IYGB - SYNOPTIC PAPER M - QUESTION 1

WORK AS FOLLOWS

$$\Rightarrow \frac{2}{x+1} < x$$

$$\Rightarrow \frac{2(x+1)}{(x+1)(x+1)} < x$$

$$\Rightarrow \frac{2(x+1)}{(x+1)^2} < x$$

MULTIPLYING ACROSS NOW IS NOT AN ISSUE AS $(x+1)^2$ CANNOT BE NEGATIVE

$$\Rightarrow 2(x+1) < x(x+1)^2$$

$$\Rightarrow 2(x+1) - x(x+1)^2 < 0$$

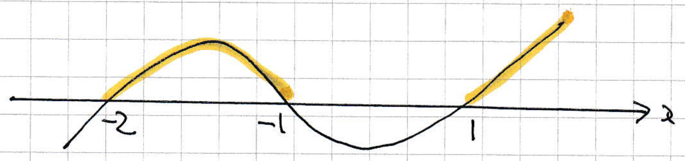
$$\Rightarrow (x+1)\left[2 - x(x+1)\right] < 0$$

$$\Rightarrow (x+1)(2 - x^2 - x) < 0$$

$$\Rightarrow (x+1)(x^2+x-2) > 0$$

$$\Rightarrow (x+1)(x-1)(x+2) > 0$$

THIS IS A "POSITIVE" CUBIC, SKETCHED BELOW



$\therefore$ $-2 < x < -1$ OR $x > 1$

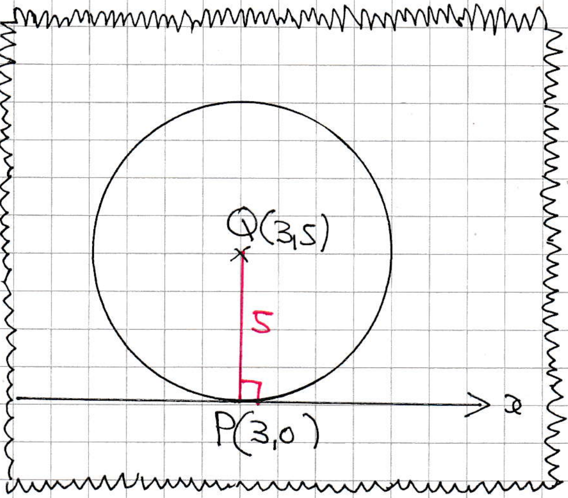
iYGB - SYNOPTIC PAPER M - QUESTION 2

a) COMPLETE THE SQUARES IN x AND IN y

$$\Longrightarrow x^2+y^2-6x-10y+k=0$$
$$\Longrightarrow x^2-6x+y^2-10y=-k$$
$$\Longrightarrow (x-3)^2-9+(y-5)^2-25=-k$$
$$\Longrightarrow (x-3)^2+(y-5)^2=34-k$$

$\therefore$ Q(3,5)

b) DRAWING A DIAGRAM



BY INSPECTION P(3,0) AND THE RADIUS IS 5

$$\Longrightarrow r^2=34-k$$
$$\Longrightarrow 5^2=34-k$$
$$\Longrightarrow 25=34-k$$
$$\Longrightarrow k=9$$

& P(3,0)

IYGB - SYNOPTIC PAPER M - QUESTION 3

PROCEED AS FOLLOWS

$$\Rightarrow f(x) = \frac{3x-1}{(1-2x)2} = (-1+3x)(1-2x)^{-2}$$

$$\Rightarrow f(x) = (-1+3x)\left[1 + \frac{-2}{1}(-2x)^{1} + \frac{-2(-3)}{1\times 2}(-2x)^{2} + \frac{-2(-3)(-4)}{1\times 2\times 3}(-2x)^{3} + O(x^4)\right]$$

$$\Rightarrow f(x) = (-1+3x)\left[1 + 4x + 12x^2 + 32x^3 + O(x^4)\right]$$

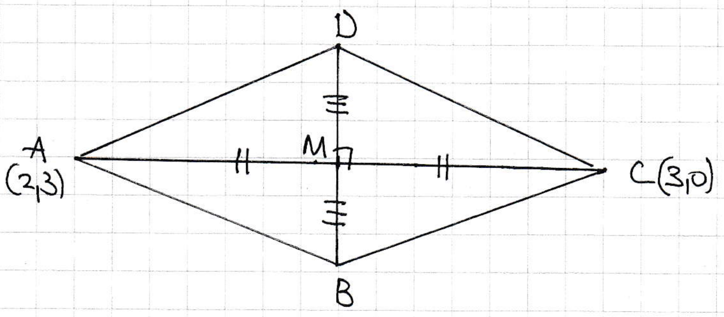
$$\Rightarrow f(x) = \begin{array}{l} -1 - 4x - \cancel{12x^2} - 32x^3 + O(x^4) \\ \qquad + 3x + \cancel{12x^2} + 36x^3 + O(x^4) \end{array}$$

$$\Rightarrow f(x) = -1 + x + 4x^3 + O(x^4)$$

// AS REQUIRED

IYGB - SYNOPTIC PAPER M - QUESTION 4

a) LOOKING AT THE GEOMETRY OF A RHOMBUS



MIDPOINT OF AC

$M\left(\frac{2+3}{2}, \frac{3+0}{2}\right)$

$M\left(\frac{5}{2}, \frac{3}{2}\right)$

GRADIENT OF AC

$m_{AC} = \frac{y_2-y_1}{x_2-x_1} = \frac{0-3}{3-2} = \frac{-3}{1} = -3$

PERPENDICULAR BD HAS GRADIENT $\frac{1}{3}$

EQUATION OF BD

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - \frac{3}{2} = \frac{1}{3}\left(x - \frac{5}{2}\right)$

$\Rightarrow y - \frac{3}{2} = \frac{1}{3}x - \frac{5}{6}$

$\Rightarrow 6y - 9 = 2x - 5.$

$\Rightarrow 0 = 2x - 6y + 4$

$\Rightarrow x - 3y + 2 = 0$

AS REQUIRED

b) SOLVING SIMULTANEOUSLY LINE_{AD} & LINE_{BD} WILL FIND D

$$\left.\begin{array}{l} x-3y+2=0 \\ 3x-4y+6=0 \end{array}\right\} \begin{array}{l} \times(-3) \\ \times 1 \end{array} \Rightarrow \begin{array}{l} -3x+9y-6=0 \\ 3x-4y+6=0 \end{array}$$

ADDING GIVES

$5y = 0$

$y = 0$

$\therefore x = -2$

$\therefore D(-2,0)$

IYGB - SYNOPTIC PAPER O - QUESTION 4

c) NOW $M\left(\frac{5}{2}, \frac{3}{2}\right)$ MUST ALSO BE THE MIDPOINT OF B & D(−2, 0)

THUS

x	-2	$\xrightarrow{+4.5}$	(2.5) $\frac{5}{2}$	$\xrightarrow{+4.5}$	7
y	0	$\xrightarrow[+1.5]{}$	$\frac{3}{2}$ (1.5)	$\xrightarrow[+1.5]{}$	3
	↑ D		↑ M		↑ B

