IYGB

Special Paper S

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $=T$, Number of non attempted questions $=N$, Percentage score $=P$.

$P=\frac{1}{2}T+N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

Question 1

Given that

$$P = \frac{5600}{7 + 25\mathrm{e}^{-0.25t}},$$

show by a detailed method that

$$\frac{dP}{dt} = \frac{P(800 - P)}{k},$$

where k is an integer to be found. **(7)**

Question 2

a) Use the compound angle identity $\cos(A+B)$ to show that

$$\cos\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}.$$ **(1)**

b) Use a suitable trigonometric substitution to find the exact value of

$$\int_{\sqrt{2}}^{\sqrt{\sqrt{6}+\sqrt{2}}} \frac{2}{x\sqrt{x^4 - 1}} \, dx.$$ **(7)**

Question 3

Solve the trigonometric equation

$$\sec x + \operatorname{cosec} x = 2\sqrt{2}, \qquad 0 \le x < 2\pi,$$

giving the answers in terms of π. **(10)**

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Question 4

The finite region R is defined by the inequalities

$$y \le \arcsin x, \; x \le 1, \; y \ge 0.$$

The region R is rotated by 2π radians in the y axis forming a solid of revolution.

Determine the exact volume of this solid. **(8)**

Question 5

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + a\mathbf{k})$$

$$\mathbf{r}_2 = 3\mathbf{i} + b\mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} - \mathbf{k}),$$

where λ and μ are scalar parameters, and a and b are scalar constants.

It is further given that the point A is the intersection of l_1 and l_2, and the acute angle between l_1 and l_2 is $60°$.

Find in any order …

… the two possible pairings for the value of a and the value b.

… the possible coordinates of A for each possible pair of a and b. **(10)**

Question 6

Using a detailed method show that

$$\sqrt{\frac{2+\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt[4]{108}}{6} + \frac{\sqrt[4]{12}}{2}.$$ **(8)**

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Question 7

A curve is given parametrically by

$$x = t^2 + t + 3, \quad y = 2t^2 - 3t + 1, \quad t \in \mathbb{R}$$

Find a Cartesian equation for the curve in the form $f(x, y) = 0$. **(7)**

Question 8

Solve the equation

$$9x^4 - 24x^3 - 2x^2 - 24x + 9 = 0, \quad x \in \mathbb{R}.$$ **(9)**

Question 9

The coefficients of x^n, x^{n+1} and x^{n+2} in the binomial expansion of $(1+x)^{23}$ are in arithmetic progression.

Determine the possible values of n. **(10)**

Question 10

Solve the following equation

$$\frac{e^{2x} + 16^x}{(4e)^x} = \frac{4+e}{2\sqrt{e}}, \quad x \in \mathbb{R}.$$ **(12)**

Question 11

$$f(x) \equiv x^5 + 3x^4 - 40x^3 - 47x^2 .$$

The polynomial $f(x)$ satisfies the relationship

$$f(x) \equiv (x-2)(x+A)(x+B)\left(x^2+3x-1\right) - 249x + 70 ,$$

where A and B are integer constants.

a) Find the value of A and the value of B. **(5)**

The polynomial $f(x)$ also satisfies the relationship

$$f(x) \equiv (x-2)^2 h(x) + Px + Q ,$$

where P and Q are constants.

b) Find the value of each of the constants P and Q. **(5)**

Question 12

A family of straight lines has equation

$$\left(a^2+a+3\right)x + \left(a^2-a-3\right)y = 7a^2 - 3a + 1 ,$$

where a is a parameter.

The point Q has coordinates $(20,7)$.

Show that this family of lines passes through a fixed point P for all values of a, and hence determine the equation of a straight line from this family of straight lines, which is furthest away from Q. **(12)**

Question 13

Solve the following quadratic in x, giving the answers in terms of k. **(9)**

$$(k+1)x^2 - (k^2 + k + 1)x + k = 0, \ k \neq 1.$$

Question 14

The three angles of a triangle are denoted by α, β and γ.

a) Show clearly that …

i. … $\sin(\alpha + \beta) = \sin\gamma$. **(2)**

ii. … $\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\gamma}{2}\right)$. **(2)**

iii. … $\sin\alpha + \sin\beta + \sin(\alpha+\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha+\beta}{2}\right) + \cos\left(\frac{\alpha-\beta}{2}\right)\right]$. **(3)**

iv. … $\sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$. **(3)**

b) By using (**iv**) with suitable values for α, β and γ, show that

$$\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}.$$ **(3)**

Question 15

A metallic component is in the shape of a right circular cone, with radius $4x$ cm and height $3x$ cm.

The metallic component is heated so that the area of its curved face is expanding at a rate inversely proportional to x.

a) Show that volume of the metallic component is increasing at a constant rate. **(7)**

b) Find the percentage rate of increase of the base area relative to the curved face area of the metallic component. **(2)**

(*You may assume that the metallic component is expanding uniformly when heated.*)

[surface area of the curved face of a cone of radius r and slant height l, is given by $\pi r l$]

[volume of a cone of radius r and height h, $\frac{1}{3}\pi r^2 h$]

Question 16

Use a suitable substitution to find the value of

$$\int_2^4 \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x)}+\sqrt{\ln(3+x)}}\, dx.$$

(8)

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Question 17

The point $A(a,0)$ lies on the circle with Cartesian equation

$$x^2 + y^2 = a^2.$$

The point P is also on the same circle, and the point Q lies on the tangent to the circle through P, so that AQP is a right angle.

Use a calculus method to show that for all possible positions of P, the largest area of the triangle AQP is

$$\frac{3\sqrt{3}}{8}a^2.$$

(10)

Question 18

Solve the trigonometric equation

$$2\arctan(x-2) + \arcsin\left(\frac{1-x}{1+x}\right) = \frac{\pi}{2}.$$

(10)

Question 19

A family of infinite geometric series S_k, has first term $\frac{k-1}{k!}$ and common ratio $\frac{1}{k}$, where $k = 3,4,5,6,...,99,100$.

Find the value of

$$\frac{10^4}{100!} + \sum_{k=3}^{100}\left[\left[(k-1)(k-2)-1\right]S_k\right].$$

(12)

Question 20

The point P and the point $R(0,1)$ lie on the curve with equation

$$f(y) = g(x), \ |y| \le 1.$$

The tangent to the curve at P meets the x axis at the point Q.

Given that $|PQ| = 1$ for all possible positions of P on this curve, determine the equation of this curve, in the form $f(y) = g(x)$.

The final answer may not contain natural logarithms. **(18)**

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