THE SHARES = 45

$$\frac{840}{n-6} - \frac{840}{n} = 45$$

$\div 15$

$$\frac{56}{n-6} - \frac{56}{n} = 3$$

$\times n(n-6)$

$$56n - 56(n-6) = 3n(n-6)$$

$$\cancel{56n} - \cancel{56n} + 336 = 3n^2 - 18n$$

$$0 = 3n^2 - 18n - 336$$

$\div 3$

$$0 = n^2 - 6n - 112$$

BY INSPECTION, QUADRATIC FORMULA OR COMPLETING THE SQUARE

$$(n+8)(n-14) = 0$$

$$n = \begin{cases} 14 \\ \cancel{-8} \end{cases}$$

$$\therefore n = 14$$

IYGB – SYNOPTIC PAPER N – QUESTION 3

a) SUPPOSE a & b ARE TWO NON CONSECUTIVE POSITIVE INTEGERS, WITH $a > b \geq 1$

THEN $\quad a^2 - b^2 = (a-b)(a+b)$

BUT $\quad a+b = 4, 5, 6, 7, 8, 9, \ldots$

$\qquad a-b = 2, 3, 4, 5, 6, 7, 8, \ldots$

HENCE a^2-b^2 CAN BE WRITTEN AS THE PRODUCT OF TWO FACTORS OF WHICH NEITHER IS 1

SO $a^2-b^2 \neq$ PRIME

b) THE ABOVE ARGUMENT CAN BE USED AS

$$(a-b)(a+b) = a^2-b^2$$

IF a & b ARE CONSECUTIVE & a^2-b^2 IS A PRIME THEN

$$(a-b)(a+b) = 163$$

$\therefore \; a-b = 1$
$\quad\; a+b = 163$ $\implies$ $2a = 164$
$\qquad a = 82 \quad (a^2 = 6724)$

$\implies b = 81 \quad (b^2 = 6561)$

i.e. THE REQUIRED NUMBERS ARE 6724 & 6561

IYGB - SYNOPTIC PAPER N - QUESTION 4

a) COMPLETING THE SQUARE IN x AND IN y

$\Rightarrow x^2+y^2-10x+4y+9=0$

$\Rightarrow x^2-10x+y^2+4y+9=0$

$\Rightarrow (x-5)^2-25+(y+2)^2-4+9=0$

$\Rightarrow (x-5)^2+(y+2)^2=20$

$\therefore C(5,-2)$

$r=\sqrt{20}$

b) "INTERSECTS THE x AXIS" $\Rightarrow y=0$

$\Rightarrow (x-5)^2+(0+2)^2=20$

$\Rightarrow (x-5)^2+4=20$

$\Rightarrow (x-5)^2=16$

$\Rightarrow x-5 = \begin{cases} 4 \\ -4 \end{cases}$

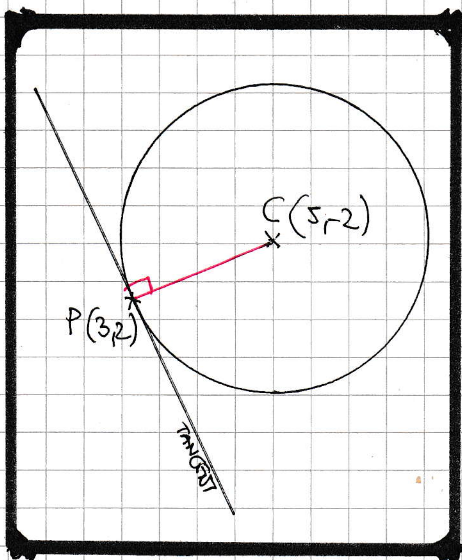
$\Rightarrow x = \begin{cases} 9 \\ 1 \end{cases}$

$\therefore (9,0)$ & $(1,0)$

c) CALCULATE THE GRADIENT CP

$m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-2-2}{5-3}=\dfrac{-4}{2}=-2$

$\therefore$ TANGENT GRADIENT IS $\frac{1}{2}$



$\therefore$ EQUATION OF TANGENT AT P(3,2)

$\Rightarrow y-y_0=m(x-x_0)$

$\Rightarrow y-2=\frac{1}{2}(x-3)$

$\Rightarrow 2y-4=x-3$

$\Rightarrow 2y-x-1=0$

$\Rightarrow x-2y+1=0$

AS REQUIRED

IYGB – SYNOPTIC PAPER N – QUESTION 5

a) SETTING $y=0$

$$y = x - 8\sqrt{x}$$
$$0 = x - 8\sqrt{x}$$
$$8\sqrt{x} = x$$
$$64x = x^2$$
$$x^2 - 64x = 0$$
$$x(x-64) = 0$$
$$x = \begin{cases} 0 \leftarrow O \\ 64 \leftarrow P \end{cases}$$

