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## IYGB GCE

## Mathematics MP1 <br> Advanced Level

Practice Paper M
Difficulty Rating: 3.2600/1.2190
Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

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## Question 1



The figure above shows the curve with equation

$$
y=(3-x)(x+1), x \in \mathbb{R} .
$$

Find the exact area of the region, bounded by the curve and the $x$ axis, shown shaded in the figure above.

## Question 2

The curve $C$ has equation

$$
y=2^{x-3} .
$$

a) Describe geometrically a single transformation that maps the graph of $y=2^{x}$ onto the graph of $C$.
b) Describe geometrically a different transformation that can also map the graph of $y=2^{x}$ onto the graph of $C$.

## Question 3

Find the coefficient of $x^{5}$ in the binomial expansion of $(2+3 x)^{9}$.

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## Question 4

The straight line joining the points $A(2,5)$ and $B(-2,9)$ is a diameter of a circle.
a) Find an equation for this circle.
b) Determine by calculation whether the point $P(1,5)$ lies inside or outside the above mentioned circle.

## Question 5

Solve the following trigonometric equation in the range given.

$$
\begin{equation*}
\cos (2 y-35)^{\circ}=0.891, \quad 0 \leq y<360 \tag{5}
\end{equation*}
$$

## Question 6

Find the real solutions of the following equation

$$
\begin{equation*}
\left(x^{2}-x-3\right)^{2}-12\left(x^{2}-x-3\right)+27=0 \tag{5}
\end{equation*}
$$

## Question 7

$$
f(x)=x^{2}-3 x+7, x \in \mathbb{R}
$$

Use the formal definition of the derivative as a limit, to show that

$$
\begin{equation*}
f^{\prime}(x)=2 x-3 . \tag{4}
\end{equation*}
$$

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## Question 8

The table below shows experimental data connecting two variables $x$ and $y$.

| $\boldsymbol{x}$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 12.0 | 14.4 | 17.3 | 20.7 | 27.0 |

It is assumed that $x$ and $y$ are related by an equation of the form

$$
y=a b^{x},
$$

where $a$ and $b$ are non zero constants.
a) Find an equation of a straight line, in terms of well defined constants, in order to investigate the validity of this assumption.
b) Plot a suitable graph to show that the assumption of part (a) is valid.
c) Use the graph of part (b) to estimate, correct to 1 decimal place, the value of $a$ and the value of $b$.
d) Estimate the value of $y$ when $x=2.5$.

## Question 9

Find the solution of the following simultaneous equations

$$
\begin{aligned}
& 2 x+2 y-z=2 \\
& z=x^{2}+y^{2}
\end{aligned}
$$

assuming that $x, y, z$ are all real numbers.

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## Question 10

A hot drink is cooling down in a room.

The temperature $T^{\circ} \mathrm{C}$ of the drink, $t$ minutes after it was made is modelled by

$$
T=22+50 \mathrm{e}^{-\frac{1}{8} t}, t \geq 0 .
$$

a) State the temperature of the drink when it was first made.
b) Assuming the drink is not consumed ...
i. ... calculate, to the nearest minute, the value of $t$ when the temperature of the drink has reached $40^{\circ} \mathrm{C}$.
ii. ... determine the value of $T$, when the drink is cooling at the rate of $2.5^{\circ} \mathrm{C}$ per minute.

## Question 11

The following quadratic equation, where $m$ is a constant, has two distinct real roots.

$$
x^{2}+(m+2) x+4 m-7=0, x \in \mathbb{R}
$$

Determine the range of the possible values of $m$.
$\qquad$

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## Question 12

The points $A, B$ and $C$ lie on a plane so that

$$
\overrightarrow{A B}=2 \mathbf{i}+7 \mathbf{j} \quad \text { and } \quad \overrightarrow{A C}=4 \mathbf{i}-5 \mathbf{j}
$$

where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

The point $D$ lies on the straight line segment $B C$, so that $|B D|:|D C|=1: 2$.
a) Determine a simplified expression, in terms of $\mathbf{i}$ and $\mathbf{j}$, for $\overrightarrow{B D}$.
b) Show that the $|\overrightarrow{A D}|$ is approximately 4 units.

## Question 13

A cubic curve and a quartic curve, are both defined for all real numbers, and have respective equations

$$
y=x^{3}-3 x^{2} \quad \text { and } \quad y=x(x-2)^{3} .
$$

a) Sketch both curves in the same set of axes, indicating the coordinates of any points where each curve meets the coordinate axes.
b) State the number of solutions of the equation

$$
\begin{equation*}
x^{3}-3 x^{2}=x(x-2)^{3}, x \in \mathbb{R} . \tag{1}
\end{equation*}
$$

c) Indicate by shading in the set of axes of part (a) the region satisfied by the following inequality.

$$
\begin{equation*}
x^{3}-3 x^{2} \leq y \leq x(x-2)^{3} \cap-1<x<4 . \tag{1}
\end{equation*}
$$

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## Question 14



The straight line $l_{1}$ passes though the points $A(15,0)$ and $B(0,30)$ ．
a）Determine an equation for $l_{1}$ ．

The straight line $l_{2}$ is perpendicular to $l_{1}$ and passes though the point $Q(0, k)$ ，where $k$ is a positive constant．

The point $P$ is the intersection between $l_{1}$ and $l_{2}$ ．
b）Find，in terms of $k$ ，the $x$ coordinate of $P$ ．
c）Given further that the area of the triangle $O Q P$ is 25 ，where $O$ is the origin， determine the possible area of the quadrilateral $O Q P A$ ．
（6）

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## Question 15

The curve $C$ has equation

$$
y=\frac{x^{3}(5 x \sqrt{x}-128)}{\sqrt{x}}, x \in \mathbb{R}, x>0 .
$$

a) Determine expressions for $\frac{d y}{d x}, \frac{d^{2} y}{d x^{2}}$ and $\frac{d^{3} y}{d x^{3}}$.
b) Show that the $y$ coordinate of the stationary point of $C$ is $-k \sqrt[3]{4}$, where $k$ is a positive integer.
c) Evaluate $\frac{d^{2} y}{d x^{2}}$ at the stationary point of $C$.

Give the answer in terms of $\sqrt[3]{2}$.
d) Find the value of $\frac{d^{3} y}{d x^{3}}$ at the point on $C$, where $\frac{d^{2} y}{d x^{2}}=0$.

