IYGB

Special Paper U

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

Question 1

Show that for all real numbers α, β and γ

$$\alpha^2 + \beta^2 + \gamma^2 \geq \alpha\beta + \beta\gamma + \gamma\alpha.$$ **(4)**

Question 2

The straight lines l_1 and l_2 have respective equations

$$y = x \quad \text{and} \quad x + y = 2.$$

The straight line l_3 is parallel to the x axis and passes through the point $P(h,k)$.

It is further given that the point B is the intersection of l_1 and l_2, the point A is the intersection of l_1 and l_3 and the point C is the intersection of l_2 and l_3,

Find the locus of P if the area of the triangle ABC is $9h^2$. **(10)**

Question 3

The points with coordinates $A(7,6,10)$, $B(6,5,6)$ and $C(1,0,4)$ are the vertices of the parallelogram $ABCD$.

a) Find …

i. … the coordinates of D. **(2)**

ii. … a vector equation of the straight line l which passes through the points A and C. **(1)**

iii. … the distance AC. **(2)**

b) Show that the shortest distance of l from B is $\sqrt{6}$ units. **(5)**

c) Hence find the exact area of the parallelogram $ABCD$. **(3)**

Question 4

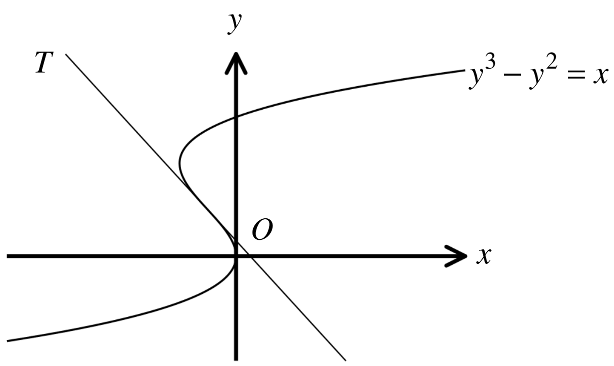
$$f(x) = x \ln\left[\left(x^2+1\right)^{\frac{1}{2}} + x\right] - \left(x^2+1\right)^{\frac{1}{2}}, \ x \in \mathbb{R}.$$

Show clearly that …

a) … $f'(x) = \ln\left[\left(x^2+1\right)^{\frac{1}{2}} + x\right]$. **(8)**

b) … $f(x)$ is an even function. **(4)**

Question 5



The figure above shows the graph of a curve with equation

$$y^3 - y^2 = x, \ x \in \mathbb{R}, \ y \in \mathbb{R}.$$

There exists a tangent to the curve T, so that this tangent **crosses** the curve at the point of tangency.

Show that an equation of T is

$$27x + 9y = 1.$$ **(7)**

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Question 6

Find the set of values of x that satisfy the inequality

$$\frac{x^2-4}{|x+5|} < 8-4x. \qquad \textbf{(10)}$$

Question 7

Use algebra to find, in terms of π, the solution of the trigonometric equation

$$x^2 - 8\pi x + 2 - 2\cos x + 16\pi^2 = 0, \ x \in \mathbb{R}. \qquad \textbf{(7)}$$

Question 8

$$y = \mathrm{e}^{kx}, \ k \neq 0.$$

Find a simplified expression for

$$\left[\frac{d^2y}{dx^2}\right]\left[\frac{d^2x}{dy^2}\right],$$

giving the answer in terms of k and e^{kx}. **(8)**

Question 9

Use trigonometric algebra to solve the equation

$$\arctan x + 2\operatorname{arccot} x = \frac{2\pi}{3}.$$

You may assume that $\operatorname{arccot} x$ is the inverse function for the part of $\cot x$ for which $0 \le x \le \pi$.

(12)

Question 10

A curve C has equation

$$y = \frac{\sqrt{18 + x^2 - x^3}}{6 - x}.$$

It is given that C has two stationary points whose x coordinates have opposite signs.

Sketch the graph of C, for the largest possible domain.

- The sketch must include, in exact form where appropriate the coordinates of any points where the graph of C meets the coordinate axes the equations of any asymptotes.
- You need not find the coordinates of the stationary points of C. **(8)**

Question 11

The curve C has parametric equations

$$x = \frac{(u+v)^2}{u^2 + v^2}, \quad y = \frac{u^2 - v^2}{u^2 + v^2},$$

where u and v are real parameters with $u^2 + v^2 \neq 0$.

By considering the tangent half angle trigonometric identities, or otherwise, show that C is a circle, stating the coordinates of its centre and the size of its radius. **(9)**

Question 12

Find as exact surds the solutions of the equation

$$x(x+1)(5x+1)(5x-4) = -4, \; x \in \mathbb{R}.$$ **(10)**

Question 13

A quadratic curve has equation

$$y = ax^2 + bx + c, \ a \neq 0,$$

where a, b and c are constants.

The curve meets the x axis at $A(-2,0)$ and has a maximum point at $B(0,1)$.

The point C lies on the curve so that AB is perpendicular to BC.

Determine the area of the finite region bounded by the curve and the straight line segment AC. **(15)**

Question 14

Find, in degrees, the solutions of the trigonometric equation

$$2\cos(x+10)° = \frac{\cos(x+22)°}{\sin(x+10)°}, \ 0° \leq x \leq 360°.$$ **(9)**

Question 15

By showing a detailed method, sum the following series.

$$\sum_{r=0}^{9}\left[(r+1)\times 11^r \times 10^{9-r}\right].$$

You may leave the answer in index form. **(12)**

Question 16

By using a reciprocal substitution, or otherwise, find the value of the following integral.

$$\int_{1}^{2} \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}\, dx \,. \qquad \textbf{(11)}$$

Question 17

At every point $P(x,y)$ which lie on the curve C, with equation $y=f(x)$, the y intercept of the tangent to C at P has coordinates $\left(0, xy^2\right)$.

Given further that the point $Q(1,1)$ also lies on C determine an equation for C, giving the answer in the form $y=f(x)$.

You might find the expression for $\frac{d}{dx}\left(\frac{x}{y}\right)$ useful in this question. **(9)**

Question 18

Find the term independent of x in the expansion of

$$\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10} . \qquad \textbf{(7)}$$

Question 19

By using symmetry arguments, find the exact value of the following integral

$$\int_{0}^{\pi} \mathrm{e}^{|\cos x|}\left[\sin(\cos x)+\cos(\cos x)\right]\sin x \;\, dx \,. \qquad \textbf{(15)}$$

Question 20

By considering the simplification of

$$\arctan(2n+1) - \arctan(2n-1),$$

determine the exact value of

$$\sum_{n=1}^{\infty}\left[\arctan\left(\frac{1}{2n^2}\right)\right].$$

(12)

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