$\therefore$ $P(64,0)$

b) OBTAIN THE y CO-ORDINATE OF Q

$$y = 4 - 8 \times \sqrt{4}$$
$$y = 4 - 8 \times 2$$
$$y = -12$$

$\therefore Q(4,-12)$

FIND THE GRADIENT AT Q

$$y = x - 8x^{\frac{1}{2}}$$
$$\frac{dy}{dx} = 1 - 4x^{-\frac{1}{2}}$$
$$\left.\frac{dy}{dx}\right|_{x=4} = 1 - 4 \times 4^{-\frac{1}{2}} = 1 - 4 \times \frac{1}{2}$$
$$\frac{dy}{dx} = -1$$

NORMAL GRADIENT IS $+1$

$$y - y_0 = m(x - x_0)$$
$$y - (-12) = +1(x-4)$$
$$y + 12 = x - 4$$
$$y = x - 16$$

IYGB – SYNOPTIC PAPER N – QUESTION 5

c) SUPPOSE THAT IT DOES MEET THE CURVE

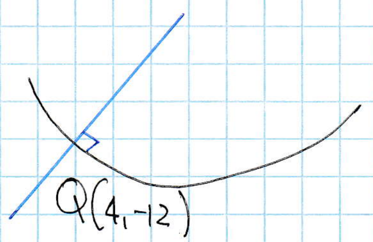
$$\left.\begin{array}{l} y = x - 16 \\ y = x - 8\sqrt{x} \end{array}\right\} \Rightarrow \left.\begin{array}{l} 16 = x - y \\ 8\sqrt{x} = x - y \end{array}\right\} \Rightarrow$$

$$\Rightarrow 16 = 8\sqrt{x}$$

$$\Rightarrow 2 = \sqrt{x}$$

$$\Rightarrow x = 4$$

∴ ONLY POINT OF INTERSECTION IS AT $x=4$, IE AT THE POINT $Q(4,-12)$, THE POINT OF NORMALITY



∴ THE NORMAL DOES NOT MEET THE CURVE AGAIN

IYGB – SYNOPTIC PAPER QUESTION 6

REMODEL INTO AN ARITHMETIC PROGRESSION

$$T = 240-5+237-5+234-5+231-5+\dots 6-5+3-5$$

$$T = (240-5)+(237-5)+(234-5)+(231-5)+\dots+(6-5)+(3-5)$$

$$T = 235+232+229+226+\dots+1+(-2)$$

THIS AN A.P. WITH $a = 235$ & $d = -3$

$$u_n = a+(n-1)d$$

$$-2 = 235+(n-1)(-3)$$

$$-2 = 235-3n+3$$

$$3n = 240$$

$$n = 80$$

USING $S_n = \frac{n}{2}(a+L)$

$$\Longrightarrow S_{40} = \frac{80}{2}\left[235-2\right]$$

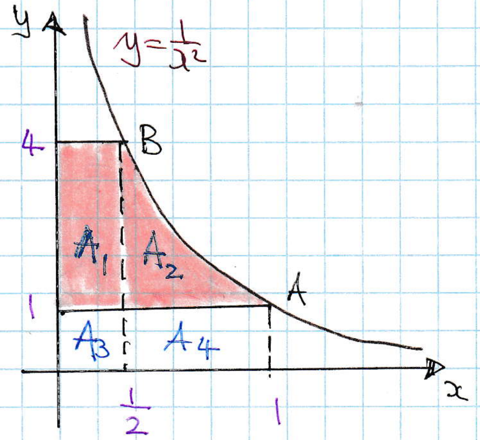
$$\Longrightarrow S_{40} = 40 \times 233$$

$$\Longrightarrow S_{40} = 9320$$

IYGB - SYNOPTIC PAPER N - QUESTION 7

METHOD A - USING INTEGRATION WITH RESPECT TO x

START BY OBTAINING THE x CO-ORDINATES OF A & B



$y = \frac{1}{x^2}$

- $4 = \frac{1}{x^2}$

  $x^2 = \frac{1}{4}$

  $x = +\frac{1}{2}$

- $1 = \frac{1}{x^2}$

  $x^2 = 1$

  $x = +1$

- $A_1 = \frac{1}{2} \times (4-1) = \frac{3}{2}$

- $A_2 + A_4 = \int_{\frac{1}{2}}^{1} \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_{\frac{1}{2}}^{1} = \left[+\frac{1}{x}\right]_{1}^{\frac{1}{2}}$

