IYGB - MP1 PAPER D - QUESTION 1

a) SWITCH EVERYTHING INTO DEFINITIONS OF FRACTIONAL POWERS

$$4 \times 4^{\frac{5}{2}} + 8^{-\frac{1}{3}} = 4 \times \left(4^{\frac{1}{2}}\right)^{5} + \frac{1}{8^{\frac{1}{3}}}$$

$$= 4 \times \left(\sqrt{4}\right)^{5} + \frac{1}{\sqrt[3]{8}}$$

$$= 4 \times 2^{5} + \frac{1}{2}$$

$$= 4 \times 32 + \frac{1}{2}$$

$$= 128 + \frac{1}{2}$$

$$= \frac{256}{2} + \frac{1}{2}$$

$$= \frac{257}{2}$$

b) SWITCH EVERYTHING INTO INDICES

$$(2pq^{2})^{4} \times 5p\sqrt{q^{6}} = 16p^{4}q^{8} \times 5p\left(q^{6}\right)^{\frac{1}{2}}$$

$$= 16p^{4}q^{8} \times 5pq^{3}$$

$$= 80p^{5}q^{11}$$

IYGB - MP1 PAPER D - QUESTION 2

OBTAIN AN EXPRESSION FOR THE DISCRIMINANT

$$(k+1)x^2 + 2kx + k = 1$$
$$(k+1)x^2 + 2kx + (k-1) = 0$$

$a = k+1,\ b = 2k,\ c = k-1$

$$\Rightarrow b^2 - 4ac = (2k)^2 - 4(k+1)(k+1)$$
$$= 4k^2 - 4(k^2-1)$$
$$= 4k^2 - 4k^2 + 4$$
$$= 4 > 0$$

ALWAYS TWO DISTINCT ROOTS EXCEPT WHEN $k=-1$

EQUATION REDUCES TO

$-2x - 2 = 0$

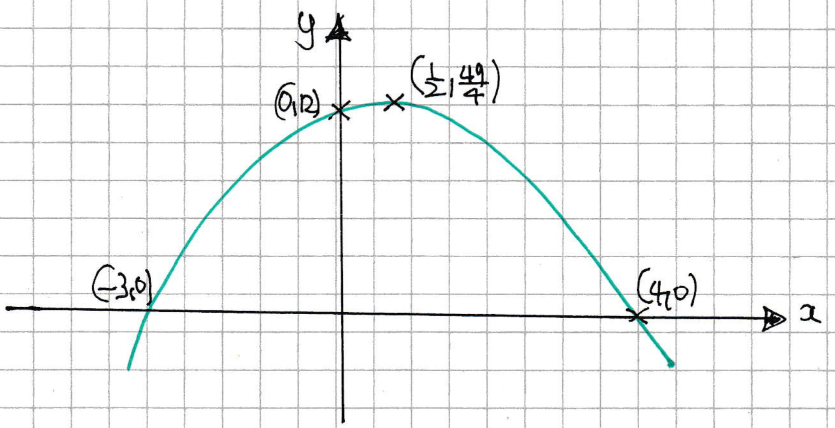
$x = -1$

IYGB - MP1 PAPER D - QUESTION 3

a) COLLECTING THE INFORMATION

- $-x^2 \Rightarrow \frown$
- $x=0 \Rightarrow y=12$ $\quad(0,12)$
- $y=0 \Rightarrow x = 4, -3$ $\quad(4,0)$, $(-3,0)$
- LINE OF SYMMETRY, BETWEEN -3 & 4, IT $\frac{1}{2}$

$$x=\frac{1}{2} \Rightarrow y=\left(4-\frac{1}{2}\right)\left(\frac{1}{2}+3\right)=\frac{7}{2}\times\frac{7}{2}=\frac{49}{4} \Rightarrow \left(\frac{1}{2},\frac{49}{4}\right)$$



