IYGB

Special Paper E

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

Question 1

Evaluate the following integral

$$\int_{-1}^{1} 2\left(x+|x|\right)-7x|x| \ dx .$$ **(4)**

Question 2

Show that $\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}$ can be expressed in the form

$$\sqrt{a+b\sqrt{2}} ,$$

where a and b are integers to be found. **(5)**

Question 3

When a man is asked how old he is, he replied.

"I am four times as old as my eldest son and five times as old as my youngest son."

He continued …

"… when my eldest son is three times as old as he is now I will be exceeding twice my youngest son's age by three years."

Determine how old the man is. **(6)**

Question 4

It is given that

$$\sin\varphi = k\sin\theta,\ k \neq 0,\ k \neq \pm 1.$$

Show, by a detailed method, that

$$1+\left(\frac{d\varphi}{d\theta}\right)^2 = \left(k^2-1\right)\sec^2\varphi. \qquad (6)$$

Question 5

The curve C has equation

$$y = \frac{x+1}{x^2+3},\ x \in \mathbb{R}.$$

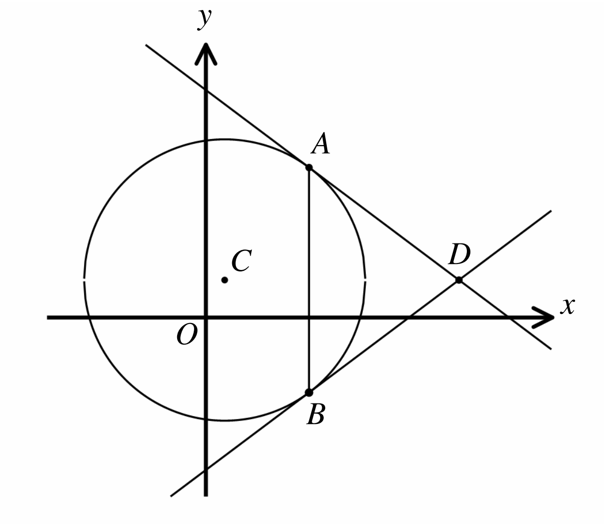
By considering the discriminant of a suitable quadratic equation, determine the range of the possible values of y. **(7)**

Question 6

Relative to a fixed origin O, the points A and B have position vectors $4\mathbf{i}+5\mathbf{j}+8\mathbf{k}$ and $6\mathbf{i}+6\mathbf{j}+7\mathbf{k}$, respectively. The straight line l_1 passes through A and , B and crosses the $y-z$ plane at the point C. The straight line l_2 passes through the point D with position vector $p\mathbf{j}+(2p+2)\mathbf{k}$, where p is a scalar constant.

Given that l_1 and l_2 are perpendicular, and intersect at C, find the value of p. **(11)**

Question 7



The figure above shows the circle with equation

$$x^2 + y^2 - 4x - 8y = 205,$$

with centre at the point C and radius r.

The straight line AB is parallel to the y axis and has length 24 units.

The tangents to the circle at A and B meet at the point D.

Find the length of AD and hence deduce the area of the kite $CADB$. **(9)**

Question 8

By suitably rewriting the numerator of the integrand, find a simplified expression for the following integral.

$$\int \frac{12\sin x - 5\cos x}{2\sin x - 3\cos x}\,dx.$$ **(7)**

I.Y.G.B www.madasmaths.com

Question 9

Fine magnetised iron fillings are falling onto a horizontal surface forming a heap in the shape of a right circular cone of height $7x$ cm and radius x cm.

The area of the curved surface of the conical heap is increasing at the constant rate of $k \text{ cm}^2\text{s}^{-1}$, $k > 0$.

Determine the value of k, given further that when $x = 5$ the volume of the heap is increasing at the rate of $24.5 \text{ cm}^3\text{s}^{-1}$.

You may assume that the volume V and curved surface area A of a right circular cone of radius r and height h are given by

$$V = \frac{1}{3}\pi r^2 h \qquad \text{and} \qquad A = \pi r\sqrt{r^2 + h^2}\,.$$ **(11)**

Question 10

The function f is defined as

$$f(x) \equiv \frac{x+1}{2x-1}, \quad x \in \mathbb{R}, \quad x \neq \frac{1}{2}.$$