  $= \frac{1}{\frac{1}{2}} - \frac{1}{1} = 2 - 1 = 1$

- $A_4 = 1 \times \left(1 - \frac{1}{2}\right) = \frac{1}{2}$

REQUIRED AREA $= A_1 + A_2 = A_1 + (A_2 + A_4) - A_4$

$= \frac{3}{2} + 1 - \frac{1}{2}$

$= 2$

IYGB - SYNOPTIC PAPER N - QUESTION 7

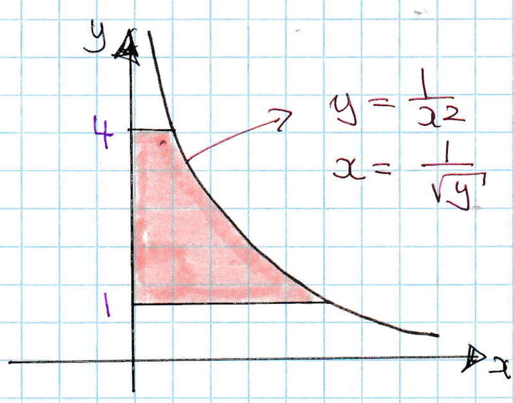
METHOD B - BY INTEGRATION WITH RESPECT TO y

- $y = \frac{1}{x^2}$

  $x^2 = \frac{1}{y}$

  $x = \pm \frac{1}{\sqrt{y}}$

  BUT $x \geqslant 0 \implies x = \frac{1}{\sqrt{y}}$



- $$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{y_1}^{y_2} x(y)\,dy$$

$$= \int_1^4 \frac{1}{\sqrt{y}}\,dy$$

$$= \int_1^4 y^{-\frac{1}{2}}\,dy$$

$$= \left[2y^{\frac{1}{2}}\right]_1^4$$

$$= 2\times4^{\frac{1}{2}} - 2\times1^{\frac{1}{2}}$$

$$= 4 - 2$$

$$= 2$$

IYGB – SYNOPTIC PAPER N – QUESTION 8

a) EXPANDING BINOMIALLY UP TO x^3

$$\frac{1}{\sqrt[3]{1+x}} = (1+x)^{-\frac{1}{3}} = 1 + \frac{-\frac{1}{3}}{1}(x)^1 + \frac{(-\frac{1}{3})(-\frac{4}{3})}{1\times 2}(x)^2 + \frac{-\frac{1}{3}(-\frac{4}{3})(-\frac{7}{3})}{1\times 2\times 3}(x)^3 + O(x^4)$$

$$\frac{1}{\sqrt[3]{1+x}} = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + O(x^4)$$

b) USING PART (a)

IF $f(x) = (1+x)^{-\frac{1}{3}}$, THEN $f(\frac{3}{4}x) = (1+\frac{3}{4}x)^{-\frac{1}{3}}$

$$\therefore \quad (1+\tfrac{3}{4}x)^{-\frac{1}{3}} = 1 - \frac{1}{3}\left(\frac{3}{4}x\right) + \frac{2}{9}\left(\frac{3}{4}x\right)^2 - \frac{14}{81}\left(\frac{3}{4}x\right)^3 + O(x^4)$$

$$(1+\tfrac{3}{4}x)^{-\frac{1}{3}} = 1 - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{7}{96}x^3 + O(x^4)$$

c) MANIPULATE AS FOLLOWS

$$\sqrt[3]{\frac{256}{4+3x}} = \sqrt[3]{\frac{64}{1+\frac{3}{4}x}} = \frac{\sqrt[3]{64}}{\sqrt[3]{1+\frac{3}{4}x}} = 4\left(1+\tfrac{3}{4}x\right)^{-\frac{1}{3}}$$

DIVIDE "TOP & BOTTOM" BY 4

$$= 4\left[1 - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{7}{96}x^3 + O(x^4)\right]$$

$$= 4 - x + \frac{1}{2}x^2 - \frac{7}{24}x^3 + O(x^4)$$

IYGB - SYNOPTIC PAPER N - QUESTION 9

WRITE THE L.H.S EXPLICITLY

$$\Longrightarrow \sum_{r=0}^{\infty} (\sin x)^{2r} = 2\tan x$$

$$\Longrightarrow 1 + \sin^2 x + \sin^4 x + \sin^6 x + \dots = 2\tan x$$

THIS IS A G.P WITH $a=1$ & $r = \sin^2 x$ & $S_\infty = \frac{a}{1-r}$

$$\Longrightarrow \frac{1}{1-\sin^2 x} = 2\tan x$$

$$\Longrightarrow \frac{1}{\cos^2 x} = 2\tan x$$

$$\Longrightarrow \sec^2 x = 2\tan x$$

USING $1+\tan^2 x \equiv \sec^2 x$

$$\Longrightarrow 1+\tan^2 x = 2\tan x$$

$$\Longrightarrow \tan^2 x - 2\tan x + 1 = 0$$

$$\Longrightarrow (\tan x - 1)^2 = 0$$

$$\Longrightarrow \tan x = 1$$

$$\therefore \quad x = \frac{\pi}{4} \pm n\pi \qquad n = 0,1,2,3,4,\dots$$

IYGB - SYNOPTIC PAPER N - QUESTION 10

SOLVE BY SEPARATION OF VARIABLES

$$\Rightarrow \frac{dy}{dx} = y^2 x^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{y^2}\,dy = x^{\frac{1}{2}}\,dx$$