b) $y=f(kx)$ IS A "HORIZONTAL STRETCH" BY SCALE FACTOR $\frac{1}{k}$

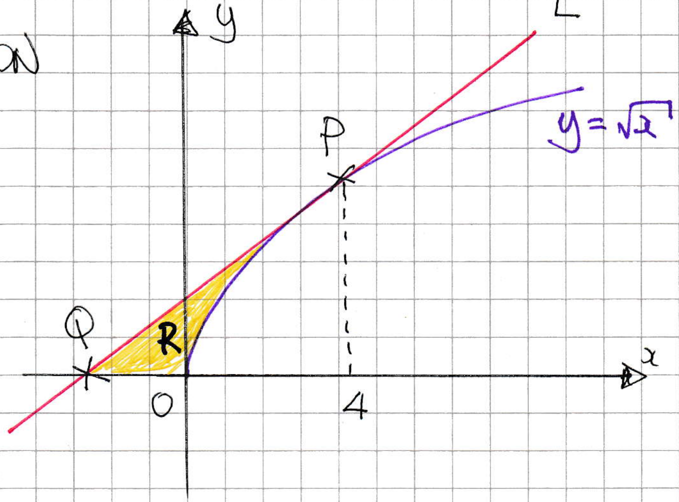
$(4,0) \longmapsto (1,0)$ $\quad f(4x)$ $\quad \therefore k=4$

$(-3,0) \longmapsto (3,0)$ $\quad f(-x)$

$(3,0) \longmapsto (1,0)$ $\quad f(-3x)$ $\quad \therefore k=-3$

IYGB - MP1 PAPER D - QUESTION 4

a) FIND THE GRADIENT FUNCTION



$$y = x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

EQUATION OF THE TANGENT AT $P(4,2)$

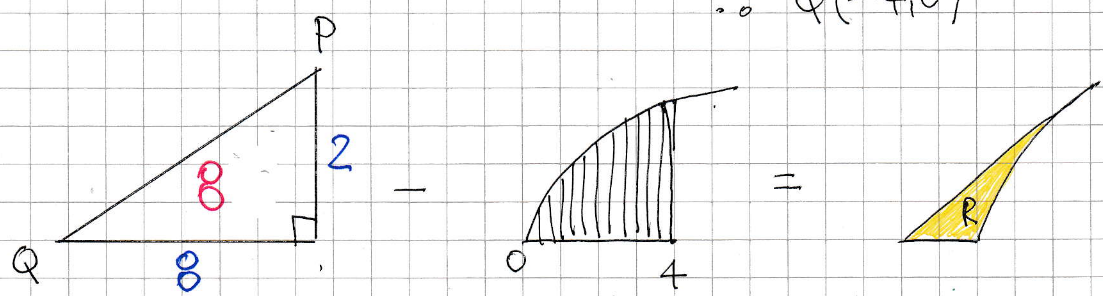
$$y - y_0 = m(x - x_0)$$
$$y - 2 = \frac{1}{4}(x-4)$$
$$4y - 8 = x - 4$$
$$4y = x + 4$$

b) START BY FINDING THE CO-ORDINATES OF Q

$y=0$ IN L $\Longrightarrow$ $0 = x+4$

$\Longrightarrow$ $x = -4$

$\therefore$ $Q(-4,0)$



AREA $= \frac{1}{2} \times 8 \times 2$

$$\int_0^4 x^{\frac{1}{2}}\,dx$$

IYGB — MP1 PAPER D — QUESTION 4

$$= \int_0^4 x^{\frac{1}{2}} \, dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^4$$

$$= \left(\frac{2}{3} \times 8 \right) - (0) = \frac{16}{3}$$

REQUIRED AREA $= 8 - \frac{16}{3} = \frac{8}{3}$

IYGB - MP1 PAPER D - QUESTION 5

PROCEED AS FOLLOWS

$$(1-2x)^2(2+kx)^4 \equiv A + Bx - 104x^2 + \dots$$

$$(1-4x+4x^2)\left[\binom{4}{0}2^4(kx)^0 + \binom{4}{1}2^3(kx)^1 + \binom{4}{2}2^2(kx)^2 + \dots\right] \equiv A + Bx - 104x^2$$