$\therefore B(7,3)$ //

IYGB - SYNOPTIC PAPER M - QUESTION 5

PROCEED AS FOLLOWS

$$\Longrightarrow \frac{2^{399}-2^{395}}{15} = 32^{k}$$

$$\Longrightarrow \frac{2^{4}\times 2^{395} - 2^{395}}{15} = \left(2^{5}\right)^{k}$$

$$\Longrightarrow \frac{16\times 2^{395} - 2^{395}}{15} = 2^{5k}$$

$$\Longrightarrow \frac{\cancel{15}\times 2^{395}}{\cancel{15}} = 2^{5k}$$

$$\Longrightarrow 2^{395} = 2^{5k}$$

$$\Longrightarrow 5k = 395$$

$$\Longrightarrow k = 79$$

//

IYGB – SYNOPTIC PAPER M – QUESTION 6

a) BY LONG DIVISION

$$\begin{array}{r|l} & 4x-10 \quad \leftarrow \text{QUOTIENT} \\ \hline x^2+2x-5 & 4x^3-2x^2+x+5 \\ & -4x^3-8x^2+20x \\ \hline & \quad -10x^2+21x+5 \\ & \quad +10x^2+20x-50 \\ \cline{2-2} & \qquad\quad 41x-45 \quad \leftarrow \text{REMAINDER} \end{array}$$

$\therefore$ QUOTIENT $4x-10$

$\therefore$ REMAINDER $41x-45$

b) BY INSPECTION IF THERE IS NO REMAINDER

$$f(x) \equiv 4x^3-2x^2+ax+b \equiv (x^2+2x-5)(4x+c)$$

- $f(0) = b = -5c$
- $f(1) = 2+a+b = -2(4+c)$
- $f(-1) = -6-a+b = -6(-4+c)$

$\Longrightarrow$ ADDING THE LAST 2 EQUATIONS

$\Longrightarrow -4+2b = -2(c+4)-6(c-4)$

$\Longrightarrow -4+2b = -2c-8-6c+24$

$\Longrightarrow -4+2b = -8c+16$

$\Longrightarrow -2+b = -4c+8$

$\Longrightarrow -2+(-5c) = -4c+8$

$\Longrightarrow -10 = c$

THUS WE NOW HAVE

- $b = -5c = -5(-10) = 50$
- $2+a+b = -8-2c$

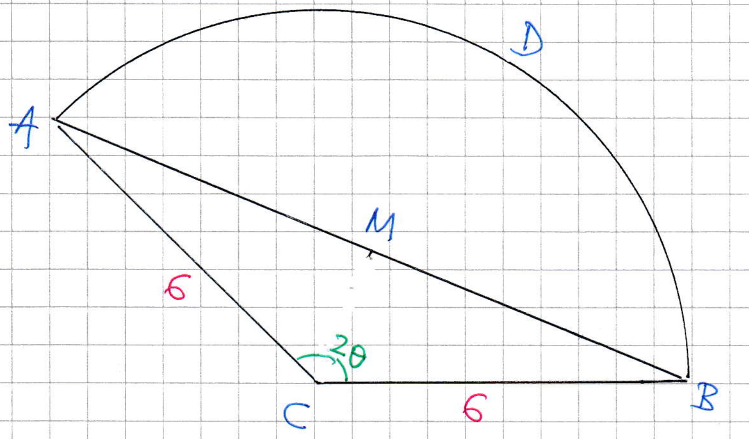
  $2+a+50 = -8-2(-10)$

  $a+52 = 12$

  $a = -40$

$\therefore a = -40$ & $b = 50$

IYGB - SYNOPTIC PAPER M - QUESTION 7



WORKING AT THE ISOSCELES TRIANGLE ACB

$$\text{AREA} = \frac{1}{2}|AC||CB|\sin 2\theta$$
$$\text{AREA} = \frac{1}{2}\times 6\times 6\times \sin 2\theta$$
$$\text{AREA} = 18\sin 2\theta$$

AREA OF SECTOR, USING "$\frac{1}{2}r^2\theta^c$"

$$\text{AREA} = \frac{1}{2}\times 6^2\times(2\theta)$$
$$\text{AREA} = 36\theta$$

AREA OF SEGMENT IS GIVEN BY

$$36\theta - 18\sin 2\theta$$

USING THE REQUIRED RATIO $\Rightarrow$ AREA OF TRIANGLE = 4× AREA OF SEGMENT

$$18\sin 2\theta = 4(36\theta - 18\sin 2\theta)$$
$$18\sin 2\theta = 144\theta - 72\sin 2\theta$$
$$0 = 144\theta - 90\sin 2\theta$$
$$\div 18$$
$$8\theta - 5\sin 2\theta = 0$$

AS REQUIRED

IYGB - SYNOPTIC PAPER M - QUESTION 8

a) FIND THE CARTESIAN CO-ORDINATES & THE VALUE OF t AT P & A

- $y = 3\sec t$
  $6 = 3\sec t$
  $2 = \sec t$
  $\cos t = \frac{1}{2}$
  $t = \frac{\pi}{3}$
- $x = 6t\sin t$
  $x = 6 \times \frac{\pi}{3} \times \sin\frac{\pi}{3}$
  $x = 2\pi \times \frac{\sqrt{3}}{2}$
  $x = \sqrt{3}\pi$

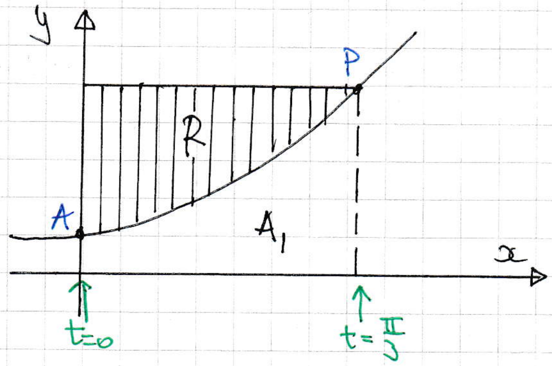
$\therefore$ $P\left(\sqrt{3}\pi, 6\right)$ WITH $t = \frac{\pi}{3}$

$x = 0$

$t = 0$ (BY INSPECTION)

$\therefore$ $A(0,3)$ WITH $t = 0$

LOOKING AT THE DIAGRAM



$$A_1 = \int_{x_1}^{x_2} y(x)\,dx$$

$$A_1 = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt$$

$$A_1 = \int_0^{\frac{\pi}{3}} (3\sec t)\underbrace{\left[6\sin t + 6t\cos t\right]}_{}\,dt \quad \frac{dx}{dt}$$