$$\Rightarrow \int y^{-2}\,dy = \int x^{\frac{1}{2}}\,dx$$

$$\Rightarrow -y^{-1} = \frac{2}{3}x^{\frac{3}{2}} + C$$

$$\Rightarrow -\frac{1}{y} = \frac{2}{3}x^{\frac{3}{2}} + C$$

$$\Rightarrow \boxed{\frac{1}{y} = -\frac{2}{3}x^{\frac{3}{2}} + C}$$

APPLY CONDITION $(1,-2)$

$$\Rightarrow -\frac{1}{2} = -\frac{2}{3} + C$$

$$\Rightarrow C = \frac{1}{6}$$

REARRANGE TO THE REQUIRED ANSWER

$$\Rightarrow \frac{1}{y} = -\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{6}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{6} - \frac{4}{6}x^{\frac{3}{2}}$$

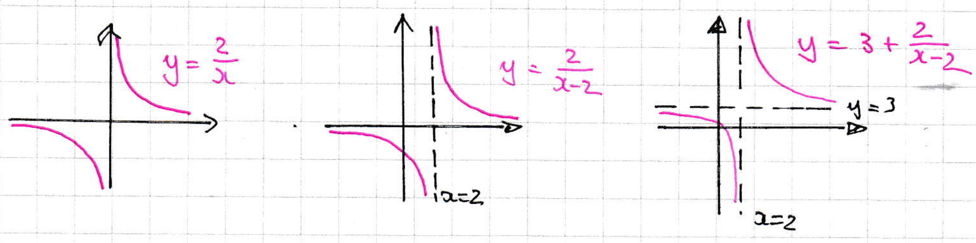
$$\Rightarrow \frac{1}{y} = \frac{1-4x^{\frac{3}{2}}}{6}$$

$$\Rightarrow y = \frac{6}{1-4x^{3/2}}$$

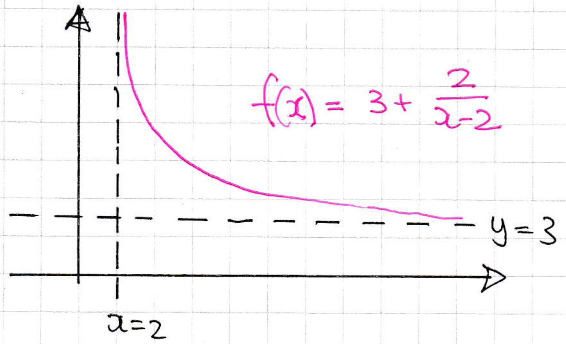
IE $A = 6$, $B = -4$

IYGB – SYNOPTIC PAPER N – QUESTION 11

a) WORKING THROUGH TRANSFORMATIONS



HENCE WE HAVE



b) APPLYING THE USUAL METHOD

$$y = 3 + \frac{2}{x-2}$$
$$y(x-2) = 3(x-2) + 2$$
$$yx - 2y = 3x - 6 + 2$$
$$yx - 3x = 2y - 4$$
$$x(y-3) = 2y - 4$$
$$x = \frac{2y-4}{y-3}$$

$$\therefore \quad f^{-1}(x) = \frac{2x-4}{x-3}$$

c) USING A "TWO WAY TABLE"