$$(1-4x+4x^2)(16 + 32kx + 24k^2x^2 + \dots) \equiv A + Bx - 104x^2 + \dots$$

MULTIPLY OUT UP TO x^2

$$\left.\begin{array}{r} 16 + 32kx + 24k^2x^2 + \dots \\ -64x - 128kx^2 + \dots \\ 64x^2 + \dots \end{array}\right\} \equiv A + Bx - 104x^2 + \dots$$

$$16 + (32k-64)x + (24k^2 - 128k + 64)x^2 \equiv A + Bx - 104x^2 + \dots$$

$\therefore A = 16$ (NOT NEEDED)

$\Rightarrow 24k^2 - 128k + 64 = -104$

$\Rightarrow 24k^2 - 128k + 168 = 0$

$\Rightarrow 3k^2 - 16k + 21 = 0$

$\Rightarrow (3k-7)(k-3) = 0$

$k = \begin{cases} 3 \\ 7/3 \end{cases}$

$32k - 64 = B$

$B = \begin{cases} 32 \times 3 - 64 = 32 \\ 32 \times \frac{7}{3} - 64 = \frac{32}{3} \end{cases}$

IYGB - MP1 PAPER D - QUESTION 6

$H = 25 - 24e^{-0.1t}$, $t \geq 0$

H = HEIGHT IN METRES

t = TIME (YEARS) SINCE PLANTING

a) WHEN $t=0$

$H = 25 - 24e^{-0.1 \times 0}$

$H = 25 - 24e^{0}$

$H = 25 - 24$

$H = 1$

IE 1 metre

b) WHEN $t=2$

$H = 25 - 24e^{-0.1 \times 2}$

$H = 25 - 24e^{-0.2}$

$H = 25 - 19.6495..$

$H \simeq 5.35\text{m}$

c) "EVENTUAL" HEIGHT MAYBE ASSOCIATED WITH ITS "FOREVER" HEIGHT, "ADULT HEIGHT", MAXIMUM HEIGHT ETC

As $t \to \infty$ $\quad e^{-0.1t} \to 0$

$24e^{-0.1t} \to 0$

$H \to 25$ ← MAXIMUM HEIGHT

80% OF 25 IS 20 METRES

$\Rightarrow 20 = 25 - 24e^{-0.1t}$

$\Rightarrow 24e^{-0.1t} = 5$

$\Rightarrow e^{-0.1t} = \frac{5}{24}$

$\Rightarrow e^{0.1t} = 24/5$

$\Rightarrow 0.1t = \ln(4.8)$

$\Rightarrow t = 10\ln(4.8)$

$\Rightarrow t \simeq 15.69$

IE 16 YEARS

IYGB - MP1 PAPER D - QUESTION 7

$$\boxed{y = \sqrt[3]{x^1} + \frac{27}{x} \qquad x \neq 0}$$

START BY WRITING IN INDEX NOTATION, BEFORE DIFFERENTIATING

$$y = x^{\frac{1}{3}} + 27x^{-1}$$

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - 27x^{-2}$$

INCREASING $\Rightarrow \frac{dy}{dx} > 0$

$$\Rightarrow \frac{1}{3}x^{-\frac{2}{3}} - 27x^{-2} > 0$$

$$\Rightarrow \frac{1}{3x^{\frac{2}{3}}} - \frac{27}{x^2} > 0$$

$$\Rightarrow \frac{1}{3x^{\frac{2}{3}}} > \frac{27}{x^2}$$

$(x^2 > 0)$

$$\Rightarrow \frac{x^2}{3x^{\frac{2}{3}}} > 27$$

$$\Rightarrow x^{\frac{4}{3}} > 81$$

$$\Rightarrow \left(x^{\frac{4}{3}}\right)^{\frac{3}{4}} > 81^{\frac{3}{4}}$$

$$\Rightarrow x^1 > \left(\sqrt[4]{81}\right)^3$$

$$\Rightarrow x > 27$$

IYGB - MP1 PAPER D - QUESTION 8

LET $f(x) = \dfrac{1}{x^2}$

$$\Rightarrow f(x+h) = \frac{1}{(x+h)^2}$$

WE REQUIRE FIRST

$$f(x+h) - f(x) = \frac{1}{(x+h)^2} - \frac{1}{x^2} = \frac{x^2-(x+h)^2}{x^2(x+h)^2}$$

