IYGB

Special Paper Q

Time: 3 hours 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

Question 1

The following information is known about 4 coplanar points.

1. B is north east of A.
2. C is on a bearing of $075°$ from A.
3. B is on a bearing of $285°$ from C.
4. D is south west of C.
5. $|AC| = 9$.
6. $|CD| = 36$.

Determine, correct to 2 decimal places, the bearing of B from D. **(7)**

Question 2

The quadratic curve C with equation

$$y = x^2 - 6x + c,$$

passes through the points with coordinates (a,b), (b,a) and $(-a,27)$, where a, b and c are constants.

Find an equation for C, given that …

i. … $a = b$.

ii. … $a \neq b$. **(10)**

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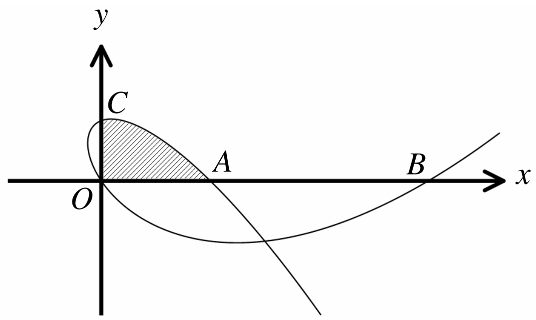
Question 3



The figure above shows a curve with parametric equations

$$x = t^2 + 2t, \quad y = t^3 - 9t, \quad t \in \mathbb{R}.$$

The curve meets the coordinate axes at the origin O and at the points A, B and C.

a) Determine the coordinates of A, B and C. **(3)**

The finite region R, shown shaded in the figure, is bounded by the curve and the coordinate axes.

b) Find the area of R. **(3)**

The finite region bounded by the curve and the y axis, for which $x < 0$, is revolved by 2π radians about the x axis, forming a solid S.

c) Calculate the volume of S. **(6)**

Question 4

A person standing at a fixed origin O observes an insect taking off from a point A on horizontal ground. The position vector of the insect $\mathbf{r}$ metres, t seconds after taking off, is given by

$$\mathbf{r} = (t+1)\mathbf{i} + \left(2t + \tfrac{1}{2}\right)\mathbf{j} + 2t\mathbf{k} .$$

All distances are in metres and the coordinates axes Ox, Oy, Oz are oriented due east, due north and vertically upwards, respectively.

a) Find …

i. … the bearing of the insect's flight path. **(2)**

ii. … the angle between the flight path and the horizontal ground. **(2)**

The roof top of a garden shed is located at $B\left(5, \frac{9}{2}, 3\right)$.

b) Calculate the shortest distance between the insect's path and the point B. **(5)**

When the insect reaches a height of 20 metres above the ground, at the point C, the insect gets eaten by a bird.

c) Determine the coordinates of C. **(1)**

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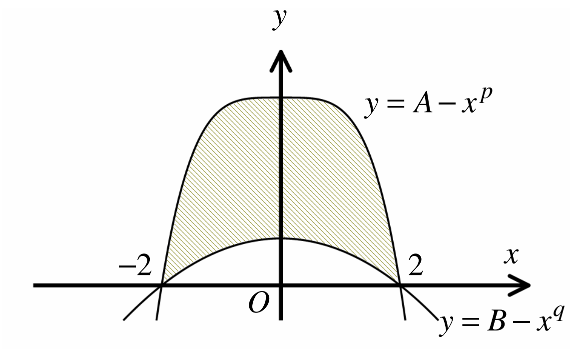
Question 5



The figure above shows the curves with equations

$$y = A - x^p \quad \text{and} \quad y = B - x^q ,$$

where A and B are positive constants with $A > B$, and p and q are positive integers with $p + q = 8$.

Both curves meet the x axis at the points $(-2,0)$ and $(2,0)$.

Find the exact value of the area bounded between the two curves. **(10)**

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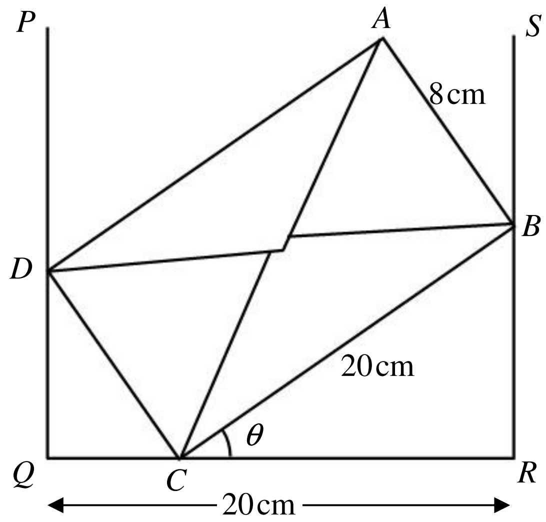
Question 6



The figure above shows the cross section of a letter inside a filling slot.

The letter $ABCD$ is modelled as a rectangle with $|AB| = 8\text{cm}$ and $|BC| = 20\text{cm}$.

The width of the filling slot QR is also 20 cm and the angle BCR is θ.

Determine the value of θ. **(8)**

Question 7

The straight line L with equation $y = kx$, where k is a positive constant, meets the curve C with equation $y = x\mathrm{e}^{-2x}$, at the point P.