	$f(x)$	$f^{-1}(x)$
DOMAIN	$x>2$	$x>3$
RANGE	$f(x)>3$	$f^{-1}(x)>2$

↑ from graph

DOMAIN OF $f^{-1}(x)$: $x>3$

RANGE OF $f^{-1}(x)$: $f^{-1}(x)>2$

IYGB - SYNOPTIC PAPER N - QUESTION 11

d) $f(x) = f^{-1}(x)$ IS EQUIVALENT TO $f(x) = x$ OR $f^{-1}(x) = x$

$$\Rightarrow f^{-1}(x) = x$$

$$\Rightarrow \frac{2x-4}{x-3} = x$$

$$\Rightarrow 2x - 4 = x^2 - 3x$$

$$\Rightarrow 0 = x^2 - 5x + 4$$

$$\Rightarrow (x-1)(x-4) = 0$$

$$\Rightarrow x = \begin{cases} \cancel{1} \\ 4 \end{cases}$$

THE DOMAINS OF $f(x)$ OR $f^{-1}(x)$ DOES NOT ALLOW IT

IYGB - SYNOPTIC PAPER N - QUESTION 12

a) USING VECTORS OR THE STANDARD FORMULA

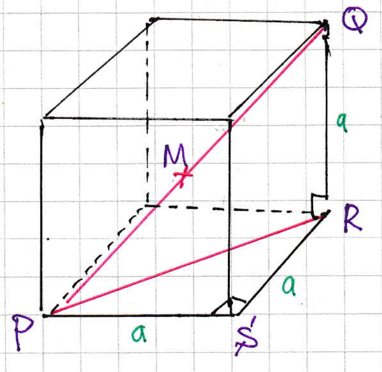
$$|PQ| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$
$$= \sqrt{(0-7)^2+(9+8)^2+(-2-11)^2}$$
$$= \sqrt{49+289+169}$$
$$= \sqrt{507}$$

b) MIDPOINT OF PQ

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = M\left(\frac{0+7}{2}, \frac{9-8}{2}, \frac{-2+11}{2}\right)$$
$$= M\left(\frac{7}{2}, \frac{1}{2}, \frac{9}{2}\right)$$

IE $\underline{m} = \frac{7}{2}\underline{i} + \frac{1}{2}\underline{j} + \frac{9}{2}\underline{k}$

c) LOOKING AT THE DIAGRAM



- $|PR|^2 = |PS|^2 + |SR|^2$
  $|PR|^2 = a^2 + a^2$
  $|PR|^2 = 2a^2$

- $|PQ|^2 = |PR|^2 + |RQ|^2$
  $(\sqrt{507})^2 = 2a^2 + a^2$
  $507 = 3a^2$
  $a^2 = 169$
  $a = +13$

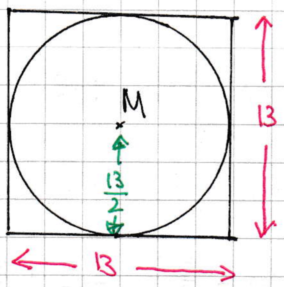
AS REQUIRED

IYGB - SYNOPTIC PAPER N - QUESTION 12

d) $M\left(\frac{7}{2}, \frac{1}{2}, \frac{9}{2}\right)$

$$|OM| = \sqrt{\frac{49}{4} + \frac{1}{4} + \frac{81}{4}} = \sqrt{\frac{131}{4}} = \frac{1}{2}\sqrt{131}$$

LOOKING AT THE DIAGRAM BELOW



IF $|OM| < \frac{13}{2}$, THEN THE POINT O MUST LIE INSIDE A SPHERE WHICH FITS TIGHTLY INSIDE THE CUBE

$|OM| = \frac{1}{2}\sqrt{131} \approx 5.72 < 6.5$

$\therefore$ "O" LIES INSIDE THE SPHERE

$\therefore$ "O" LIES INSIDE THE CUBE

IYGB - SYNOPTIC PAPER N - QUESTION 13

AS THE SINE AND COSINE FUNCTIONS "OSCILLATE" BETWEEN -1 & 1, IT IS EVIDENT THAT

$$\sin 6\theta + \cos 4\phi = -2 \implies \begin{cases} \sin 6\theta = -1 \\ \cos 4\phi = -1 \end{cases}$$

HENCE WE HAVE

- $\sin 6\theta = -1$

$$\begin{cases} 6\theta = -90 \pm 360n \\ 6\theta = 270 \pm 360n \end{cases} \quad n = 0,1,2,3,\ldots$$

$$\begin{cases} \theta = -15 \pm 60n \\ \theta = 45 \pm 60n \end{cases}$$

$$\theta = 105^\circ$$
$$\theta = 165$$

- $\cos 4\phi = -1$

$$\begin{cases} 4\phi = 180 \pm 360n \\ 4\phi = 180 \pm 360n \end{cases}$$

$$\left(\phi = 45 \pm 90n \right. \quad n = 0,1,2,3,\ldots$$

$$\phi = 135^\circ$$

$$\therefore \quad (\theta, \phi) = (105^\circ, 135^\circ) \cup (165^\circ, 135^\circ)$$

SYNOPTIC PAPER N – QUESTION 14

a) TWO SENSIBLE METHODS

- $e^{1-x} = 3e$

$$\ln(e^{1-x}) = \ln(3e)$$
$$1-x = \ln 3 + \ln e$$
$$1-x = \ln 3 + 1$$
$$-x = \ln 3$$
$$x = -\ln 3$$