$$= \frac{x^2-(x^2+2xh+h^2)}{x^2(x^2+2xh+h^2)} = \frac{-2xh-h^2}{x^4+2x^3h+x^2h^2}$$

USING THE FORMAL DEFINITION OF THE DERIVATIVE

$$\Rightarrow f'(x) = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$\Rightarrow f'(x) = \lim_{h \to 0}\left[\frac{\frac{-2xh-h^2}{x^4+2x^3h+x^2h^2}}{h}\right]$$

$$\Rightarrow f'(x) = \lim_{h \to 0}\left[\frac{-2xh-h^2}{x^4-2x^3h+x^2h^2} \div h\right]$$

$$\Rightarrow f'(x) = \lim_{h \to 0}\left[\frac{-\cancel{h}(2x+h)}{x^4-2x^3h+x^2h^2} \times \frac{1}{\cancel{h}}\right]$$

$$\Rightarrow f'(x) = \lim_{h \to 0}\left[\frac{-(2x+h)}{x^4-2x^3h+x^2h^2}\right]$$

TAKING THE LIMIT AS $h \to 0$

$$\Rightarrow f'(x) = -\frac{2x+\cancel{h}}{x^4\cancel{-2x^3h}+\cancel{x^2h^2}} = -\frac{2x}{x^4} = -\frac{2}{x^3}$$

AS REQUIRED

IYGB − MP1 PAPER D − QUESTION 9

START BY MULTIPLYING BY x

$$\Rightarrow x - \frac{14}{x} = 6\sqrt{2}$$

$$\Rightarrow x^2 - 14 = 6\sqrt{2}x$$

$$\Rightarrow x^2 - 6\sqrt{2}x - 14 = 0$$

BY THE QUADRATIC FORMULA OR BY COMPLETING THE SQUARE

$$\Rightarrow x = \frac{6\sqrt{2} \pm \sqrt{(6\sqrt{2})^2 - 4\times1\times(-14)}}{2\times1}$$

$$\Rightarrow x = \frac{6\sqrt{2} \pm \sqrt{72+56}}{2}$$

$$\Rightarrow x = \frac{6\sqrt{2} \pm \sqrt{128}}{2}$$

$$\Rightarrow x = \frac{6\sqrt{2} \pm 8\sqrt{2}}{2} = \begin{cases} \frac{6\sqrt{2}+8\sqrt{2}}{2} = 7\sqrt{2} \\ \frac{6\sqrt{2}-8\sqrt{2}}{2} = -\sqrt{2} \end{cases}$$

OR BY COMPLETING THE SQUARE

$$\Rightarrow x^2 - 6\sqrt{2}x - 14 = 0$$

$$\Rightarrow (x - 3\sqrt{2})^2 - (3\sqrt{2})^2 - 14 = 0$$

$$\Rightarrow (x - 3\sqrt{2})^2 - 18 - 14 = 0$$

$$\Rightarrow (x - 3\sqrt{2})^2 = 32$$

$$\Rightarrow x - 3\sqrt{2} = \begin{cases} \sqrt{32} = 4\sqrt{2} \\ -\sqrt{32} = -4\sqrt{2} \end{cases}$$

$$\Rightarrow x = \begin{cases} 7\sqrt{2} \\ -\sqrt{2} \end{cases}$$

IYGB – MP1 PAPER D – QUESTION 10

AS $A(-3,0)$, $x+3$ MUST BE ONE OF THE FACTORS

$$\Rightarrow f(x) = -x^3+5x^2+17x-21$$
$$\Rightarrow -f(x) = x^3-5x^2-17x+21$$

BY LONG DIVISION OR MANIPULATIONS

$$\Rightarrow -f(x) = x^2(x+3) - 8x(x+3) + 7(x+3)$$
$$\Rightarrow -f(x) = (x+3)(x^2-8x+7)$$
$$\Rightarrow -f(x) = (x+3)(x-1)(x-7)$$
$$\Rightarrow f(x) = (x+3)(1-x)(x-7)$$

