IYGB

Special Extension Paper G

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core and the Advanced Level Further Pure Mathematics Syllabi of recent years.

Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

Question 1

Use algebra to solve the following simultaneous equations

$$\frac{1}{x}+\frac{1}{y}=5 \qquad \text{and} \qquad \frac{1}{x^3}+\frac{1}{y^3}=35,$$

given further that $x \in \mathbb{R}$ and $y \in \mathbb{R}$. **(10)**

Question 2

Find the Maclaurin expansion, up and including the term in x^4, for $y=\sin(\cos x)$.

(7)

Question 3

Find the value of the following limit

$$\lim_{x \to 0}\left[\frac{1-\cos\left(x^2\right)}{x^2 \tan^2 x}\right].$$

(10)

Question 4

$$I=\int_1^a \frac{1}{\left(x^{\frac{4}{3}}+7x\right)^{\frac{2}{3}}}\,dx.$$

Given that $I=9$, determine the value a. **(8)**

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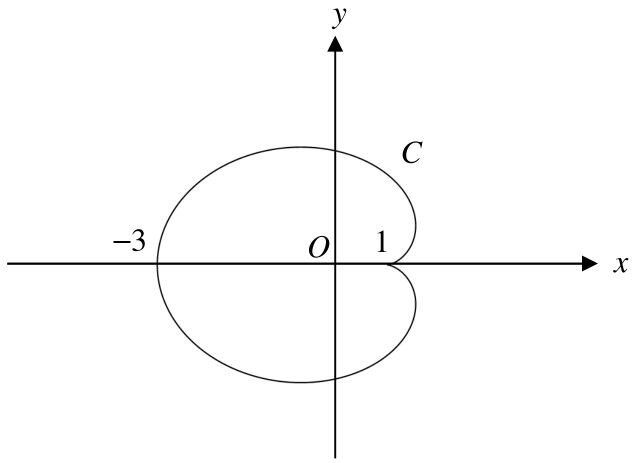
Question 5

Use an algebraic method justifying each step, to find the greatest value of k, $k \in \mathbb{N}$, which satisfies the following inequality.

$$\sum_{r=k+1}^{80}\left[\frac{r-1}{\log_{8^r}(16)}\right] > 100\,000 .$$ **(11)**

Question 6



The figure above shows the cardioid C with parametric equations

$$x = 2\cos\theta - \cos 2\theta, \quad y = 2\sin\theta - \sin 2\theta, \quad 0 \le \theta < 2\pi .$$

The curve is revolved by a full turn in the x axis, forming a surface of revolution.

Find in exact simplified form the area of this surface. **(10)**

Question 7

$$5\tanh 2x - \frac{3\tan 2x}{\tanh x} = 5\tanh x - 3 .$$

Find, as an exact natural logarithm, the real solution of the above equation. **(9)**

Question 8

Two distinct complex numbers z_1 and z_2 are such so that $|z_1| = |z_2| = r \neq 0$.

Show clearly that $\dfrac{z_1 + z_2}{z_1 - z_2}$ is purely imaginary.

You may find the result $z\overline{z} = |z|^2 = r^2$ useful. **(8)**

Question 9

$$I_n \equiv \int e^{2x} \sin^n x \, dx \, , \ n \in \mathbb{N} \, , \ n \geq 2 \, .$$

Use integration by parts twice to show

$$\left(n^2 + 4\right) I_n = n(n-1) I_{n-2} + (2\sin x - n\cos x) e^{2x} \sin^{n-1} x \, .$$ **(10)**

Question 10

A curve C and a straight line L have respective equations

$$C: \ y = 4x\sqrt{x} - \frac{25x^2}{16} \quad \text{and} \quad L: \ x + 2y = 18 \, .$$

a) Show that L is a tangent to C, at some point to be found. **(7)**

b) Verify the answer of part **(a)** by an alternative method. **(7)**

c) Show further that L does not meet C again. **(7)**

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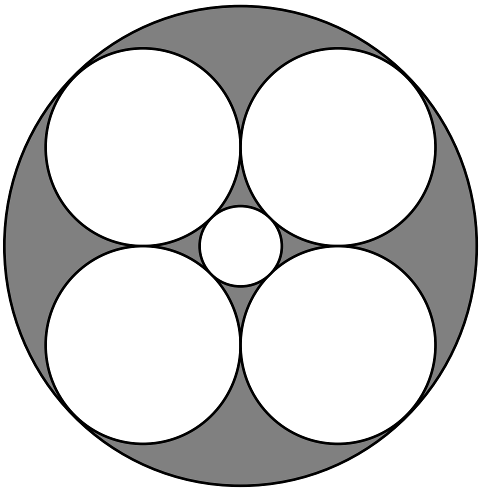
Question 11



The figure above shows 4 identical circles touching each other so that their centres form a square.

A smaller circle is touching all 4 of the identical circles externally, and all 4 of the identical circles are touching internally a larger circle.

Determine, correct to 1 decimal place, the fraction of the larger circle not occupied, by the other 5 circles, shown shaded in the figure.

You may assume that $\sqrt{2} \approx 1.4142$. **(10)**

Question 12

The function with equation $y = f(x)$ satisfies the differential equation

$$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 = 2y\ln 3, \quad y(0)=1, \quad \frac{dy}{dx}(0) = 2\ln 3.$$

Solve the above differential equation by using the substitution $p = \frac{dy}{dx}$, to show that

$$y = 3^{x^2+2x}.$$ **(12)**

Question 13

Sketch the graph of the curve with equation

$$x = y^2 \ln y .$$

The sketch must include ...

- ... the coordinates of any intersections with the axes.
- ... the coordinates of any points where the tangent to the curve is parallel to the coordinate axes.
- ... the coordinates of any points of inflexion. **(10)**

Question 14

$$\theta = \arctan\left[\frac{\cos 66° - \sin 48°}{\cos 48° - \sin 66°}\right].$$

Show by detailed working that $\theta = -12°$. **(8)**

Question 15

A straight line L, whose gradient is $-\frac{3}{11}$, is a tangent to the curve with polar equation

$$r = 25\cos 2\theta , \ 0 \le \theta \le \tfrac{1}{2}\pi$$

Show that the area of the finite region bounded by the curve, the straight line L and the initial line is

$$\frac{25}{12}\left[46 - 75\arctan\tfrac{1}{3}\right].$$ **(14)**

Question 16

Solve the following equation

$$3|z|z + 20zi = 125, \quad z \in \mathbb{C}.$$

Give the answer in the form $x + iy$, where x and y are real. **(8)**

Question 17

Show that the exact value of

$$\int_0^1 \frac{(1-x)e^x}{x^2 + e^{2x}} \, dx,$$

can be written as

$$\operatorname{arccot}(e). \quad \textbf{(9)}$$

Question 18

A sequence u_1, u_2, u_3, u_5, u_6, … is generated by the recurrence relation

$$n^2 u_{n+1} = (n+1)u_n, \quad n = 1, 2, 3, 4, \ldots$$

It is further given that

$$\sum_{n=1}^{\infty} u_n = 1.$$

Find in exact form the value of u_1. **(9)**

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Question 19

Solve the following differential equation

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} = 0, \quad y(0)=2, \quad \frac{dy}{dx}(0) = -\frac{1}{2}.$$

Give the answer in the form $y^2 = f(x)$. **(9)**

Question 20

By using the definition of e as an infinite convergent series, prove **by contradiction** that e is irrational.

(7)
