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IYGB

Special Paper G

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

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Question 1

Given that

$$y=\frac{(x-1)^4(x-2)^2}{(x+1)^3},\ x\in\mathbb{R},\ x\neq -1,$$

find the value of $\frac{dy}{dx}$ at $x=3$. **(7)**

Question 2

A curve C has equation

$$y=e^{1-\left(\frac{x}{e}\right)^2},\ x\in\mathbb{R},$$

The finite region bounded by C, the y axis and straight line with equation $y=1$, is revolved by 2π radians about the y axis, forming a solid of revolution.

Find an exact simplified value for the volume of this solid. **(8)**

Question 3

$$f(x)\equiv\frac{1-x}{1+x+x^2+x^3},\ -1<x<1.$$

Show that $f(x)$ can be written in the form

$$f(x)=g(x)\sum_{r=0}^{\infty}\left(x^{4r}\right),$$

where $g(x)$ is a simplified function to be found. **(9)**

Question 4

$$f(x) = \frac{4x^2 - 10x + 7}{x^2 - 3x + 2}, \ x \in \mathbb{R}, \ x \neq 1, \ x \neq 2.$$

Determine the range of $f(x)$. **(8)**

Question 5

The following three numbers

$$\log_{10} 2, \quad \log_{10}\left(2^x - 1\right), \quad \log_{10}\left(2^x + 3\right),$$

are consecutive terms in an arithmetic progression.

Determine the value of x as an exact logarithm, of base 2. **(8)**

Question 6

Solve the following trigonometric equation.

$$\cos 4x° = \cos 40° + \cos 80°, \quad 0° \leq x \leq 180°. \quad \textbf{(7)}$$

Question 7

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}\left[u_i\right] = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.$$

Evaluate, showing a clear method

$$\prod_{r=2}^{\infty}\left[1 - \frac{2}{r(r+1)}\right]. \quad \textbf{(7)}$$

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Question 8

The point $A(1,3)$ is one of the vertices of a rhombus whose centre is $B(3,7)$.

One of the sides of the rhombus lies on the straight line with equation

$$y = 5 - 2x.$$

Determine the coordinates of the other three vertices of this rhombus. **(8)**

Question 9

The points P and Q are the points of tangency of the common tangent to each of the curves with equations

$$y^2 = 4ax \quad \text{and} \quad ay = 2x^2,$$

where a is a positive constant.

Show that $|PQ|$ is $7\frac{1}{2}$ times the distance of the common tangent from the origin O. **(14)**

Question 10

By using an appropriate substitution or substitutions, show that

$$\int_0^1 \frac{\ln(x+1)}{1+x^2}\, dx = \frac{\pi \ln 2}{8}.$$

(12)

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Question 11

With respect to a fixed origin O, the points A, B and C have position vectors

$$\mathbf{a} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 8 \\ 2 \\ 7 \end{pmatrix} \text{ and } \mathbf{c} = \begin{pmatrix} 11 \\ 0 \\ 1 \end{pmatrix}.$$

a) Determine the volume of the cube, with vertices the points A, B and C. **(4)**

The points P, Q and R are vertices of a different cube, so that

$$\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 1 \\ 7 \end{pmatrix} \text{ and } \overrightarrow{PR} = \begin{pmatrix} k \\ 4 \\ 3 \end{pmatrix},$$

where k is a positive constant.

b) Given that $\measuredangle QPR = 60°$, determine …

i. … the value of k. **(4)**

ii. … the length of the diagonal of the second cube. **(4)**

Question 12

The curve with equation $y = f(x)$, lies entirely in the first quadrant. The point P, whose x coordinate is a lies on this curve.

The tangent to the curve at P meets the x axis at the point A and the y axis at the point C.

The normal to the curve at P meets the x axis at the point B and the y axis at the point D.

Given further that the gradient at P is positive, show that the difference between the areas of the triangle PAB and the triangle PCD is given by

$$\frac{1+\left[f'(a)\right]^2}{2f'(a)}\left|\left[f(a)\right]^2 - a^2\right|.$$ **(12)**

Question 13

It is given that the following series converges to a limit L.

$$\sum_{r=1}^{\infty}\left[\frac{2x-1}{x+2}\right]^{r}.$$

Determine with full justification the range of possible values of L. **(11)**

Question 14

A right circular cone, of radius r and semi-vertical angle θ, lies with one of its generators in contact with a horizontal surface.

The cone is then rolled on the horizontal surface with its vertex at rest, so that the rolling circumference of its base completes a full circle on the surface, while the cone completes N revolutions about its own axis.

Show that $N = \operatorname{cosec}\theta$. **(8)**

Question 15

The point P lies on the curve C with equation $y = f(x)$.

It is further given that C passes through the origin O and lies in the first quadrant.

The normal to C at P meets the x axis at the point A.

The point B is the foot of the perpendicular of P onto the x axis.

Given that for all positions of P,

$$|OA|^2 = 9|OB|,$$

determine in simplified form an equation of C. **(10)**

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Question 16

By using a suitable trigonometric substitution, or otherwise, find

$$\int \frac{(3x^2+5x)\sqrt{x}}{(x+1)^2}\,dx.$$ **(12)**

Question 17

$$f(p) \equiv \left(p-\sqrt{2}\right)^2 + \left(\frac{1}{p}-\sqrt{2}\right)^2, \quad p \in \mathbb{R}, \quad p \neq 0.$$

Given that $p+\frac{1}{p}<\sqrt{2}$, find $\sqrt{f(p)}$ in its simplest form. **(6)**

Question 18

A curve has Cartesian equation

$$y = \frac{1}{2}x^2, \quad x \in \mathbb{R}.$$

The points P and Q both lie on the curve so that POQ is a right angle, where O is the origin.

The point M represents the midpoint of PQ.

Show that as the position of P varies along the curve, M traces the curve with equation

$$y = x^2 - 2.$$ **(11)**

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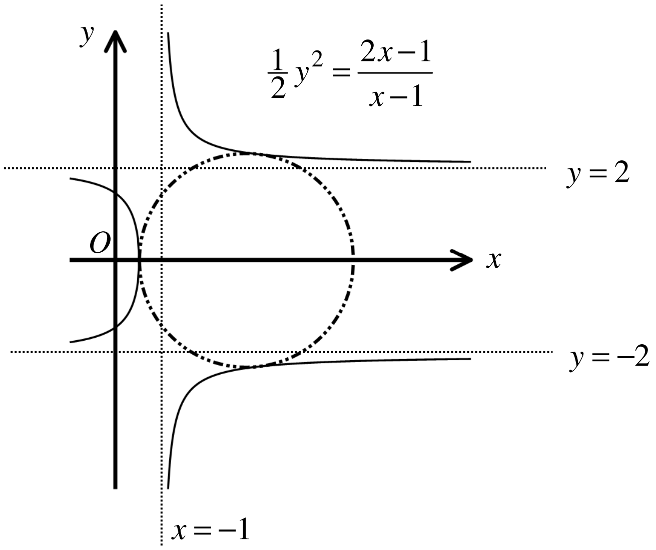
Question 19

Solve the equation

$$14x - 11y = 29,$$

given further that $x \in \mathbb{N}$, $y \in \mathbb{N}$, and $x + y < 100$. **(10)**

Question 20



The figure above shows the curve with equation $\frac{1}{2}y^2 = \frac{2x-1}{x-1}$, whose three asymptotes are marked with dotted lines.

A circle centred at the point C and of radius r is drawn, so that it touches all three branches of the curve, as shown in the figure.

Determine the coordinates of C and the value of r. **(20)**