- $e^{1-x} = 3e$

$$\frac{e^{1-x}}{e^1} = 3$$
$$e^{1-x-1} = 3$$
$$e^{-x} = 3$$
$$-x = \ln 3$$
$$x = \ln 3$$

AS BEFORE

b) THIS IS A "HIDDEN QUADRATIC"

$$e^w - 3 = \frac{8}{e^w - 1} \implies (e^w-3)(e^w-1) = 8$$
$$\implies e^{2w} - e^w - 3e^w + 3 = 8$$
$$\implies e^{2w} - 4e^w - 5 = 0$$
$$\implies (e^w+1)(e^w-5) = 0$$
$$\implies e^w = \cancel{-1},\ 5$$
$$\implies w = \ln 5$$

IYGB - SYNOPTIC PAPER N - QUESTION 15

a) START BY CONVERTING THE LIMITS

- $x = \frac{2}{3} \implies \frac{1}{1+t} = \frac{2}{3}$

$\implies t+1 = \frac{3}{2}$

$\implies t = \frac{1}{2}$

- $x = 2 \implies \frac{1}{1+t} = 2$

$\implies t+1 = \frac{1}{2}$

$\implies t = -\frac{1}{2}$

SET UP THE INTEGRAL FOR THE AREA FROM CARTESIAN INTO PARAMETRIC

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt$$

$$= \int_{\frac{1}{2}}^{-\frac{1}{2}} \frac{1}{1-t}\left(-\frac{1}{(1+t)^2}\right)dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1-t}\left(+\frac{1}{(1+t)^2}\right)dt$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{(1-t)(1+t)^2}\,dt$$

AS REQUIRED

BY PARTIAL FRACTIONS

$$\frac{1}{(1-t)(1+t)^2} \equiv \frac{A}{1-t} + \frac{B}{(1+t)^2} + \frac{C}{1+t}$$

$$\boxed{1 \equiv A(1+t)^2 + B(1-t) + C(1+t)(1-t)}$$

- IF $t=1$

$1 = 4A$

$A = \frac{1}{4}$

- IF $t=-1$

$1 = 2B$

$B = \frac{1}{2}$

- IF $t=0$

$1 = A + B + C$

$1 = \frac{1}{4} + \frac{1}{2} + C$

$C = \frac{1}{4}$

IYGB - SYNOPTIC PAPER N - QUESTION 15

FINALLY THE AREA CAN BE FOUND

$$\Rightarrow \text{AREA} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\frac{1}{4}}{1-t} + \frac{\frac{1}{4}}{1+t} + \frac{\frac{1}{2}}{(1+t)^2}\, dt$$

$$\Rightarrow \text{AREA} = \left[\tfrac{1}{4}\ln|1+t| - \tfrac{1}{4}\ln|1-t| - \frac{\frac{1}{2}}{1+t} \right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$\Rightarrow \text{AREA} = \tfrac{1}{4}\left[\ln(1+t) - \ln(1-t) - \frac{2}{t+1} \right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$\Rightarrow \text{AREA} = \tfrac{1}{4}\left[\left(\ln\tfrac{3}{2} - \ln\tfrac{1}{2} - \tfrac{4}{3}\right) - \left(\ln\tfrac{1}{2} - \ln\tfrac{3}{2} - 4\right) \right]$$

$$\Rightarrow \text{AREA} = \tfrac{1}{4}\left[2\ln\tfrac{3}{2} - 2\ln\tfrac{1}{2} + \tfrac{8}{3} \right]$$

$$\Rightarrow \text{AREA} = \tfrac{1}{2}\left[\ln\tfrac{3}{2} - \ln\tfrac{1}{2} + \tfrac{4}{3} \right]$$

$$\Rightarrow \text{AREA} = \tfrac{1}{2}\left[\ln 3 + \tfrac{4}{3} \right]$$

$$\Rightarrow \text{AREA} = \tfrac{2}{3} + \tfrac{1}{2}\ln 3$$

b) **ELIMINATE THE PARAMETER**

$$\Rightarrow x = \frac{1}{t+1}$$

$$\Rightarrow t+1 = \frac{1}{x}$$

$$\Rightarrow t = \frac{1}{x} - 1 \quad \Rightarrow \quad y = \frac{1}{1-\left(\frac{1}{x}-1\right)}$$

$$\Rightarrow \quad y = \frac{1}{2-\frac{1}{x}}$$

$$\Rightarrow \quad y = \frac{x}{2x-1}$$

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RE-ATTEMPTING THE AREA BY CARTESIAN INTEGRATION

$$\Rightarrow \text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{\frac{2}{3}}^{2} \frac{x}{2x-1}\,dx$$