SIMPLIFYING THE INTEGRAND

$$A_1 = \int_0^{\frac{\pi}{3}} \frac{18}{\cos t}\left(\sin t + t\cos t\right)dt = 18\int_0^{\frac{\pi}{3}} \frac{\sin t}{\cos t} + \frac{t\cos t}{\cos t}\,dt$$

$$= 18\int_0^{\frac{\pi}{3}} \tan t + t\,dt$$

AS REQUIRED

IYGB - SYNOPTIC PAPER M - QUESTION 8

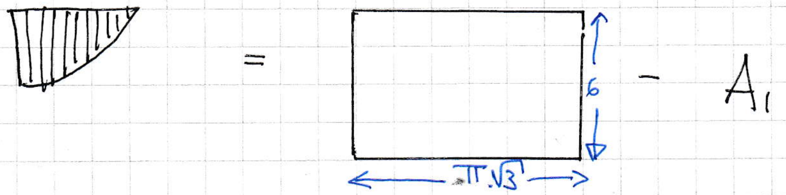
b) FINISHING THE INTEGRAL

$$A_1 = 18\int_0^{\pi/3} t + \tan t \, dt = 18\left[\tfrac{1}{2}t^2 + \ln|\sec t|\right]_0^{\frac{\pi}{3}}$$

$$= 18\left[\left[\tfrac{1}{2}\times\tfrac{\pi^2}{9} + \ln\left(\sec\tfrac{\pi}{3}\right)\right] - \left[0 + \ln\left(\cancel{\sec 0}\right)\right]\right]$$

$$= 18\left[\tfrac{\pi^2}{18} + \ln 2\right] = \pi^2 + 18\ln 2$$

LOOKING AT THE PREVIOUS DIAGRAM



$$= 6\pi\sqrt{3} - \left(18\ln 2 + \pi^2\right)$$

$$= 6\pi\sqrt{3} - \pi^2 - 18\ln 2$$

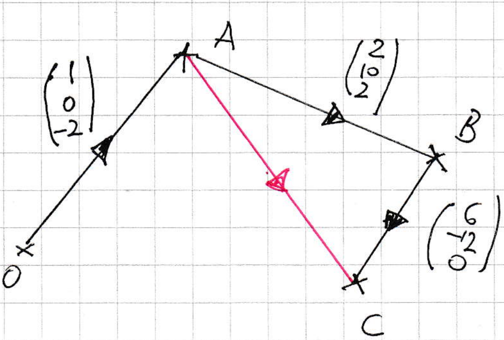
10.30213491

$$\simeq 10.3$$

// AS REQUIRED

IYGB – SYNOPTIC PAPER M – QUESTION 9

a) STARTING WITH A DIAGRAM



$$\vec{AC} = \vec{AB} + \vec{BC}$$
$$= \begin{pmatrix}2\\10\\2\end{pmatrix} + \begin{pmatrix}6\\-12\\0\end{pmatrix}$$
$$= \begin{pmatrix}8\\-2\\2\end{pmatrix}$$

WORKING OUT MODULI

$$\Longrightarrow |\vec{AB}| = \begin{vmatrix}2\\10\\2\end{vmatrix} = \sqrt{4+100+4} = \sqrt{108}$$

$$\Longrightarrow |\vec{BC}| = \begin{vmatrix}6\\-12\\0\end{vmatrix} = \sqrt{36+144+0} = \sqrt{180}$$

$$\Longrightarrow |\vec{AC}| = \begin{vmatrix}8\\-2\\2\end{vmatrix} = \sqrt{64+4+4} = \sqrt{72}$$

$$\Longrightarrow |AC|^2 + |AB|^2 = \sqrt{72}^2 + \sqrt{108}^2 = 72 + 108 = 180 = |BC|^2$$

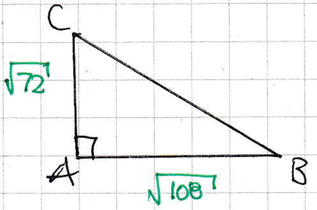
$\therefore$ AC $\perp$ AB

ALTERNATIVE BY DOT PRODUCT

$$\begin{pmatrix}2\\10\\2\end{pmatrix} \cdot \begin{pmatrix}8\\-2\\2\end{pmatrix} = 2\times8 + 10(-2) + 2\times2 = 16 - 20 + 4 = 0$$

INDEED PERPENDICULAR

b) USING THE LENGTHS FOUND

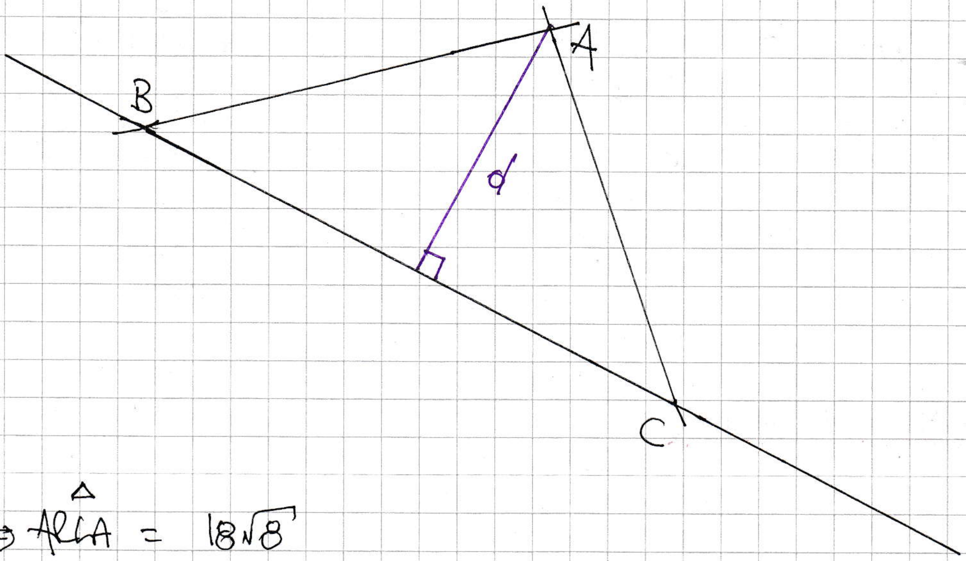


$$\text{AREA} = \frac{1}{2}\sqrt{72} \times \sqrt{108}$$
$$= \frac{1}{2}(6\sqrt{2})(6\sqrt{3})$$
$$= 18\sqrt{6}$$

AS REQUIRED

IYGB - SYNOPTIC PAPER M - QUESTION 9

c) LOOKING AT A DIAGRAM AGAIN



$$\Rightarrow \overset{\triangle}{\text{AREA}} = 18\sqrt{8}$$

$$\Rightarrow \frac{1}{2}|BC| \times d = 18\sqrt{6}$$

$$\Rightarrow \frac{1}{2} \times \sqrt{180} \times d = 18\sqrt{6}$$

$$\Rightarrow \sqrt{180}\,d = 36\sqrt{6}$$

$$\Rightarrow \cancel{\sqrt{6}}\sqrt{30}\,d = 36\cancel{\sqrt{6}}$$

$$\Rightarrow d = \frac{36}{\sqrt{30}} \quad //$$

$$\left[\text{OR RATIONALIZE TO } \frac{36\sqrt{30}}{30} = \frac{6}{5}\sqrt{30}\right]$$

