IYGB

Special Paper M

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

Question 1

The curve C_1, with equation $y = f(x)$, undergoes 3 transformations in the order given below.

1. A translation of 2 units, in the negative x direction.

2. An enlargement parallel to the x axis, with scale factor 2.

3. A translation of 1 unit in the positive y direction.

The resulting curve C_2 has equation

$$y = \frac{x^2 + 9x + 22}{x + 4}, \; x \in \mathbb{R}, \; x \neq -4 .$$

Determine in its simplest form an equation for C_1. **(5)**

Question 2

By using the substitution $\tan\theta = \sqrt{x^3 - 1}$, or otherwise, find an exact value for the following integral.

$$\int_1^{\sqrt[3]{2}} \frac{\sqrt{x^3 - 1}}{\frac{1}{6}x} \, dx .$$ **(7)**

Question 3

Sum each of the following double series.

a) $\displaystyle\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[\frac{1}{2^{m+n}}\right]$. **(3)**

b) $\displaystyle\sum_{m=0}^{\infty}\sum_{n=0}^{m}\left[\frac{1}{2^{m+n}}\right]$. **(5)**

Question 4

Two thin rigid vertical poles AB and CD are standing on level horizontal ground.

- AB has length a and the point B is level with the ground.
- CD has length b, $b < a$, and the point C is level with the ground.

A taut string is connecting A to C and another taut string is connecting B to D.

The two strings cross each other at the point E.

Find, in terms of a and b, the vertical height of E above the ground. **(8)**

Question 5

A population of bacteria P is growing exponentially with time t and the table below shows some of these values.

t	12	36	60
P	576	2304	a

Show clearly that $a = 9216$. **(8)**

Question 6

$$x^2 + 2x + 1 + k = 0, \ x \in \mathbb{R},$$

where k is a real constant.

Given that the above equation has distinct real roots, determine the nature of the roots of the following equation

$$(k+2)\left(x^2 + 2x + 1 + k\right) = 2k\left(x^2 + 1\right).$$ **(8)**

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Question 7

The straight line l_1, where λ is a scalar parameter, has vector equation

$$\mathbf{r} = 10\mathbf{i} + 8\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k}).$$

The points $A(4,1,3)$ and $B(6,5,-3)$ lie on the straight line l_2.

a) Given that l_1 and l_2 lie on the same plane, show that l_1 is perpendicular to l_2. **(2)**

The points C and D lie on l_1 so that the resulting quadrilateral $ACBD$ is a kite, whose line of symmetry is l_2.

b) Given further that the area of the kite is $8\sqrt{42}$ square units, determine the possible coordinates of the points C and D. **(10)**

Question 8

$$f(x) = \left(\frac{1}{x}\right)^{\sqrt{x}}, \quad x \in \mathbb{R}, \; x > 0.$$

Determine in exact simplified form the coordinates of the stationary point of $f(x)$, fully justifying its nature. **(10)**

Question 9

Use algebra to solve the following simultaneous equations

$$x^3 - y^3 = \frac{7}{16} \qquad \text{and} \qquad x - y = 1,$$

given further that $x \in \mathbb{R}$ and $y \in \mathbb{R}$. **(9)**

Question 10

Eliminate θ from the following pair of equation.

$$\tan\theta + \cot\theta = x^3$$

$$\sec\theta - \cos\theta = y^3$$

Write the answer in the form

$$f(x,y) = 1.$$

(9)

Question 11

The points A and B are stationary points of the curve with equation

$$y = 2x^3 + 3x^2 - 12x + k, \; x \in \mathbb{R},$$

where k is a constant.

Given that $y = \frac{37}{2}$ at the point of inflexion of the curve, show that the area of the region bounded by the curve and the straight line through A and B is $5\frac{1}{16}$. **(12)**

Question 12

An extended ladder AB, of length 20 m, has one end A on level horizontal ground and the other end B resting against a vertical wall.

The end A begins to slip away from the wall with constant speed 0.3 ms^{-1}, and the end B slips down the wall.

Determine the speed of the end B, when B has reached a height of 12 m above the ground.

(7)

Question 13

A water tank has the shape of a hollow inverted hemisphere with a radius of 1 m, which is initially full.

Water is leaking from a hole at the bottom of the tank, in m^3 per hour, at a rate proportional to the volume of the water left in the tank at that time.

Show that if the height of the water in the tank is $h\text{ m}$, then

$$3h^2 - h^3 = 2\mathrm{e}^{-kt},$$

where k is a positive constant. **(12)**

$\left[\text{volume of a sphere of radius } r \text{ is given by } \frac{4}{3}\pi r^3\right]$

Question 14

A curve C has equation

$$y = \frac{x\mathrm{e}^{3x}}{2x+k}, \quad x \in \mathbb{R}, \quad x \neq k,$$

where k is a non zero constant.

It is given that C has a single turning point at P.

Find the exact coordinates of P. **(12)**

Question 15

By using an appropriate trigonometric substitution, or otherwise, find an exact value for the following integral.

$$\int_7^9 \sqrt{\frac{x-7}{11-x}}\ dx.$$ **(12)**

Question 16

$$f(a,b,c) \equiv a^4(b-c)+b^4(c-a)+c^4(a-b).$$

Factorize $f(a,b,c)$ into a product of 3 linear factors and 1 quadratic factor. **(8)**

Question 17

Given that $0<\theta<\frac{1}{2}\pi$, $0<\varphi<\frac{1}{2}\pi$, solve the following simultaneous equations.

$$5\cos\theta+2\tan\varphi=5 \qquad \text{and} \qquad 5\sin\theta+\cot\varphi=5.$$

Give the answers in exact form in terms of inverse trigonometric functions. **(13)**

Question 18

$$f(x) \equiv x^2+\frac{2x}{2+\sqrt{3}}-1,\ x \in \mathbb{R}.$$

Factorize $f(x)$ into a product of 2 simple linear factors. **(12)**

Question 19

It is given that θ, α and β are distinct real numbers which satisfy.

$$\tan(\theta-\alpha)+\tan(\theta-b)=x$$

$$\cot(\theta-\alpha)+\cot(\theta-b)=y.$$

Find, in exact simplified form, an expression for $\tan(\alpha-\beta)$, in terms of x and y. **(12)**

Question 20

The function f is defined in terms of the real constants, a, b and c, by

$$f(x) = \left(a + bx + cx^2\right)(1-x)^{-3}, \quad x \in \mathbb{R}, \quad |x| < 1.$$

a) Show that

$$f(x) = a + (3a+b)x + \frac{1}{2}\sum_{n=2}^{\infty}\Big[\big[a(n+1)(n+2) + bn(n+1) + cn(n-1)\big]\, x^n\Big]. \qquad \mathbf{(8)}$$

b) Use the expression of part **(a)** to deduce the value of

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}. \qquad \mathbf{(8)}$$