IYGB - MP1 PAPER A - QUESTION 1

FIND THE EQUATION OF l_1, THROUGH $A(3,20)$ & $B(13,0)$

$$\text{GRAD } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-20}{13-3} = \frac{-20}{10} = -2$$

EQUATION OF l_1:

$$y - y_0 = m(x - x_0)$$
$$y - 0 = -2(x - 13)$$
$$y = -2x + 26$$

THE EQUATION OF l_2 (USING $y = mx + c$) IS GIVEN BY

$$y = \frac{1}{3}x + 5$$

SOLVING SIMULTANEOUSLY l_1 & l_2 TO FIND D

$$\left.\begin{aligned} y &= -2x + 26 \\ y &= \tfrac{1}{3}x + 5 \end{aligned}\right\} \Longrightarrow \tfrac{1}{3}x + 5 = -2x + 26$$
$$\Longrightarrow x + 15 = -6x + 78$$
$$\Longrightarrow 7x = 63$$
$$\Longrightarrow x = 9$$
$$\Longrightarrow y = 8$$

$$\therefore D(9,8)$$

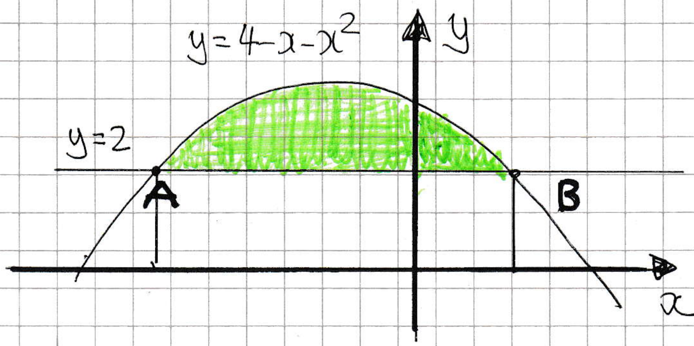
FINALLY THE DISTANCE AD CAN BE FOUND

$$\Rightarrow d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\Rightarrow |AD| = \sqrt{(3-9)^2 + (20-8)^2}$$
$$\Rightarrow |AD| = \sqrt{36 + 144} = \sqrt{180} = \sqrt{36}\sqrt{5} = 6\sqrt{5}$$

$$\therefore k = 6$$

IYGB – MP1 PAPER A – QUESTION 2

a) START BY FINDING A & B



$$\left.\begin{aligned} y &= 4-x-x^2 \\ y &= 2 \end{aligned}\right\} \Rightarrow$$

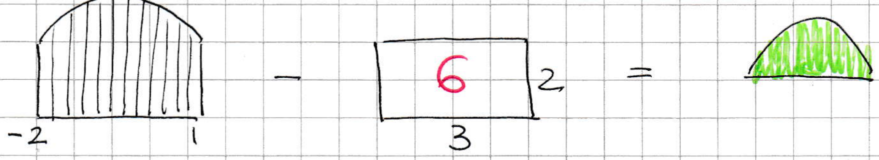
$$2 = 4-x-x^2$$

$$x^2+x-2=0$$

$$(x-1)(x+2)$$

$$\therefore\ A(-2,2) \qquad B(1,2)$$

b)



$$\int_{-2}^{1} 4-x-x^2\,dx = \left[4x-\tfrac{1}{2}x^2-\tfrac{1}{3}x^3\right]_{-2}^{1}$$