$A(-3,0)$ $B(1,0)$ $C(7,0)$

AND WHEN $x=0$ $y=-21$, IE $D(0,-21)$

$\therefore A(-3,0), B(1,0), C(7,0), D(0,-21)$

IYGB - MP1 PAPER D - QUESTION 11

a) FACTORIZE BY INSPECTION

$$2CT - 2C + T - 1 = 2C(T-1) + (T-1)$$
$$= (T-1)(2C+1)$$

b) USING THE RESULT OF PART (a)

$$2\cos\theta\tan\theta - 2\cos\theta + \tan\theta = 1$$
$$2\cos\theta\tan\theta - 2\cos\theta + \tan\theta - 1 = 0$$
$$2\cos\theta(\tan\theta - 1) + (\tan\theta - 1) = 0$$
$$(\tan\theta - 1)(2\cos\theta + 1) = 0$$

EITHER $\tan\theta = 1$

$\arctan(1) = 45°$

$\left(\begin{array}{l} \theta = 45 \pm 180n \quad n = 0,1,2,3\ldots \\ \theta = \text{N/A} \end{array}\right.$

OR $\cos\theta = \frac{1}{2}$

$\arccos(-\frac{1}{2}) = 120°$

$\left(\begin{array}{l} \theta = 120 \pm 360n \quad n = 0,1,2,3\ldots \\ \theta = 240 \pm 360n \end{array}\right.$

COLLECTING THE RESULTS

$$\theta = 45°, 225°, 120°, 240°$$

IYGB - MP1 PAPER D - QUESTION 12

$$\left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)=1 \qquad xy+2=0$$

a) REARRANGE THE SECOND EQUATION & SUBSTITUTE INTO THE FIRST

$\Rightarrow xy = -2$

$\Rightarrow y = -\frac{2}{x}$

$\Rightarrow \frac{1}{y} = -\frac{x}{2} \longrightarrow \left(\frac{1}{x}+1\right)\left(-\frac{x}{2}+1\right)=1$

$\Rightarrow -\frac{1}{2}+\frac{1}{x}-\frac{x}{2}\cancel{+1}=\cancel{1}$

$\Rightarrow -\frac{1}{2}+\frac{1}{x}-\frac{x}{2}=0 \quad)\times 2$

$\Rightarrow -1+\frac{2}{x}-\frac{x}{2}=0 \quad)\times x$

$\Rightarrow -x+2-x^2=0$

$\Rightarrow 0=x^2+x-2$

$\Rightarrow 0=(x+2)(x-1)$

$\Rightarrow x = \begin{cases}-2\\1\end{cases} \quad y = \begin{cases}1\\-2\end{cases}$