The function g is suitably defined so that

$$f(g(x)) \equiv \frac{3x+2}{3x-5}, \quad x \in \mathbb{R}, \quad x \neq \frac{5}{3}.$$

a) Determine an expression for $g(x)$. **(5)**

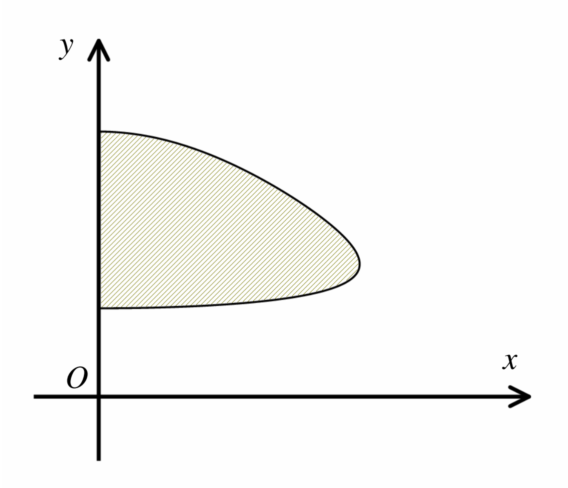
The function h is suitably defined so that

$$h(f(x)) \equiv \frac{2x-7}{x-2}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

b) Determine an expression for $h(x)$. **(7)**

I.Y.G.B. www.madasmaths.com

Question 11



The figure above shows a curve given parametrically by

$$x = \sin 3t, \quad 1 + y\cos 3t = 2y, \quad t \in \mathbb{R}, \quad 0 \le t \le \tfrac{1}{3}\pi.$$

The finite region bounded by the curve and the y axis, shown shaded in the figure is revolved by 2π radians about the y axis, forming a solid of revolution.

Determine an exact simplified value for the volume of this solid. **(12)**

Question 12

A large water tank is in the shape of a cuboid with a rectangular base measuring 10 m by 5 m, and a height of 5 m.

Let h m be the height of the water in the tank and t the time in hours.

At a certain instant, water begins to pour into the tank at the constant rate of 50 m^3 per hour and at the same time water begins to drain from a tap at the bottom of the tank at the rate of $10h$ m^3 per hour.

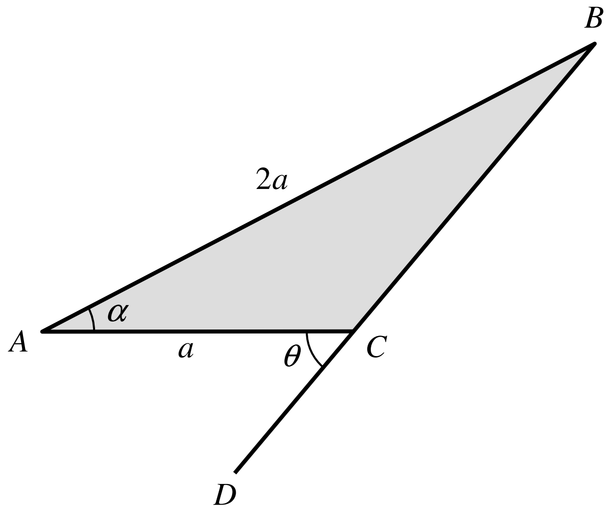
Show that it takes $5\ln 3$ hours for the height of the water to rise from 2 m to 4 m. **(10)**

Question 13

Solve following equation

$$4+\sqrt{x^2-6x+13}=x+\sqrt{2x-5}\,,\quad x\in\mathbb{R}\,,\ x\geq\frac{5}{2}\,.\qquad (12)$$

Question 14



The figure above shows a triangle ABC, where $|AB|=a$ and $|AC|=2a$.

The angle BAC is α, where $\tan\alpha=\frac{3}{4}$.

The side BC is extended to the point D so that the angle ACD is denoted by θ.

Show clearly that

$$\theta=\arctan 2\,.\qquad (10)$$

Question 15

Use partial fractions to sum the following series.

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^4+2n^3+n^2}.$$

You may assume the series converges. **(8)**

Question 16

Solve the trigonometric equation

$$(\cos 4x + \cos x)^2 + (\sin 4x + \sin x)^2 = 2\sqrt{3}\sin 3x\,,\ 0 \le x < \pi\,,$$

giving the answers in terms of π. **(11)**

Question 17

A sphere of radius r, whose centre is at O, is fixed on a horizontal plane.

A thin right circular conical shell, **without** a base is placed over the sphere.

The axis of the conical shell is vertical and passes through O. The circumference of the missing base of the conical shell is at the same horizontal level as O.

Show that the minimum value of the outer surface area of the conical shell is

$$\frac{3\sqrt{3}\pi r^2}{2}.$$ **(15)**

Question 18

By using an appropriate substitution or substitutions, show that

$$\int_0^{\pi} \ln(\sin x)\ dx = -\pi \ln 2\,.$$ **(11)**

Question 19

The first three terms of a series S are

$$S = 7 + 9x + 8x^2 + ...$$

The n^{th} term of S is given by

$$A\left(\frac{3}{4}x\right)^n + B\left(\frac{1}{3}x\right)^n,$$

where A and B are non zero constants.

Given that the sum to infinity of S is 19, determine the value of x. **(14)**

Question 20

A curve is given parametrically by

$$x = \frac{1}{3}t^2, \quad y = \frac{2}{3}t, \quad t \in \mathbb{R}.$$

The normal to the curve at the point P meets the curve again at the point Q.

Show that the minimum value of $|PQ|$ is $\sqrt{12}$. **(19)**

I.Y.G.B www.madasmaths.com