IYGB

Special Paper D

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

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Question 1

When a man is asked how old he is, he replied.

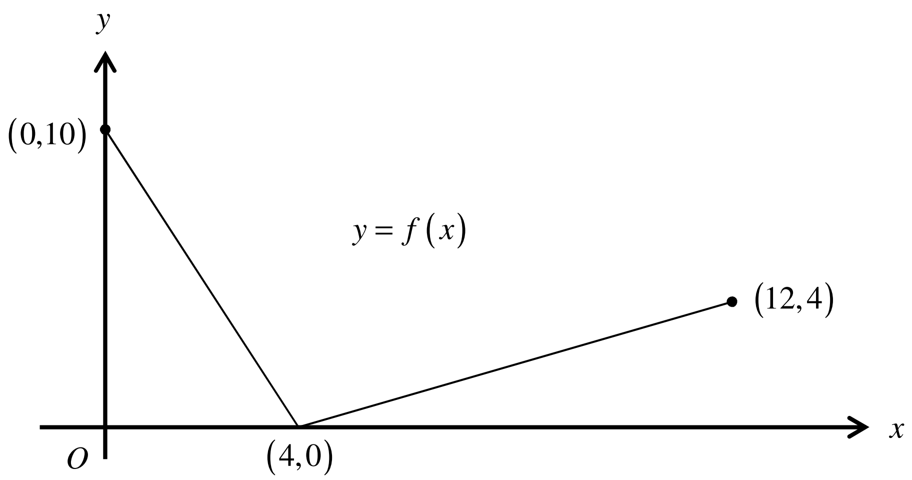
"Ten years ago I was five times as old as my son."

He continued …

"… in twenty years time I will be twice as old as my son."

Determine how old the man is. **(5)**

Question 2



The graph of the function $f(x)$ consists of two straight line segments joining the point $(0,10)$ to $(4,0)$ and the point $(12,4)$ to $(4,0)$, as shown in the figure above.

a) Find the value of $ff(2)$. **(3)**

The function g is defined as

$$g(x) \equiv \frac{2x+1}{x-1}, \; x \in \mathbb{R}, \; x \neq 1.$$

b) Determine the solutions of the equation $gf(x) = 3$. **(6)**

Question 3

The figure above shows the graph of the curve with equation

$$y = \frac{1}{2}\text{arcsec}\sqrt{x}, \ x \geq 1, \ 0 \leq y < \frac{\pi}{4},$$

where $\text{arcsec}(u)$ is the inverse function of $\sec(u)$.

Show clearly that …

a) … $\frac{dy}{dx} = \frac{1}{4x\sqrt{x-1}}$. **(6)**

b) … $\frac{d^2y}{dx^2} = \frac{2-3x}{8x^2(x-1)^{\frac{3}{2}}}$. **(5)**

Question 4

Prove the validity of the following trigonometric identity.

$$\frac{1+\tan\theta\tan 3\theta}{1+\tan 2\theta\tan 3\theta} \equiv \frac{\cos^2 2\theta}{\cos^2\theta}.$$ **(6)**

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Question 5

The points $A(-3,9)$, $B(4,5)$ and $C(7,-6)$ are three vertices of the kite $ABCD$.

Determine the coordinates of D. **(9)**

Question 6

It is given that

$$\sum_{r=1}^{3} \log_a x^r = \sum_{r=1}^{3} (\log_a x)^r ,$$

where a and x are positive numbers such that $x \neq a$, $x \neq 1$ and $a > 1$.

Show clearly that

$$x = a^{\frac{-1\pm\sqrt{21}}{2}} .$$ **(9)**

Question 7

A function has equation

$$f(x) = x^2 + 6x + 20 + k\left(x^2 - 3x - 12\right), \; x \in \mathbb{R} ,$$

where k is a non zero constant.

a) State the value of k if $f(x)$ represents a straight line. **(1)**

b) Find the value of k if the equation $f(x) = 0$ two equal in magnitude roots, but of opposite signs. **(2)**

c) Determine the value of k and the value of p, given that the graph of $f(x)$ has a maximum at $(2, p)$. **(5)**

Question 8

The coordinates in this question are relative to a fixed origin O at $(0,0,0)$.

The straight line l_1 has vector equation

$$\mathbf{i}+3\mathbf{j}+5\mathbf{k}+\lambda(-\mathbf{i}+3\mathbf{j}+\mathbf{k}),$$

where λ is a scalar parameter.

The straight line l_2 passes thought the point with coordinates $(6,0,6)$ and is in the direction $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$.

a) Verify that $A(4,3,5)$ is the intersection of l_1 and l_2, and show further that $B(12,-9,9)$ lies on l_2. **(3)**

The point $C(6,-3,3)$ lies on l_1.

The straight line l_3 passes through B and C.

The straight line l_4 is parallel to l_2 and passes through C.