$\therefore (-2,1)$ & $(1,-2)$

b) • $C: xy+2=0$

$xy=-2$

$y=-\frac{2}{x}$

• $L: \left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)=1$

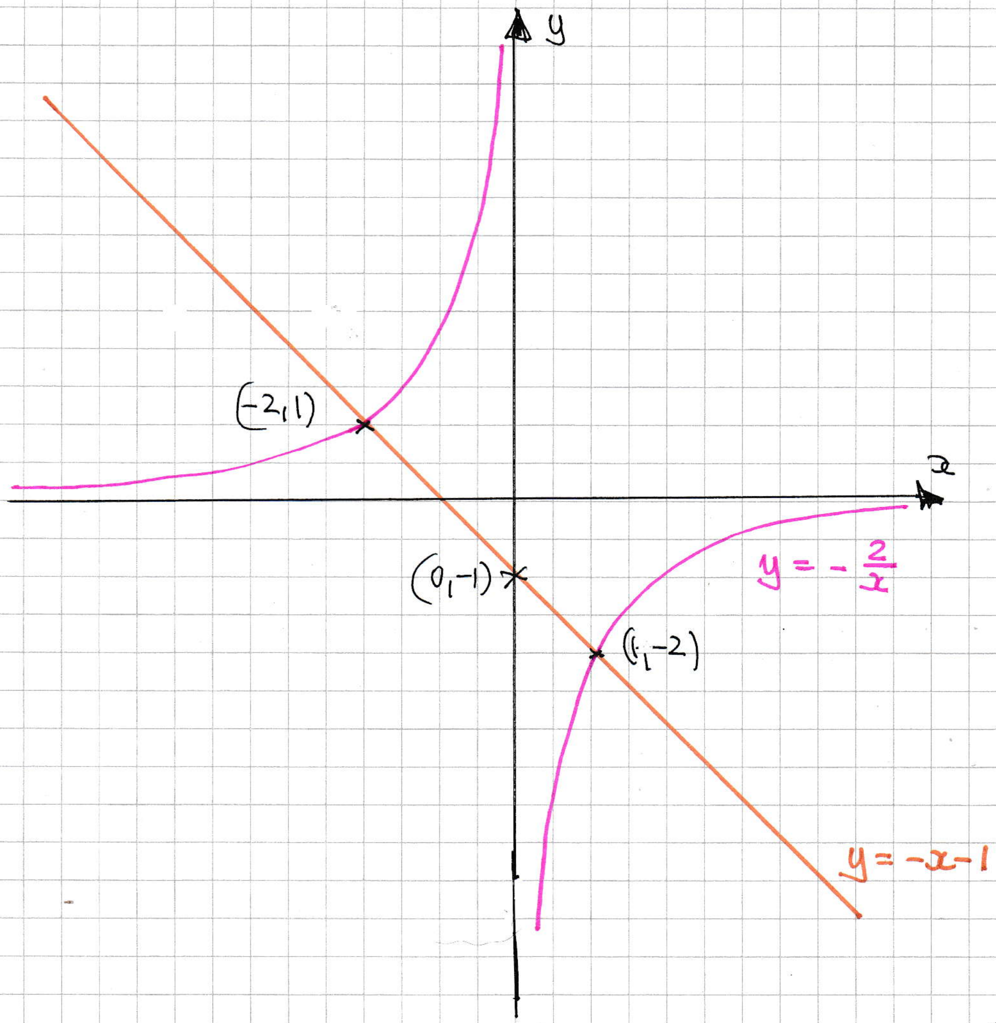
$\frac{1}{xy}+\frac{1}{x}+\frac{1}{y}\cancel{+1}=\cancel{1}$

$\frac{1}{xy}+\frac{1}{x}+\frac{1}{y}=0 \quad)\times xy$

$1+y+x=0$

IYGB - MP1 PAPER D - QUESTION 12

SKETCHING $y=-x-1$ & $y=-\frac{2}{x}$



IYGB - MP1 PAPER D - QUESTION 13

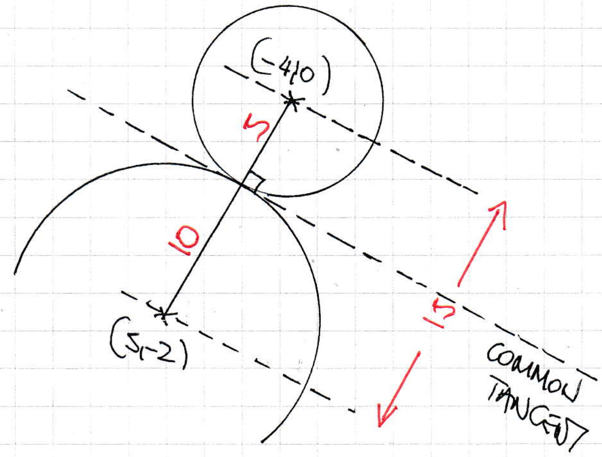
a) COMPLETE THE SQUARE

$$\Longrightarrow x^2+y^2-10x+4y=71$$
$$\Longrightarrow x^2-10x+y^2+4y=71$$
$$\Longrightarrow (x-5)^2-25+(y+2)^2-4=71$$
$$\Longrightarrow (x-5)^2+(y+2)^2=100$$