The tangent to C at P meets the x axis at the point Q.

Given that $|OP| = |PQ|$, find in exact simplified form the area of the **finite** region bounded by C and L. **(10)**

Question 8

At time t seconds, a spherical balloon has radius r cm, surface area S cm^2 and volume V cm^3. The surface area of the balloon is increasing at a constant rate of $24\pi \ \text{cm}^2\text{s}^{-1}$.

Show that

$$\frac{dV}{dt} = \sqrt[3]{1296\pi^2 V} \ ,$$

and given further that the initial volume of the balloon was $64\pi \ \text{cm}^3$, find an exact simplified value for V when $t = \sqrt[3]{36}$.

$\left[\text{volume of a sphere of radius } r \text{ is given by } \frac{4}{3}\pi r^3\right]$

$\left[\text{surface area of a sphere of radius } r \text{ is given by } 4\pi r^2\right]$ **(10)**

Question 9

A cubic curve has equation

$$f(x) \equiv x^3 - x^2(1 + \sin t + 2\cos t) + x(2\sin t\cos t + 2\cos t + \sin t) - 2\sin t\cos t \ ,$$

where $x \in \mathbb{R}$ and t is a constant such that $0 \le t < 2\pi$.

Given that the equation $f(x) = 0$ has exactly 2 equal real roots, determine the possible values of t. **(8)**

Question 10

Use algebra to solve the following simultaneous equations

$$x^4 + y^4 = 97 \qquad \text{and} \qquad x + y = 5 \ ,$$

given further that $x \in \mathbb{R}$ and $y \in \mathbb{R}$. **(9)**

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Question 11

The point P lies on the curve C with equation

$$y = \frac{1}{1+x^2}, \quad x \in \mathbb{R}, \quad x \geq 0.$$

The straight line L is the tangent to the C at P.

Determine an equation for L, given further that L meets C at the point $(0,1)$. **(10)**

Question 12

Two parallel straight lines, L_1 and L_2, have respective equations

$$y = 2x+5 \quad \text{and} \quad y = 2x-1 \ .$$

L_1 and L_2, are tangents to a circle centred at the point C .

A third line L_3 is perpendicular to L_1 and L_2, and meets the circle in two distinct points, A and B.

Given that L_3 passes through the point $(9,0)$, find, in exact simplified surd form, the coordinates of C. **(14)**

Question 13

A curve has equation

$$y = \frac{8}{x^2-4x+8}, \quad x \in \mathbb{R}.$$

The finite region R is bounded by the curve, the y axis and the tangent to the curve at the stationary point of the curve.

Determine, in simplified exact form, the volume of the solid formed when R is fully revolved about the y axis. **(14)**

Question 14

Find an equation of a cubic function, with integer coefficients, whose graph crosses the x axis at the point $\left(3+2^{\frac{2}{3}}+2^{\frac{5}{3}}\ ,\ 0\right)$. **(8)**

Question 15

It is given that

$$\cot x - 2\cot 2x \equiv \tan x\ .$$

a) Prove the validity of the above trigonometric identity. **(2)**

b) Hence, or otherwise, show that

$$\sum_{r=1}^{10} \frac{1}{2^{r-1}}\tan\left(\frac{x}{2^r}\right) = \frac{1}{512}\cot\left(\frac{x}{1024}\right) - 2\cot x\ .$$ **(6)**

Question 16

The functions f and g are each defined in the largest possible real number domain and given by

$$f(x) = \sqrt{x - \sqrt{x^2 - x - 2}} \qquad \text{and} \qquad g(x) = \sqrt{x - \sqrt{x+6}}\ .$$

By considering the domains of f and g, show that $fg(x)$ cannot be formed. **(10)**

Question 17

Use trigonometric algebra to fully simplify

$$2\arctan\left(\frac{1}{5}\right) + \arccos\left(\frac{7}{5\sqrt{2}}\right) + \arctan\left(\frac{1}{8}\right),$$

giving the final answer in terms of π. **(10)**

Question 18

$$g(x) \equiv \sum_{r=0}^{\infty} f(x,r) \; - \; \frac{1-x}{\sqrt{1-x^2}\,\sqrt[3]{1-x^3}}, \quad -1 < x < 1.$$

Given that the first term of the series expansion of $g(x)$ is $\frac{1}{5}x^5$, determine in exact simplified form a simplified expression of $f(x,r)$. **(10)**

Question 19

The point P lies on the curve given parametrically as

$$x = t^2, \quad y = t^2 - t, \quad t \in \mathbb{R}.$$

The tangent to the curve at P meets the y axis at the point A and the straight line with equation $y = x$ at the point B.

P is moving along the curve so that its x coordinate is increasing at the constant rate of 15 units of distance per unit time.

Determine the rate at which the area of the triangle OAB is increasing at the instant when the coordinates of P are $(36,30)$. **(11)**

Question 20

$$f(x) \equiv \frac{1}{k}\left(x^2-1\right)\left(x^2-9\right), \quad x \in \mathbb{R}, \; k \in \mathbb{N}.$$

Determine the solution interval (n,k), $n \in \mathbb{N}$, so that the equation

$$|f(x)| = n,$$

has exactly n distinct real roots. **(11)**