IYGB - SYNOPTIC PAPER M - QUESTION 10

BY SUBSTITUTION

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$ & $x = e^5 \longmapsto u = \ln e^5 = 5$

$dx = x\,du$ $x = e^3 \longmapsto u = \ln e^3 = 3$

TRANSFORMING THE INTEGRAL

$$\int_{e^3}^{e^5} \frac{5}{2x\left[(\ln x)^2 + \ln x - 6\right]}\,dx = \int_3^5 \frac{5}{2\cancel{x}(u^2+u-6)}(\cancel{x}\,du)$$

$$= \int_3^5 \frac{5}{2(u^2+u-6)}\,du = \frac{5}{2}\int_3^5 \frac{1}{(u+3)(u-2)}\,du$$

BY PARTIAL FRACTIONS

$$\frac{1}{(u+3)(u-2)} \equiv \frac{A}{u+3} + \frac{B}{u-2}$$

$$1 \equiv A(u-2) + B(u+3)$$

IF $u = 2$ $1 = 5B$

IF $u = -3$ $1 = -5A$

$\therefore A = -\frac{1}{5}$ & $B = \frac{1}{5}$

FINALLY WE HAVE

$$\therefore \frac{5}{2}\int_3^5 \frac{-\frac{1}{5}}{u+3} + \frac{\frac{1}{5}}{u-2}\,du = \frac{1}{2}\int_3^5 \frac{-1}{u+3} + \frac{1}{u-2}\,du$$

$$= \frac{1}{2}\Big[\ln|u-2| - \ln|u+3|\Big]_3^5 = \frac{1}{2}\Big[(\ln 3 - \ln 8) - (\cancel{\ln 1} - \ln 6)\Big]$$

$$= \frac{1}{2}\Big[\ln 3 - \ln 8 + \ln 6\Big] = \frac{1}{2}\ln\frac{18}{8} = \frac{1}{2}\ln\frac{9}{4} = \frac{1}{2}\ln\left(\frac{3}{2}\right)^2$$

$$= \ln\frac{3}{2}$$

IYGB - SYNOPTIC PAPER M - QUESTION 11

$$\boxed{f(x) \equiv \ln(1+\sin x)}$$

MANIPULATE AS FOLLOWS

$$f(x) - f(-x) = \ln(1+\sin x) - \ln(1+\sin(-x))$$

$\sin(-x) \equiv -\sin x$

$$= \ln(1+\sin x) - \ln(1-\sin x)$$

$$= \ln\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$= \ln\left[\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}\right]$$

$$= \ln\left[\frac{(1+\sin x)^2}{1-\sin^2 x}\right]$$

$$= \ln\left[\frac{(1+\sin x)^2}{\cos^2 x}\right]$$

$$= \ln\left[\frac{1+\sin x}{\cos x}\right]^2$$

$$= 2\ln\left(\frac{1+\sin x}{\cos x}\right)$$

$$= 2\ln\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$$

$$= 2\ln(\sec x + \tan x)$$

//

IYGB - SYNOPTIC PAPER M - QUESTION 12

TIDY THE EQUATION

$$\Longrightarrow \sqrt{3}\left(x+\frac{6}{x}\right)=9$$

$$\Longrightarrow x+\frac{6}{x}=\frac{9}{\sqrt{3}}$$

$$\frac{9}{\sqrt{3}}=\frac{9\sqrt{3}}{\sqrt{3}\sqrt{3}}=\frac{9\sqrt{3}}{3}=3\sqrt{3}$$

$$\Longrightarrow x+\frac{6}{x}=3\sqrt{3}$$

$$\Longrightarrow x^2+6=3\sqrt{3}x$$

$$\Longrightarrow x^2-3\sqrt{3}x+6=0$$

BY THE QUADRATIC FORMULA

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Longrightarrow x=\frac{3\sqrt{3}\pm\sqrt{(-3\sqrt{3})^2-4\times1\times6}}{2\times1}$$

$$\Longrightarrow x=\frac{3\sqrt{3}\pm\sqrt{9\times3-24}}{2}$$

$$\Longrightarrow x=\frac{3\sqrt{3}\pm\sqrt{3}}{2}$$

$$\Longrightarrow x=\begin{cases}\frac{4\sqrt{3}}{2}\\ \frac{2\sqrt{3}}{2}\end{cases}$$

$$\Longrightarrow x=\begin{cases}2\sqrt{3}\\ \sqrt{3}\end{cases}$$

OR BY COMPLETING THE SQUARE

$$\Longrightarrow \left(x-\frac{3\sqrt{3}}{2}\right)^2-\left(\frac{3\sqrt{3}}{2}\right)^2+6=0$$

$$\Longrightarrow \left(x-\frac{3\sqrt{3}}{2}\right)^2-\frac{9\times3}{4}+6=0$$

$$\Longrightarrow \left(x-\frac{3\sqrt{3}}{2}\right)^2-\frac{27}{4}+6=0$$

$$\Longrightarrow \left(x-\frac{3\sqrt{3}}{2}\right)^2=\frac{27}{4}-\frac{24}{4}$$

$$\Longrightarrow \left(x-\frac{3\sqrt{3}}{2}\right)^2=\frac{3}{4}$$

$$\Longrightarrow x-\frac{3}{2}\sqrt{3}=\begin{cases}\sqrt{\frac{3}{4}}\\ -\sqrt{\frac{3}{4}}\end{cases}$$

$$\Longrightarrow x-\frac{3}{2}\sqrt{3}=\begin{cases}\frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2}\end{cases}$$

$$\Longrightarrow x=\begin{cases}2\sqrt{3}\\ \sqrt{3}\end{cases}$$

IYGB - SYNOPTIC PAPER M - QUESTION 13

a) OBTAIN THE GRADIENT AT R(2,0)

$$y = 2x^3 + 3x^2 - 11x - 6$$

$$\frac{dy}{dx} = 6x^2 + 6x - 11$$

$$\left.\frac{dy}{dx}\right|_{x=2} = 6\times2^2 + 6\times2 - 11 = 24 + 12 - 11 = 25$$