CENTRE $(5,-2)$, RADIUS 10

b) LOOKING AT THE DIAGRAM

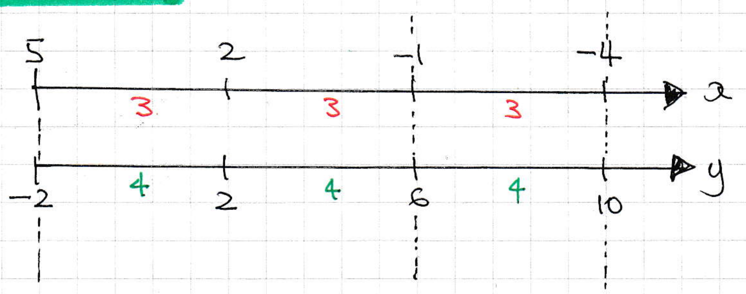
$(-4,10)$ & $(5,-2)$



$$d=\sqrt{(-4-5)^2+(10+2)^2}$$
$$d=\sqrt{81+144}$$
$$d=15$$

$\therefore$ CIRCLES TOUCH EACH OTHER EXTERNALLY AS THE SUM OF THE RADII EQUALS THE DISTANCE BETWEEN THE CENTRES

c) BY INSPECTION



$P(-1,6)$

IYGB – MP1 PAPER D – QUESTION 13

d) GRADIENT OF THE LINE JOINING THE CENTRES $(5,-2)$ & $(-4,10)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-(-2)}{-4-5} = \frac{12}{-9} = -\frac{4}{3}$$

REQUIRED GRADIENT IS $\frac{3}{4}$

USING $(-1,6)$

$$y - y_0 = m(x - x_0)$$
$$y - 6 = \tfrac{3}{4}(x+1)$$
$$4y - 24 = 3x + 3$$
$$4y = 3x + 27$$

AS REQUIRED

SEE DIAGRAM FROM EARLIER

ALTERNATIVE – NOT SENSIBLE HERE – FOR PART (c)(d)

$$\left.\begin{aligned} x^2+y^2-10x+4y &= 71 \\ (x+4)^2+(y-10)^2 &= 25 \end{aligned}\right\} \Rightarrow \begin{aligned} x^2+y^2-10x+4y &= 71 \\ x^2+y^2+8x-20y &= -91 \end{aligned}$$

⇓ SUBTRACT

$$-18x + 24y = 162$$
$$-3x + 4y = 27$$
$$4y = 3x + 27 \leftarrow \text{PART d}$$

AND TO FIND P (PART c)

$$\Rightarrow (x+4)^2 + (y-10)^2 = 25$$
$$\Rightarrow 16(x+4)^2 + 16(y-10)^2 = 400$$
$$\Rightarrow 16(x+4)^2 + (4y-40)^2 = 400$$
$$\Rightarrow 16(x+4)^2 + (3x+27-40)^2 = 400$$
$$\Rightarrow 16(x+4)^2 + (3x-13)^2 = 400$$
$$\Rightarrow \begin{pmatrix} 16x^2+128x+256 \\ 9x^2-78x+169 \end{pmatrix} = 400$$

$$\Rightarrow 25x^2 + 50x + 25 = 0$$
$$\Rightarrow x^2 + 2x + 1 = 0$$
$$\Rightarrow (x+1)^2 = 0$$
$$\Rightarrow x = -1$$
$$y = 6 \quad \swarrow \; 4y = 3x+27$$

$$\therefore P(-1,6)$$