BY SUBSTITUTION OR MANIPULATIONS

$$\Rightarrow \text{AREA} = \frac{1}{2}\int_{\frac{2}{3}}^{2} \frac{2x}{2x-1}\,dx = \frac{1}{2}\int \frac{(2x-1)+1}{(2x-1)}\,dx$$

$$\Rightarrow \text{AREA} = \frac{1}{2}\int_{\frac{2}{3}}^{2} 1 + \frac{1}{2x-1}\,dx$$

$$\Rightarrow \text{AREA} = \frac{1}{2}\left[x + \frac{1}{2}\ln|2x-1|\right]_{\frac{2}{3}}^{2}$$

$$\Rightarrow \text{AREA} = \frac{1}{2}\left[\left(2 + \frac{1}{2}\ln 3\right) - \left(\frac{2}{3} + \frac{1}{2}\ln\frac{1}{3}\right)\right]$$

$$\Rightarrow \text{AREA} = \frac{1}{2}\left[\frac{4}{3} + \frac{1}{2}\ln 3 - \frac{1}{2}\ln\frac{1}{3}\right]$$

$$\Rightarrow \text{AREA} = \frac{1}{2}\left[\frac{4}{3} + \frac{1}{2}\ln 3 + \frac{1}{2}\ln 3\right]$$

$$\Rightarrow \text{AREA} = \frac{1}{2}\left[\frac{4}{3} + \ln 3\right]$$

$$\Rightarrow \text{AREA} = \frac{2}{3} + \frac{1}{2}\ln 3$$ ✓ AS BEFORE

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START BY OBTAINING THE GRADIENT FUNCTION

$$y = 3\tan^3 2x$$

$$\frac{dy}{dx} = 9\tan^2 2x\,(\sec^2 2x) \times 2$$

$$\frac{dy}{dx} = 18\tan^2 2x \sec^2 2x$$

OR $\frac{dy}{dx} = \frac{18\tan^2 2x}{\cos^2 2x}$

NOW EVALUATION OF $\frac{dy}{dx}$ AT $\arctan\frac{1}{2}$

LET $x = \arctan\frac{1}{2}$

$\tan x = \frac{1}{2}$

$$\Rightarrow \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\Rightarrow \tan 2x = \frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}$$

$$\Rightarrow \tan 2x = \frac{1}{3/4}$$

$$\Rightarrow \tan 2x = \frac{4}{3}$$

NOW USING THE STANDARD TRIANGLE OR AN IDENTITY

$$\Rightarrow 1 + \tan^2 2x = \sec^2 2x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 2x$$

$$\Rightarrow \sec^2 2x = \frac{25}{9}$$

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COLLECTING ALL THE RESULTS

$$\frac{dy}{dx} = 18\tan^2 2x\sec^2 2x$$

$$\left.\frac{dy}{dx}\right|_{x=\arctan\frac{1}{2}} = 18 \times \left(\frac{4}{3}\right)^2 \times \frac{25}{9}$$

$$\left.\frac{dy}{dx}\right|_{x=\arctan\frac{1}{2}} = \cancel{18}^{\,2} \times \frac{16}{\cancel{9}} \times \frac{25}{9}$$

$$\left.\frac{dy}{dx}\right|_{x=\arctan\frac{1}{2}} = \frac{16 \times 50}{9} = \frac{800}{9}$$

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a) DIFFERENTIATE THE ENTIRE EQUATION WITH RESPECT TO x

$$\Rightarrow x^2 + 4xy + 2y^2 + 18 = 0$$

$$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(4xy) + \frac{d}{dx}(2y^2) + \frac{d}{dx}(18) = \frac{d}{dx}(0)$$

$$\Rightarrow 2x + \left[4y + 4x\frac{dy}{dx}\right] + 4y\frac{dy}{dx} + 0 = 0$$

$$\Rightarrow 4x\frac{dy}{dx} + 4y\frac{dy}{dx} = -2x - 4y$$

$$\Rightarrow 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -x - 2y$$

$$\Rightarrow (2x+2y)\frac{dy}{dx} = -(x+2y)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x+2y}{2x+2y}$$

// AS REQUIRED

b) SOLVING $\frac{dy}{dx} = 0$

$$x + 2y = 0$$

$$x = -2y$$

SUBSTITUTE INTO THE EQUATION OF THE CURVE

$$\Rightarrow x^2 + 4xy + 2y^2 + 18 = 0$$

$$\Rightarrow (-2y)^2 + 4(-2y)y + 2y^2 + 18 = 0$$

$$\Rightarrow 4y^2 - 8y^2 + 2y^2 + 18 = 0$$

$$\Rightarrow 18 = 2y^2$$

$$\Rightarrow y^2 = 9$$

$$\Rightarrow y = \begin{cases} 3 \\ -3 \end{cases} \qquad x = \begin{cases} -6 \\ 6 \end{cases}$$