$$= \left(4-\tfrac{1}{2}-\tfrac{1}{3}\right)-\left(-8-2+\tfrac{8}{3}\right)$$

$$= \frac{19}{6}-\left(-\frac{22}{3}\right)$$

$$= \frac{21}{2}$$

$$\therefore\ \text{REQUIRED AREA} = \frac{21}{2}-6 = \frac{9}{2}$$

IYGB – MP1 PAPER A – QUESTION 3

a) $x^2 - 8x + y^2 - 2y = 0$

COMPLETING THE SQUARE

$(x-4)^2 - 16 + (y-1)^2 - 1 = 0$

$(x-4)^2 + (y-1)^2 = 17$

$\therefore$ $P(4,1)$ & RADIUS $= \sqrt{17}$

b) WHEN $x=0$

$y^2 - 2y = 0$

$y(y-2) = 0$

$\therefore$ $(0,0)$ & $(0,2)$

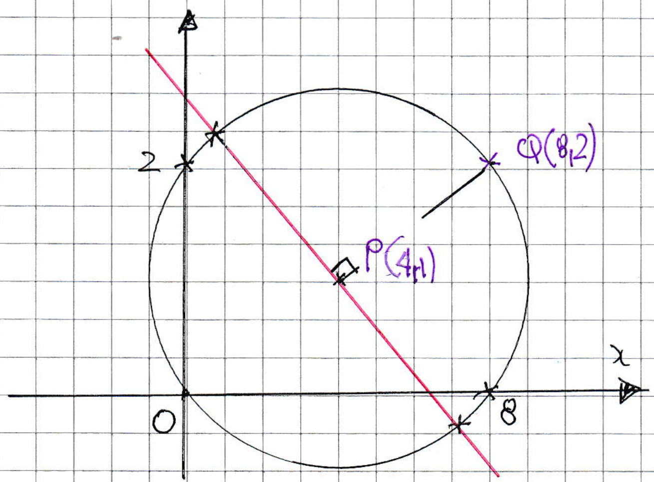
WHEN $y=0$

$x^2 - 8x = 0$

$x(x-8) = 0$

$\therefore$ $(0,0)$ & $(8,0)$

c) LOOKING AT THE DIAGRAM BELOW



- GRADIENT $PQ = \dfrac{2-1}{8-4} = \dfrac{1}{4}$
- GRADIENT $AB = -4$
- EQUATION OF LINE THROUGH A & B

$y - 1 = -4(x-4)$

$y - 1 = -4x + 16$

$y = 17 - 4x$

- SOLVING SIMULTANEOUSLY

$(x-4)^2 + (y-1)^2 = 17$

$(x-4)^2 + (17-4x-1)^2 = 17$

$(x-4)^2 + (16-4x)^2 = 17$

IYGB - MP1 PAPER A - QUESTION 3

$\Longrightarrow \quad x^2 - 8x + 16 + 256 - 128x + 16x^2 = 17$

$\Longrightarrow \quad 17x^2 - 136x + 255 = 0$

$\Longrightarrow \quad x^2 - 8x + 15 = 0$

$\Longrightarrow \quad (x-3)(x-5) = 0$

$x = \begin{cases} 3 \\ 5 \end{cases} \qquad y = \begin{cases} 5 \\ -3 \end{cases}$

$\therefore \quad A(3,5) \; \& \; B(5,-3)$ (IN ANY ORDER)

IYGB – MP1 PAPER A – QUESTION 4

a) USING THE CONDITIONS GIVEN INTO $f(x) = 2x^3 + ax^2 + bx + c$

$f(2) = 0$

$16 + 4a + 2b + c = 0$

$4a + 2b + c = -16$

$f(-1) = 0$

$-2 + a - b + c = 0$

$a - b + c = 2$

$f(1) = -14$

$2 + a + b + c = -14$

$a + b + c = -16$

SUBTRACT

$-2b = 18$

$b = -9$

THUS WE NOW HAVE

$4a - 18 + c = -16$

$4a + c = 2$

$a + c = 2 + b$

$a + c = -7$

SUBTRACT

$3a = 9$

$a = 3$

& $3 + c = -7$

$c = -10$

b) USING THESE VALUES $f(x) = 2x^3 + 3x^2 - 9x - 10$

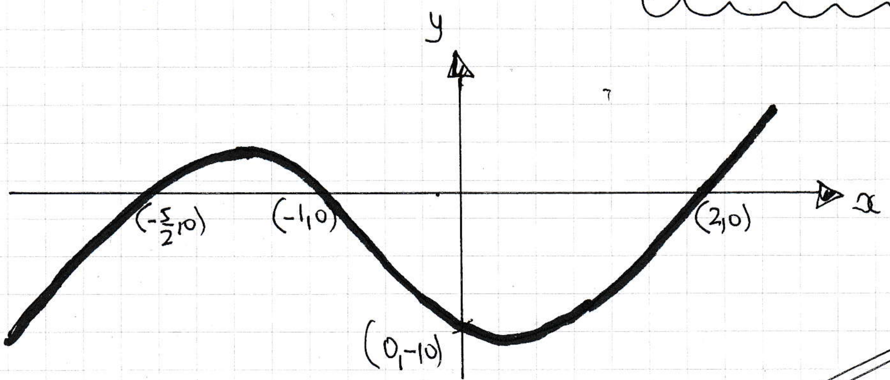
$f(x) = (x-2)(x+1)(2x+5)$

$f(2)=0$ $\quad f(-1)=0$ $\quad$ BY INSPECTION

$+2x^3 + \dots$

$y = 0 \quad x = 2, \ -1, \ -\frac{5}{2} \quad (2,0) \ (-1,0) \ (-\frac{5}{2},0)$