EQUATION OF TANGENT AT R

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 0 = 25(x-2)$$

$$\Rightarrow y = 25(x-2)$$

b) START BY OBTAINING THE CO-ORDINATES OF P

$$\begin{array}{r|l} & 2x^2 + 7x + 3 \\ \hline x-2 & 2x^3 + 3x^2 - 11x - 6 \\ & -2x^3 + 4x^2 \\ \hline & \quad 7x^2 - 11x - 6 \\ & \quad -7x^2 + 14x \\ \hline & \qquad 3x - 6 \\ & \qquad -3x + 6 \\ \hline & \qquad\quad 0 \end{array}$$

$$\therefore y = (x-2)(2x^2+7x+3)$$

$$y = (x-2)(2x+1)(x+3)$$

$$\therefore P(-3,0)$$

IYGB - SYNOPTIC PAPER M - QUESTION 13

FIND THE GRADIENT AT P(-3,0)

$$\frac{dy}{dx} = 6x^2 + 6x - 11$$

$$\left.\frac{dy}{dx}\right|_{x=-3} = 6(-3)^2 + 6(-3) - 11 = 54 - 18 - 11 = 25$$

NORMAL AT P HAS GRADIENT $-\frac{1}{25}$

$\therefore$ TANGENT AT R HAS GRADIENT 25
NORMAL AT P HAS GRADIENT $-\frac{1}{25}$) NEGATIVE RECIPROCALS

$\therefore$ THEY MEET AT RIGHT ANGLES $\Longrightarrow$ $P\hat{S}R = 90°$ //

IYGB - SYNOPTIC PAPER M - QUESTION 14

a) USING THE INFO GIVEN, TO SET AN EQUATION

$$S_4 = \frac{a(r^4-1)}{r-1}$$

$$\Longrightarrow \quad S_4 = 5S_2$$

$$\Longrightarrow \quad \frac{a(r^4-1)}{r-1} = 5 \times \frac{a(r^2-1)}{r-1}$$

$$\Longrightarrow \quad \cancel{\frac{a}{r-1}}(r^4-1) = \cancel{\frac{a}{r-1}} \times 5(r^2-1)$$

AS $a \neq 0$, $r > 1$ WE MAY DIVIDE WITHOUT LOSING SOLUTIONS

$$\Longrightarrow \quad r^4 - 1 = 5r^2 - 5$$

$$\Longrightarrow \quad r^4 - 5r^2 + 4 = 0$$

$$\Longrightarrow \quad (r^2-4)(r^2-1) = 0$$

$$\Longrightarrow \quad r^2 = \begin{cases} 4 \\ 1 \end{cases} \quad \Longrightarrow \quad r = \begin{cases} 2 \\ \cancel{-2} \\ \cancel{1} \\ \cancel{-1} \end{cases} \qquad r > 1$$

b) START BY FINDING THE VALUE OF a

$$\Longrightarrow \quad S_3 = 21$$

$$\Longrightarrow \quad \frac{a(2^3-1)}{2-1} = 21$$

$$\Longrightarrow \quad 7a = 21$$

$$\Longrightarrow \quad a = 3$$

$$\therefore \quad S_{10} = \frac{3(2^{10}-1)}{2-1}$$

$$S_{10} = 3 \times 1023$$

$$S_{10} = 3069$$

IYGB - SYNOPTIC PAPER M - QUESTION 15

a) BY THE STANDARD METHODOLOGY

$$y = 2 + \frac{1}{x+1}$$

$$y(x+1) = 2(x+1) + 1$$

$$yx + y = 2x + 3$$

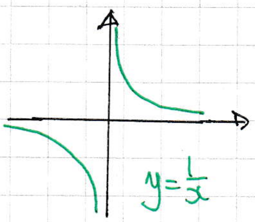
$$yx - 2x = 3 - y$$

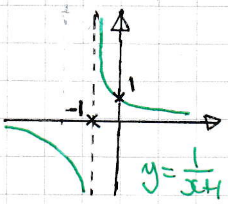
$$x(y-2) = 3 - y$$

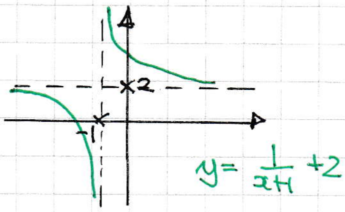
$$x = \frac{3-y}{y-2}$$

$$\therefore f^{-1}(x) = \frac{3-x}{x-2}$$

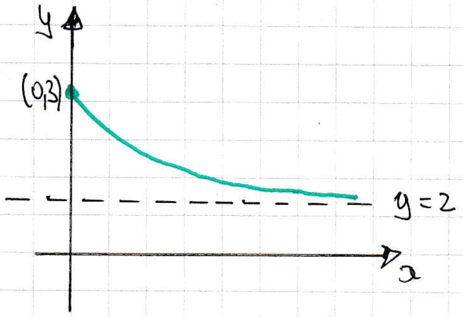
b) START BY SKETCHING THE GRAPH OF $f(x)$ VIA TRANSFORMATIONS







HENCE THE GRAPH OF $f(x)$ CAN BE SKETCHED



	$f(x)$	$f^{-1}(x)$
DOMAIN	$x \geq 0$	$2 < x \leq 3$
RANGE	$2 < f(x) \leq 3$	$f^{-1}(x) \geq 0$

$\therefore$ DOMAIN FOR $f^{-1}(x)$ IS $2 < x \leq 3$

RANGE OF $f^{-1}(x)$ IS $f^{-1}(x) \geq 0$

IYGB - SYNOPTIC PAPER M - QUESTION 16

BY ANY SENSIBLE SUBSTITUTION

$e^{2y} = x-4$ & $2y = 1 + \ln(x+1)$

$2y = \ln(x-4)$

$$\Longrightarrow \ln(x-4) = 1 + \ln(x+1)$$
$$\Longrightarrow \ln(x-4) - \ln(x+1) = 1$$
$$\Longrightarrow \ln\left(\frac{x-4}{x+1}\right) = 1$$
$$\Longrightarrow \frac{x-4}{x+1} = e$$
$$\Longrightarrow x+4 = ex+e$$
$$\Longrightarrow x - ex = e+4$$
$$\Longrightarrow x(1-e) = e+4$$
$$\Longrightarrow x = \frac{e+4}{1-e}$$

or $x = \dfrac{4+e}{1-e}$

SUBSTITUTING INTO $2y = \ln(x-4)$

$$2y = \ln\left(\frac{4+e}{1-e} - 4\right) = \ln\left(\frac{4+e-4(1-e)}{1-e}\right) = \ln\left(\frac{\cancel{4}+e-\cancel{4}+4e}{1-e}\right)$$

$$2y = \ln\left(\frac{5}{1-e}\right)$$

$$y = \frac{1}{2}\ln\left(\frac{5}{1-e}\right)$$

NO REAL SOLUTION AS EVIDENTLY THE ARGUMENT OF THE LOGARITHM IN $y = \frac{1}{2}\ln\left(\frac{5}{1-e}\right)$ IS NEGATIVE

IYGB - SYNOPTIC PAPER M - QUESTION 16

ALTERNATIVE

$x = e^{2y} + 4$ $\qquad\qquad$ $\ln(x+1) = 2y - 1$

$$\Longrightarrow \ln\left(e^{2y} + 4 + 1\right) = 2y - 1$$

$$\Longrightarrow \ln\left(e^{2y} + 5\right) = 2y - 1$$

$$\Longrightarrow e^{2y} + 5 = e^{2y-1}$$

$$\Longrightarrow 5 = e^{2y-1} - e^{2y}$$

$$\Longrightarrow 5 = e^{2y}\left(e^{-1} - 1\right)$$

$$\Longrightarrow e^{2y} = \frac{5}{e^{-1} - 1}$$

$$\Longrightarrow e^{2y} = \frac{5e^{1}}{e^{-1}e^{1} - 1e^{1}}$$

$$\Longrightarrow e^{2y} = \frac{5e}{1 - e}$$

$$\Longrightarrow 2y = \ln\left(\frac{5e}{1-e}\right)$$

$$\Longrightarrow y = \frac{1}{2}\ln\left(\frac{5e}{1-e}\right)$$

AND SINCE $e^{2y} = \frac{5e}{1-e}$

$$x = \frac{5e}{1-e} + 4 = \frac{5e + 4(1-e)}{1-e} = \frac{5e + 4 - 4e}{1-e} = \frac{e+4}{1-e}$$