The straight line l_5 is perpendicular to l_3 and passes through A.

b) Given that l_4 and l_5 intersect at the point D, find the coordinates of D. **(9)**

Question 9

Find, in terms of π, the solutions of the trigonometric equation

$$\cos 2x+3\cos x-2\cos^2 x-\sqrt[3]{\cos x}=1,\ 0\le x\le 2\pi. \qquad \textbf{(7)}$$

Question 10

By showing a detailed method, sum the following series.

$$\frac{2}{1}+\frac{3}{2}+\frac{4}{4}+\frac{5}{8}+\frac{6}{16}+\frac{7}{32}\ ...$$ (7)

Question 11

Show clearly that

$$\frac{d}{dx}\left(\frac{\cos 2x}{\sqrt{1+\sin 2x}}\right)=\begin{cases}-\sin x-\cos x & 0\le x\le \alpha\pi \\ \sin x+\cos x & \alpha\pi\le x\le \beta\pi \\ -\sin x-\cos x & \beta\pi\le x\le 2\pi\end{cases}$$

where α and β are constants to be found. (8)

Question 12

$$2x\frac{dy}{dx}=x-y+3\ ,\ \ x>0\ .$$

Determine a general solution of the above differential equation, by using the substitution $u=y\sqrt{x}$. (10)

Question 13

Use suitable integration techniques to show that

$$\int_{-\frac{1}{6}\ln 3}^{\frac{1}{6}\ln 3} 6\mathrm{e}^{-3x}\arctan\left(\mathrm{e}^{3x}\right)\ dx\ \ =\ \ \ln 3+\frac{\pi\sqrt{3}}{9}.$$ (10)

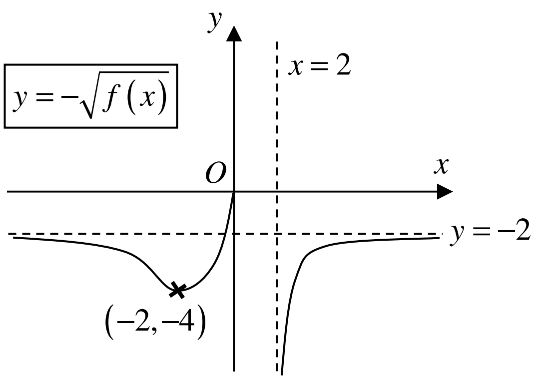
Question 14

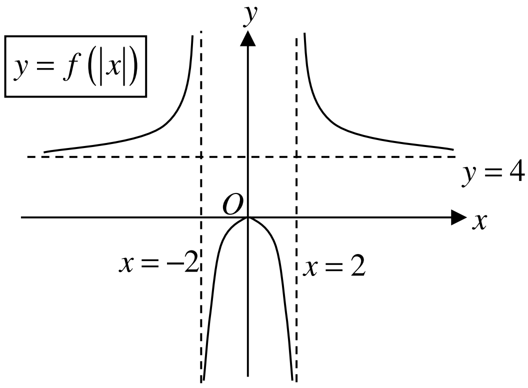
If $x \in \mathbb{R}$, $y \in \mathbb{R}$, find the non-trivial solution the following simultaneous equations.

$$36y^2(x+1)+36x^2(y+1)=7x^2y^2 \quad \text{and} \quad 6x+6y+xy=0.$$

Question 15

The graph of $y=-\sqrt{f(x)}$ and the graph of $y=f(|x|)$ are shown below, in two separate set of axes.



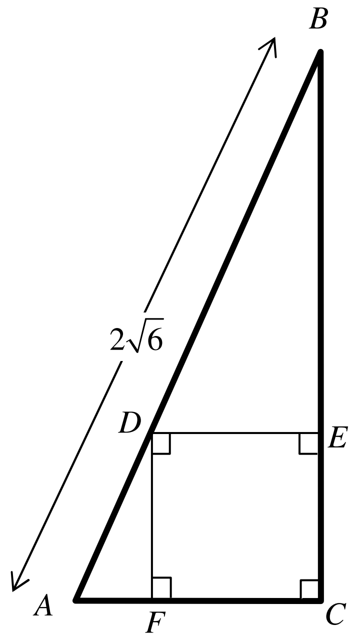


Sketch on separate set of axes a detailed graph of …

a) … $y=f(x)$. **(6)**

b) … $y=f'(x)$. **(3)**

Question 16



The figure above shows a right angled triangle ABC, where $|AB| = 2\sqrt{6}$.

A square $DECF$, of side length 1, is drawn inside ABC, so that D lies on AB, E lies on BC and F lies on AC.

Determine, in exact simplified surd form, the possible values of the tangent of the angle BAC.

(12)

Question 17

$$\frac{1}{2}\sin^4 x + \frac{1}{3}\cos^4 x = \frac{1}{5}.$$

Show that the above trigonometric equation is equivalent to

$$\tan^2 x = \frac{2}{3}.$$

(12)

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Question 18

Use suitable integration techniques to show that

$$\int_0^1 \frac{x^2}{\left(x^2+1\right)^3}\, dx = \frac{\pi}{32}.$$

(10)

Question 19

Two curves are defined in the largest possible real number domain and have equations

$$y^2 = \frac{4(4-x)}{x} \qquad \text{and} \qquad x^2 = \frac{4(4-y)}{y}.$$

a) Show that the two curves have one, and only one, common point which is also a point of common tangency. **(8)**

b) Find the exact value of the area enclosed by the common tangent to the curves, and either of the two curves. **(10)**

Question 20

It is given that for $x \in \mathbb{R}$, $-\frac{1}{k} < x < \frac{1}{k}$, $k > 0$,

$$f(x,k) \equiv \frac{k+1}{(1-x)(1+kx)}.$$

Given further that

$$f(x,k) \equiv \sum_{r=0}^{\infty} \left[a_r x^r \right],$$

where a_r are functions of k, show that

$$\sum_{r=0}^{\infty} \left[a_r^{\,2} x^r \right] = \frac{(1-kx)(1+k)^2}{(1-x)(1+kx)\left(1-k^2 x\right)}.$$ **(18)**

You may assume that $\sum_{r=0}^{\infty} \left[a_r^{\,2} x^r \right]$ converges.