IYGB

Special Paper B

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

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Question 1

Find the coefficient of x^2 in the binomial expansion of

$$\left(2x^2+3x+1\right)^7 .$$ **(4)**

Question 2

It is known that a box contains 10 coins of which some are gold, some are silver and some are bronze.

The combined weight of the 10 coins is 116 grams

Each gold coin weighs 23 grams, each silver coin weighs 13 grams and each bronze coin weighs 7 grams.

Determine the number of each type of coin. **(6)**

Question 3

It is given that

$$\sum_{r=1}^{n} u_r \;=\; 3n^2-2n+4+(3n-2)\times 2^{n+1},$$

where u_n is the n^{th} term of a sequence.

Find a simplified expression for u_n. **(7)**

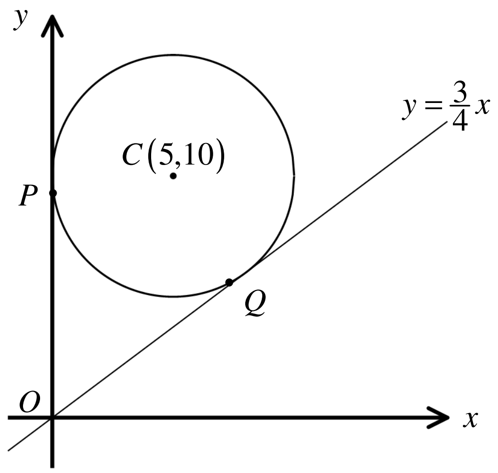
Question 4

Solve the trigonometric equation

$$\arcsin 2x+\arccos x=\frac{5\pi}{6}.$$ **(10)**

Question 5

The figure below shows the circle with centre at $C(5,10)$ and radius 5.



The straight lines with equations, $x = 0$ and $y = \frac{3}{4}x$ are tangents to the circle at the points P and Q respectively.

Show that the area of the triangle PCQ is 10 square units. **(10)**

Question 6

Solve the following trigonometric equation, for $0 < x < \frac{1}{2}\pi$.

$$4\cos x \cos 2x \cos 5x + 1 = 0.$$ **(10)**

Question 7

The straight line L_1 passes through the points A and B, whose respective position vectors relative to a fixed origin O are

$$\begin{pmatrix}1\\2\\5\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}2\\1\\6\end{pmatrix}.$$

The point C has position vector $\begin{pmatrix}3\\0\\1\end{pmatrix}$.

The straight line L_2 passes through C and is parallel to L_1.

The points P and Q both lie on L_2 so that $|CP| = |CQ| = 2|AB|$.

Find the area of the quadrilateral with vertices at A, B, P and Q. **(10)**

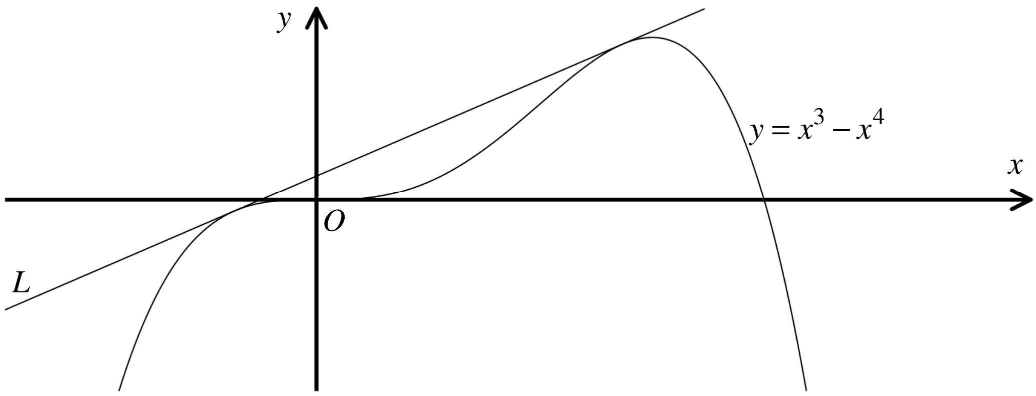
Question 8

The curve has parametric equations

$$x = \frac{t^2+5}{t^2+1}, \quad y = \frac{4t}{t^2+1}, \quad t \in \mathbb{R}.$$

Show by eliminating the parameter t that the curve is a circle, stating the coordinates of its centre, and the size of its radius. **(8)**

Question 9



The figure above shows the curve C with equation

$$y = x^3 - x^4 .$$

The straight line L is a tangent to the C, at two distinct points.