IYGB - SYNOPTIC PAPER M - QUESTION 17

a) BY LONG DIVISION OR MANIPULATION

$$x-2\,\overset{2}{\overline{\big)\,2x+3}}$$
$$\underline{-\cancel{2x}+4}$$
$$7$$

OR

$$\frac{2x+3}{x-2}=\frac{2(x-2)+7}{(x-2)}$$
$$=\frac{2(x-2)}{x-2}+\frac{7}{x-2}$$
$$=2+\frac{7}{x-2}$$

AS REQUIRED

$$\therefore\ \frac{2x+3}{x-2}=2+\frac{7}{x-2}$$

AS REQUIRED

b) SOLVING THE EQUATION FOR $x=0$ & $y=0$

- $x=0$

$$y=\frac{0+3}{0-2}=-\frac{3}{2}$$

$\therefore\ \left(0,-\frac{3}{2}\right)$

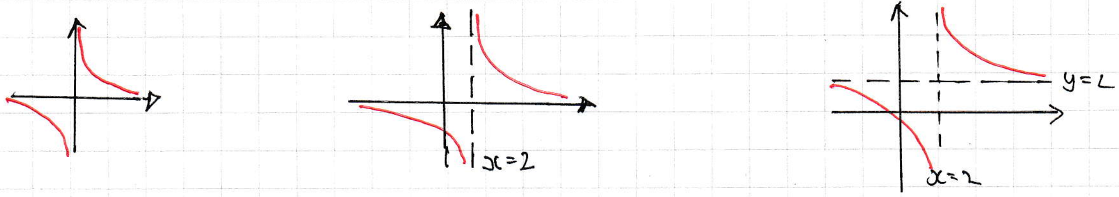
- $y=0$

$$0=\frac{2x+3}{x-2}$$
$$2x+3=0$$
$$x=-\frac{3}{2}$$

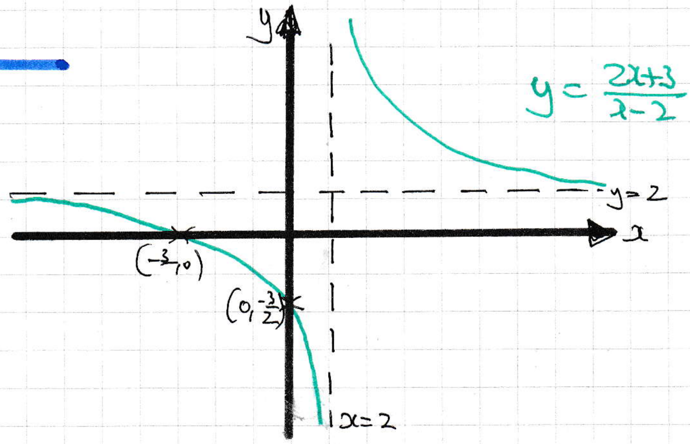
$\therefore\ \left(-\frac{3}{2},0\right)$

c) WORKING WITH TRANSFORMATIONS

$$\frac{7}{x}\longmapsto\frac{7}{(x-2)}\longmapsto\left(\frac{7}{x-2}\right)+2$$



HENCE WE OBTAIN



$$y=\frac{2x+3}{x-2}=2+\frac{7}{x-2}$$

IYGB – SYNOPTIC PAPER M – QUESTION 17

d) SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\left.\begin{array}{l} y = 7x - 12 \\ y = \dfrac{2x+3}{x-2} \end{array}\right\} \implies \frac{2x+3}{x-2} = 7x - 12$$

$$\implies 2x + 3 = (7x-12)(x-2)$$

$$\implies 2x + 3 = 7x^2 - 14x - 12x + 24$$

$$\implies 0 = 7x^2 - 28x + 21$$

$$\implies 0 = x^2 - 4x + 3$$

$$\implies (x-3)(x-1) = 0$$

$$\implies x = \begin{cases} 1 \\ 3 \end{cases} \qquad y = \begin{cases} 1 \times 7 - 12 = -5 \\ 7 \times 3 - 12 = 9 \end{cases}$$

$$\therefore \ (1,-5) \ \& \ (3,9)$$

IYGB - SYNOPTIC PAPER M - QUESTION 18

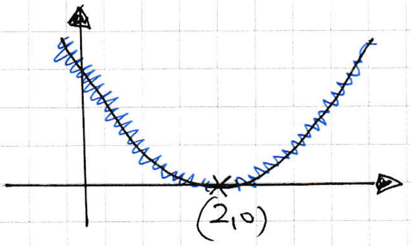
TWO DISTINCT REAL ROOTS $\Rightarrow b^2-4ac>0$