$\therefore$ $(-6, 3)$ & $(6, -3)$ //

IYGB - SYNOPTIC PAPER N - QUESTION 18

START WITH THE DENOMINATOR, BY AN "R-TRANSFORMATION" OR A MANIPULATION

$$\cos x + \sqrt{3}\sin x = 2\left[\tfrac{1}{2}\cos x + \tfrac{\sqrt{3}}{2}\sin x\right]$$
$$= 2\left[\cos\tfrac{\pi}{3}\cos x + \sin\tfrac{\pi}{3}\sin x\right]$$
$$= 2\cos\left(\tfrac{\pi}{3} - x\right)$$
$$= 2\cos\left(x - \tfrac{\pi}{3}\right)$$

HENCE THE INTEGRAL BECOMES

$$\int_0^{\frac{\pi}{3}} \frac{1}{(\cos x + \sqrt{3}\sin x)^2}\,dx = \int_0^{\frac{\pi}{3}} \frac{1}{\left[2\cos\left(x - \frac{\pi}{3}\right)\right]^2}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{4\cos^2\left(x - \frac{\pi}{3}\right)}\,dx = \int_0^{\frac{\pi}{3}} \tfrac{1}{4}\sec^2\left(x - \tfrac{\pi}{3}\right)\,dx$$

$$= \left[\tfrac{1}{4}\tan\left(x - \tfrac{\pi}{3}\right)\right]_0^{\frac{\pi}{3}} = \tfrac{1}{4}\left[\tan 0 - \tan\left(-\tfrac{\pi}{3}\right)\right]$$

$$= \tfrac{1}{4}\left(0 + \tan\tfrac{\pi}{3}\right) = \tfrac{1}{4}\sqrt{3}$$

IYGB - SYNOPTIC PAPER N - QUESTION 19

USING THE FORMAL DEFINITION OF THE DERIVATIVE

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$\frac{d}{dx}(\tan x) = \lim_{h \to 0}\left[\frac{\tan(x+h) - \tan x}{h}\right]$$

WRITE AS SINES AND COSINES

$$\frac{d}{dx}(\tan x) = \lim_{h \to 0}\left[\frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\frac{\sin(x+h)\cos x - \cos(x+h)\sin x}{\cos x\cos(x+h)}}{h}\right]$$

COMPOUND ANGLE IDENTITY FOR $\sin(A-B)$

$$= \lim_{h \to 0}\left[\frac{\frac{\sin[(x+h)-x]}{\cos x\cos(x+h)}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\frac{\sin(h)}{\cos x\cos(x+h)}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\sin(h)}{\cos x\cos(x+h)} \div h\right]$$

$$= \lim_{h \to 0}\left[\frac{\sin(h)}{\cos x\cos(x+h)} \times \frac{1}{h}\right]$$

AS $h \to 0$ $\quad \frac{\sin(h)}{h} \longrightarrow 1 \qquad \sin x \approx x$ FOR SMALL x

$$= \lim_{h \to 0}\left[\frac{1}{\cos x\cos(x+h)} \times \cancel{\frac{\sin h}{h}}\right] = \frac{1}{\cos^2 x} = \sec^2 x$$

IYGB - SYNOPTIC PAPER N - QUESTION 20

$x=b$ IS A SOLUTION OF THE EQUATION

$$\Rightarrow ax^3+ax^2+ax+b=0$$
$$\Rightarrow ab^3+ab^2+ab+b=0$$
$$\Rightarrow ab^2+ab+a+1=0$$

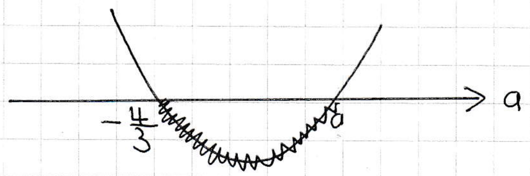
$\div b$ (as $b \neq 0$)

THIS IS A QUADRATIC IN b

$$\Rightarrow ab^2+ab+(a+1)=0$$

IT MUST HAVE REAL SOLUTIONS IN b

$$\Rightarrow B^2-4AC \geqslant 0$$
$$\Rightarrow a^2-4\times a\times(a+1) \geqslant 0$$
$$\Rightarrow a^2-4a^2-4a \geqslant 0$$
$$\Rightarrow -3a^2-4a \geqslant 0$$
$$\Rightarrow 3a^2+4a \leqslant 0$$
$$\Rightarrow a(3a+4) \leqslant 0$$



$$-\frac{4}{3} \leqslant a \leqslant 0$$