IYGB

Special Paper A

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

Question 1

$$y = 2\left\{ e^{2x} + 3\ln\left[x + \left(e^{x} + 1 \right)^{2} \right] \right\}^{2}.$$

Show that the value of $\frac{dy}{dx}$ at $x=0$ is $23(1+6\ln 2)$ **(6)**

Question 2

Solve the following simultaneous logarithmic equations

$$y^{\log x} = 100$$

$$\log\sqrt{\frac{xy}{10}} = 1,$$

given further that x, $y \in \mathbb{R}$, with $x > 0$, $y > 0$. **(8)**

Question 3

The rate of change, with respect to x, of the gradient of a curve is constant.

The curve passes through the points with coordinates $(1,2)$ and $(-3,0)$, the gradient at the former point being $-\frac{1}{2}$.

Show that the area of the finite region bounded between the curve and the straight line with equation $y = 2x$ is $\frac{125}{3}$. **(12)**

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Question 4

The straight lines L_1 and L_2 have respective equations

$$y = -3x - 10 \quad \text{and} \quad y = 5x - 4.$$

The point A has coordinates $(4,2)$.

The point B lies on L_1, so that the midpoint M of the straight line segment AB, lies on L_2.

Determine the coordinates of B and the coordinates of M. **(10)**

Question 5

The function f satisfies

$$2f(x) + 3f\left(\frac{2x+3}{x-2}\right) = 3x + 1, \ x \in \mathbb{R}.$$

Find the value of $f(9)$. **(7)**

Question 6

Simplify, showing all steps in the calculation, the expression

$$\arctan 8 + \arctan 2 + \arctan \frac{2}{3},$$

giving the answer in terms of π. **(12)**

Question 7

The straight line l has vector equation

$$\mathbf{r} = 3\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}),$$

where λ is a scalar parameter.

The point A has coordinates $(3,3,-3)$, relative to a fixed origin O.

The points P and Q lie on the l so that $|AP| = |AQ|$.

Given further that $\measuredangle PAQ = 90°$, find the coordinates of P and the coordinates of Q.

(11)

Question 8

Solve the following simultaneous equations, to find in exact form where appropriate, the value or values of x and k.

$$x^2 - 9x + 10 = 0 \quad \text{and} \quad k = \sqrt{x - \sqrt{x+6}}\ .$$

(10)

Question 9

$$I = \int_0^1 \left(x^{\frac{7}{6}} + 4x^{\frac{2}{3}}\right)^{-\frac{3}{4}} \, dx\ .$$

Use appropriate integration techniques to show that

$$I = 8\left[\sqrt[4]{5} - \sqrt{2}\right].$$

(8)

Question 10

The acute angles θ, ψ and α satisfy the following equations.

$$4\tan\theta = \tan\alpha$$

$$(5+3\cos 2\alpha)\tan\psi = 3\sin 2\alpha .$$

Express $\theta+\psi$, in terms of α. **(12)**

Question 11

Fungus is spreading on a wall and when it was first noticed $\frac{1}{3}$ of the wall was already covered by this fungus.

Let x represent the proportion of the wall not yet covered by the fungus, t weeks after the fungus was first required. The rate at which x is changing is proportional to the square root of the proportion of the wall not yet covered by the fungus.

When the fungus was first noticed it was spreading at rate that if it this rate was to remain constant from that instant onwards the fungus would have covered the entire wall in 4 weeks.

Determine the proportion of the wall covered by the fungus, 4 weeks after it was first noticed.

(12)

Question 12

Find the set of values of x that satisfy the inequality

$$\left|\frac{4x}{x+2}\right| \geq 4-x .$$ **(10)**

Question 13

Find in exact form the equations of the common tangents to the curves with equations

$$(x-2)^2+(y+1)^2=4 \quad \text{and} \quad y=x^2-4x+11 .$$ **(13)**

Question 14

$$S = 1 - \frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 8} - \frac{1 \cdot 3 \cdot 5}{4 \cdot 8 \cdot 12} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12 \cdot 16} - \ldots$$

Find the sum to infinity of S, by considering the binomial series expansion of $(1+x)^n$ for suitable values of x and n. **(9)**

Question 15

A water tank is fed by one inlet pipe which feeds into the tank at constant rate

The tank has 6 outlet pipes, each having the same constant drainage rate. The drainage rate of one of the outlet pipes is greater than the inflow rate of the inlet pipe.

- When the inlet pipe and all 6 outlet pipes are turned on, it takes 3 hours to empty the full tank.
- When the inlet pipe and 3 outlet pipes are turned on, it takes 7 hours to empty the full tank.

Determine the number of hours it takes to empty a full tank with the inlet pipe and just one of the outlet pipes turned on. **(10)**

Question 16

Solve the equation

$$13x + 11y = 414,$$

given further that $x \in \mathbb{N}$, $y \in \mathbb{N}$. **(9)**

Question 17

Solve the following trigonometric equation, for $0 < x < 2\pi$.

$$\sin x \sin 2x + \sin 2x \sin 3x + \sin 3x \sin 4x = 0.$$ **(12)**

Question 18

$$I = \int_0^{\frac{1}{4}\pi} \frac{1}{9\cos^2 x - \sin^2 x} \, dx .$$

By using a tangent substitution, or otherwise, show that

$$I = \frac{1}{6}\ln 2 .$$ **(9)**

Question 19

The variables x, y and z satisfy the following relationships.

$$x = \ln(z+1) \qquad \text{and} \qquad \frac{d^2 y}{dz^2} = \frac{y}{e^{2x}} .$$

Show that

$$\frac{d^2 y}{dx^2} = \frac{dy}{dx} + y .$$ **(9)**

Question 20

$$\sum_{r=0}^{\infty} \left[\frac{1}{r!} \right] .$$

It is given that the above series converges to a finite limit L.

By using this fact, or otherwise, find, in terms of L, the sum to infinity of the following convergent series

$$\sum_{r=1}^{\infty} \left[\frac{r^3}{r!} \right] .$$ **(11)**