$\Rightarrow (k+2)^2-4\times2\times k>0$

$\Rightarrow k^2+4k+4-8k>0$

$\Rightarrow k^2-4k+4>0$

$\Rightarrow (k-2)^2>0$

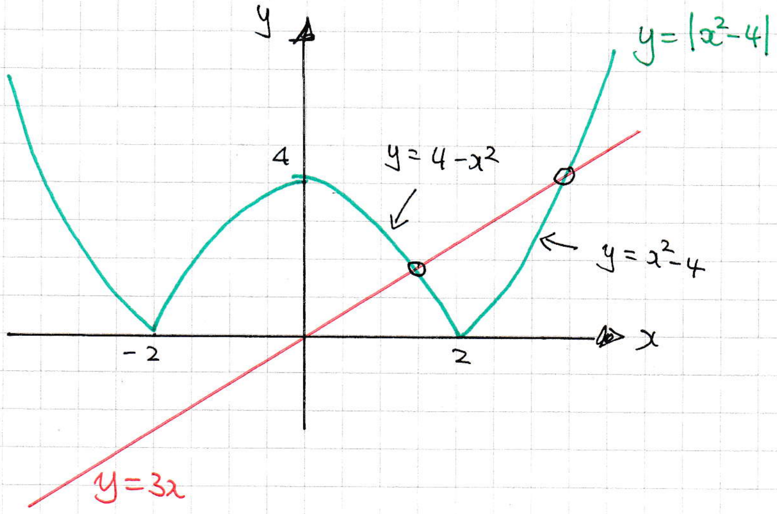


$\therefore$ ALL REAL NUMBER EXCEPT 2

$\therefore \quad k\in\mathbb{R}, k\neq2$

IYGB - SYNOPTIC PAPER N - QUESTION 19

START WITH A SKETCH



SOLVING EQUATIONS NOTING BOTH SOLUTIONS ARE POSITIVE

$x^2 - 4 = 3x$

$x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$x = \cancel{-1}$ or 4

- $x = 4$

$4 - x^2 = 3x$

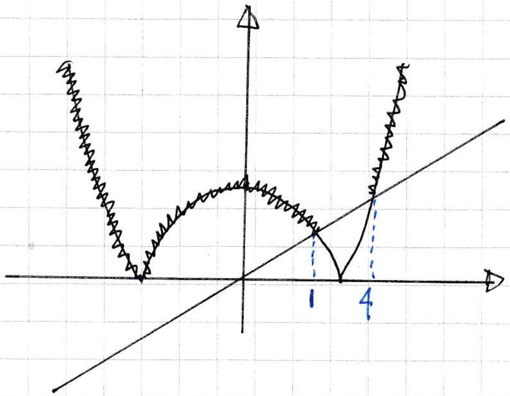
$0 = x^2 + 3x - 4$

$0 = (x-1)(x+4)$

$x = 1$ or $\cancel{-4}$

- $x = 1$

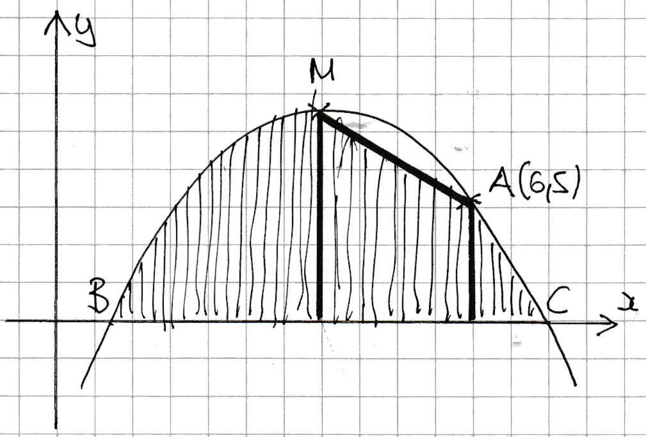
LOOKING AT THE DIAGRAM AGAIN



$x < 1$ or $x > 4$

IYGB - SYNOPTIC PAPER M - QUESTION 20

FIND THE CO.ORDS OF M, BY COMPLETING THE SQUARE (OR CALCULUS)



$\Rightarrow \quad y = -x^2+8x-7$

$\Rightarrow \quad -y = x^2-8x+7$

$\Rightarrow \quad -y = (x-4)^2-16+7$

$\Rightarrow \quad -y = (x-4)^2-9$

$\Rightarrow \quad y = 9-(x-4)^2$

$\therefore$ M(4,9)

ALSO THE CO.ORDINATES OF B & C ARE NEEDED

$\Rightarrow \quad y=0$

$\Rightarrow \quad -x^2+8x-7=0$

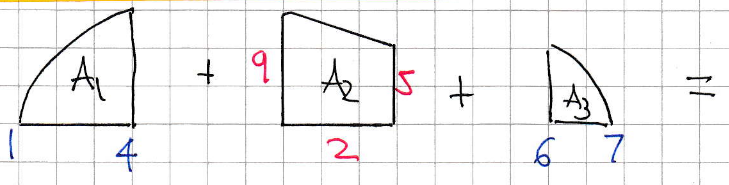
$\Rightarrow \quad x^2-8x+7=0$

$\Rightarrow \quad (x-1)(x-7)=0$

$\Rightarrow \quad x = \left\langle \begin{matrix} 1 \\ 7 \end{matrix} \right.$

$\therefore$ B(1,0) C(7,0)

HENCE THE REQUIRED AREA CAN BE FOUND



$$A_1 = \int_1^4 -x^2+8x-7 \, dx \qquad A_2 = \frac{9+5}{2} \times 2 \qquad A_3 = \int_6^7 -x^2+8x-7 \, dx$$

$$A_2 = 14$$

IYGB - SYNOPTIC PAPER M - QUESTION 20

$$A_1 = \int_1^4 -x^2+8x-7\,dx = \left[-\tfrac{1}{3}x^3+4x^2-7x\right]_1^4$$

$$= \left(-\tfrac{64}{3}+64-28\right)-\left(-\tfrac{1}{3}+4-7\right)$$

$$= \tfrac{44}{3}-\left(-\tfrac{10}{3}\right)$$

$$= 18$$

$$A_3 = \int_6^7 -x^2+8x-7\,dx = \left[-\tfrac{1}{3}x^3+4x^2-7x\right]_6^7$$

$$= \left(-\tfrac{343}{3}+196-49\right)-\left(-72+144-42\right)$$

$$= \tfrac{98}{3}-30$$

$$= \tfrac{8}{3}$$

THE AREA OF THE SHADED REGION IS $A_1+A_2+A_3$

$$18+14+\tfrac{8}{3} = \tfrac{104}{3}$$

IYGB – SYNOPTIC PAPER M – QUESTION 21

FORMING A DIFFERENTIAL EQUATION

$$\frac{dA}{dt} = +k\sqrt{A}$$

A = AREA COVERED (m^2)
t = TIME (days)

SQUARE ROOT OF THE AREA ALREADY COVERED

PROPORTIONAL

RATE

GROWING

APPLY STRAIGHT AWAY THE CONDITION $\left.\frac{dA}{dt}\right|_{\substack{t=0\\A=1}} = 0.25$

$$\Longrightarrow 0.25 = k \times \sqrt{1}$$

$$\Longrightarrow k = \frac{1}{4}$$

$$\Longrightarrow \frac{dA}{dt} = \frac{1}{4}A^{\frac{1}{2}}$$

SOLVE BY SEPARATION OF VARIABLES

$$\Longrightarrow dA = \frac{1}{4}A^{\frac{1}{2}}\,dt$$

$$\Longrightarrow \frac{1}{A^{\frac{1}{2}}}\,dA = \frac{1}{4}\,dt$$

$$\Longrightarrow \int A^{-\frac{1}{2}}\,dA = \int \frac{1}{4}\,dt$$

$$\Longrightarrow 2A^{\frac{1}{2}} = \frac{1}{4}t + C$$

APPLY THE CONDITION $t=0$, $A=1$

$$\Longrightarrow 2 \times 1^{\frac{1}{2}} = 0 + C$$

$$\Longrightarrow C = 2$$

IYGB - SYNOPTIC PAPER M - QUESTION 21

$$\Longrightarrow \quad 2A^{\frac{1}{2}} = \frac{1}{4}t + 2$$

WHEN THE WHOLE LAWN IS INFECTED WHEN $A = 225$

$$\Longrightarrow \quad 2 \times 225^{\frac{1}{2}} = \frac{1}{4}t + 2$$

$$\Longrightarrow \quad 2 \times 15 = \frac{1}{4}t + 2$$

$$\Longrightarrow \quad 28 = \frac{1}{4}t$$

$$\Longrightarrow \quad t = 112 \text{ days}$$

IYGB - SYNOPTIC PAPER M - QUESTION 22

START BY RELATING DERIVATIVES

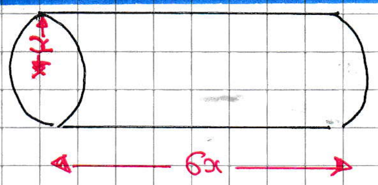
$$\frac{dA}{dt} = \frac{dA}{dV} \times \frac{dV}{dt}$$