Determine an equation of L. **(9)**

Question 10

$$f(x) \equiv x^4 - 16x^3 + 68x^2 - 32x + 3, \quad x \in \mathbb{R} .$$

Factorize $f(x)$ into a product of 4 linear factors. **(10)**

Question 11 **(*****)**

Use algebra to solve the following simultaneous equations

$$xy(5 - xy) = 4 \qquad \text{and} \qquad x^2 + 9y^2 = 10 ,$$

given further that $x \in \mathbb{R}$ and $y \in \mathbb{R}$. **(10)**

Question 12

By using an appropriate substitution followed by trigonometric identities, show that

$$\int_0^{\pi} \frac{x\tan x}{\tan x+\sec x}\,dx = \tfrac{1}{2}\pi(\pi-2)$$ **(10)**

Question 13

By using the substitution $\sqrt[3]{10\pm 6\sqrt{3}} = u\pm\sqrt{v}$, where $u\in\mathbb{Q}$, $v\in\mathbb{Q}$, simplify fully the following cubic radical expression.

$$\sqrt[3]{10+6\sqrt{3}} + \sqrt[3]{10-6\sqrt{3}}\,.$$ **(10)**

Question 14

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 1\,.$$

Given that $y=\frac{dy}{dx}=0$ at $x=0$, show that

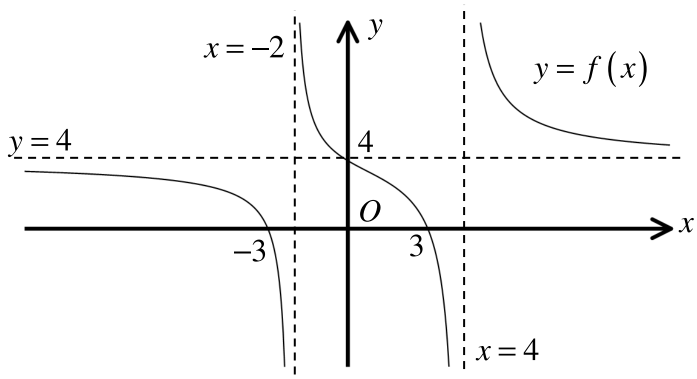
$$y=-x+\ln\left[\tfrac{1}{2}\left(1+e^{2x}\right)\right].$$ **(14)**

Question 15

Solve the following trigonometric equation, for $0\le\theta<2\pi$.

$$3\cos^2\theta-\sin^2\theta-\sqrt{3}\cos\theta-\sin\theta=0\,.$$ **(10)**

Question 16



A sketch of the curve with equation $y = f(x)$ is shown above.

Important information about the curve, such as the equations of its asymptotes and its intercepts with the coordinate axes are marked in the diagram.

Sketch on separate detailed diagrams the graph of …

a) … $y = f\left(\sqrt{x}\right)$. **(3)**

b) … $y = \sqrt{f(x)}$. **(3)**

c) … $y = f\left(x^2\right)$. **(3)**

d) … $y = f'(x)$. **(3)**

Question 17

Find an exact simplified value for

$$\int_{\sqrt{e}}^{e} \ln(\ln x) + \frac{1}{(\ln x)^2} \, dx .$$ **(10)**

Question 18

The function $y = f(x)$ satisfies the following relationship.

$$4x\frac{d^2y}{dx^2} + 4x\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} - 1 = 0.$$

It is further given that $x = t^2$ and $y = \ln v$.

Show that

$$\frac{d^2v}{dt^2} = v.$$

(9)

Question 19

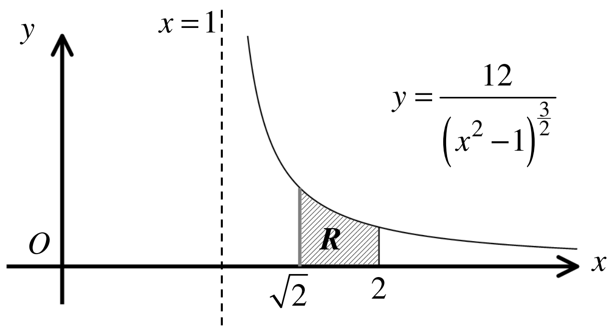
A lump of metal, of volume 76 cubic units is **moulded** into the shape of a cuboidal box, with a square base, rectangular sided and no lid.

All the faces of the box are 1 unit thick.

All the metal is moulded in the construction of this box, and the construction is such so that the box has maximum capacity.

If the internal width of the box is x, find the value of x which maximises the capacity of the box, and hence determine this maximum capacity. **(16)**

Question 20



The figure above shows the curve with equation

$$y = \frac{12}{\left(x^2-1\right)^{\frac{3}{2}}}, \ x > 1.$$

The region R, bounded the curve, the x axis and the straight lines with equations $x=\sqrt{2}$ and $x=2$, is revolved by a full turn about the x axis, forming a solid S.

a) Show that the volume of S is given by

$$144\pi \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \operatorname{cosec}\theta \cot^4\theta \ d\theta .$$ **(3)**

b) Hence find an exact simplified expression for the volume of S. **(12)**