$$\frac{dA}{dt} = \frac{dA}{dV} \times \frac{6}{7}\pi$$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dV} \times \frac{6}{7}\pi$$

$$\frac{dA}{dt} = (28\cancel{\pi x})\left(\frac{1}{18\cancel{\pi x}^2}\right)\left(\frac{6}{7}\pi\right)$$

$$\frac{dA}{dt} = \frac{4\pi}{3x}$$



- VOLUME $= (\pi x^2)(6x)$

$V = 6\pi x^3$

$$\boxed{\frac{dV}{dx} = 18\pi x^2}$$

- SURFACE AREA $= A$

$A = 2\pi x^2 + (2\pi x)(6x)$

$A = 2\pi x^2 + 12\pi x^2$

$A = 14\pi x^2$

$$\boxed{\frac{dA}{dx} = 28\pi x}$$

NOW WE REQUIRE THE VALUE OF x WHEN t=14

- AS THE $\frac{dV}{dt} = \frac{6}{7}\pi$ ← CONSTANT
- AFTER 14 SECONDS $\frac{6}{7}\pi \times 14 = 12\pi$
- VOLUME AFTER 14 s $= 36\pi + 12\pi = 48\pi$
- USING $V = 6\pi x^3 \Longrightarrow 48\pi = 6\pi x^3$

$$\Longrightarrow 8 = x^3$$

$$\Longrightarrow x = 2$$

FINALLY WE HAVE

$$\left.\frac{dA}{dt}\right|_{t=14} = \left.\frac{dA}{dt}\right|_{x=2} = \frac{4\pi}{3\times 2} = \frac{2\pi}{3} \approx 2.09 \text{ cm}^2\text{s}^{-1}$$

IYGB - SYNOPTIC PAPER M - QUESTION 23

BY IMPLICIT DIFFERENTIATION NOTING THAT $y=y(x)$ & $x=x(t)$

$$\Longrightarrow x = t^{\frac{1}{2}}$$

$$\frac{dx}{dy} = \frac{d}{dy}\left(t^{\frac{1}{2}}\right) = \frac{1}{2}t^{-\frac{1}{2}} \times \frac{dt}{dy} = \frac{1}{2t^{\frac{1}{2}}}\frac{dt}{dy}$$

$$\therefore \boxed{\frac{dy}{dx} = 2t^{\frac{1}{2}}\frac{dy}{dt}}$$

DIFFERENTIATE THE ABOVE RESULT WITH RESPECT TO x NOW

$$\Longrightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(2t^{\frac{1}{2}}\frac{dy}{dt}\right)$$

$$\Longrightarrow \frac{d^2y}{dx^2} = \underbrace{t^{-\frac{1}{2}}\frac{dt}{dx} \times \frac{dy}{dt} + 2t^{\frac{1}{2}}\frac{d}{dx}\left(\frac{dy}{dt}\right)}_{\text{PRODUCT RULE}}$$

$$\Longrightarrow \frac{d^2y}{dx^2} = \frac{1}{t^{\frac{1}{2}}}\frac{dy}{dx} + 2t^{\frac{1}{2}}\frac{d^2y}{dt^2} \times \frac{dt}{dx}$$

NOW IF $x = t^{\frac{1}{2}}$

$$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$$

$$\frac{dt}{dx} = 2t^{\frac{1}{2}}$$

$$\Longrightarrow \frac{d^2y}{dx^2} = \frac{1}{t^{1/2}}\overset{\frac{dy}{dx}}{\left(2t^{\frac{1}{2}}\frac{dy}{dt}\right)} + 2t^{\frac{1}{2}}\frac{d^2y}{dt^2}\overset{\frac{dt}{dx}}{\left(2t^{\frac{1}{2}}\right)}$$

$$\Longrightarrow \frac{d^2y}{dx^2} = 2\frac{dy}{dt} + 4t\frac{d^2y}{dt^2}$$

AS REQUIRED

IYGB - SYNOPTIC PAPER M - QUESTION 24

$$\boxed{f(x) = 3\sin x - \cos x + 3 \qquad g(x) = \sin x + \cos x}$$

a) DIFFERENTIATE $g(x)$

$$g'(x) = \cos x - \sin x$$

EQUATE & COMPARE COEFFICIENTS

$$\Rightarrow f(x) \equiv A \times g(x) + B \times g'(x) + 3$$

$$\Rightarrow 3\sin x - \cos x + \cancel{3} \equiv A(\sin x + \cos x) + B(\cos x - \sin x) \cancel{+3}$$

$$\Rightarrow 3\sin x - \cos x \equiv (A - B)\sin x + (A + B)\cos x$$

$$\left\{\begin{array}{l} A - B = 3 \\ A + B = -1 \end{array}\right\} \quad \therefore 2A = 2$$

$$\underline{A = 1} \quad \& \quad \underline{B = -2}$$

$$\Rightarrow f(x) = g(x) - 2g'(x) + 3$$

b) $g(x) = \sin x + \cos x$

$$\Rightarrow g(x) = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$$

$$\Rightarrow g(x) = \sqrt{2}\left(\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x\right)$$

$$\Rightarrow g(x) = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$$

I.E $R = \sqrt{2}$ $\phi = \frac{\pi}{4}$

CAN BE DONE ALSO BY COMPARING

$$\sin x + \cos x \equiv R\cos\left(x - \frac{\pi}{4}\right)$$

IYGB - SYNOPTIC PAPER M - QUESTION 24

c) $$\int \frac{f(x)}{g(x)}\,dx = \int \frac{g(x) - 2g'(x) + 3}{g(x)}\,dx$$

$$= \int 1 + \frac{2g'(x)}{g(x)} + \frac{3}{g(x)}\,dx$$

$$= \int 1\,dx + 2\int \frac{g'(x)}{g(x)}\,dx + \int \frac{3}{g(x)}\,dx$$

$$= x + 2\ln|g(x)| + \int \frac{3}{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}\,dx$$

$$= x + 2\ln|g(x)| + \frac{3}{\sqrt{2}}\int \sec\left(x-\frac{\pi}{4}\right)\,dx$$

$$\boxed{\text{NOTING THAT } \int \sec x\,dx = \ln|\sec x + \tan x| + C}$$

$$\Longrightarrow \int \frac{f(x)}{g(x)}\,dx = x + 2\ln|\sin x + \cos x| + \frac{3}{2}\sqrt{2}\ln\left|\sec\left(x-\frac{\pi}{4}\right) + \tan\left(x-\frac{\pi}{4}\right)\right| + C$$