$x = 0 \quad y = -10 \quad (0,-10)$



IYGB – MP1 PAPER A – QUESTION 5

$$\frac{5\cos 2x + \sin 2x}{3\sin 2x} = 7 \qquad -90 \leqslant x \leqslant 90$$

PROCEED BY MULTIPLYING THE DENOMINATOR THROUGH

$$\Rightarrow 5\cos 2x + \sin 2x = 21\sin 2x$$
$$\Rightarrow 5\cos 2x = 20\sin 2x$$
$$\Rightarrow 5 = \frac{20\sin 2x}{\cos 2x}$$
$$\Rightarrow 5 = 20\tan 2x$$
$$\Rightarrow \tan 2x = \frac{1}{4}$$

$$\arctan\left(\tfrac{1}{4}\right) \simeq 14.036\ldots$$

$$\Rightarrow 2x = 14.036 \pm 180n \qquad n = 0, 1, 2, 3, \ldots$$
$$\Rightarrow x = 7.02^\circ \pm 90n.$$

ONLY SOLUTIONS IN RANGE ARE 7.0° & -83.0°

ALTERNATIVE APPROACH

$$\frac{5\cos 2x + \sin 2x}{3\sin 2x} = 7$$

$$\frac{5\cos 2x}{3\sin 2x} + \frac{\cancel{\sin 2x}}{3\cancel{\sin 2x}} = 7$$

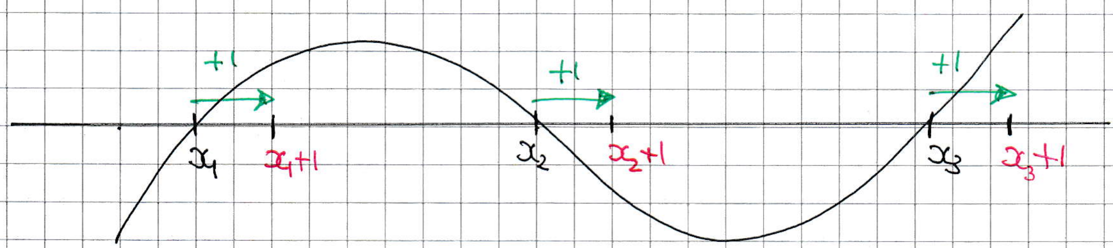
$$\frac{5}{3}\left(\frac{1}{\tan 2x}\right) + \frac{1}{3} = 7$$

$$\frac{5}{3}\left(\frac{1}{\tan 2x}\right) = \frac{20}{3}$$

$$\frac{1}{\tan 2x} = 4 \qquad \therefore \tan 2x = \frac{1}{4} \text{ ETC.}$$

IYGB - MP1 PAPER A - QUESTION 6

THERE IS NO NEED TO FIND THE ROOTS OF THE CUBIC



LOOKING AT THE ABOVE GRAPH

$$f(x) = x^3 - 4x + 1$$

TRANSLATION "1 UNIT TO THE RIGHT"

$$f(x-1) = (x-1)^3 - 4(x-1) + 1$$

$$f(x-1) = (x-1)(x-1)^2 - 4x + 4 + 1$$

$$f(x-1) = (x-1)(x^2-2x+1) - 4x + 5$$

$$f(x-1) = \begin{array}{l} x^3 - 2x^2 + x \\ \quad -x^2 + 2x - 1 \end{array} - 4x + 5$$

$$f(x-1) = x^3 - 3x^2 + 3x - 1 - 4x + 5$$

$$f(x-1) = x^3 - 3x^2 - x + 4$$

$$\therefore \quad x^3 - 3x^2 - x + 4 = 0$$

IYGB - MP1 PAPER A - QUESTION 7

a) START BY OBTAINING THE GRADIENT FUNCTION

$$\Rightarrow y = 4x^3 + 7x^2 + x + 11$$

$$\Rightarrow \frac{dy}{dx} = 12x^2 + 14x + 1$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=-1} = 12(-1)^2 + 14(-1) + 1 = -1$$

OBTAIN THE FULL CO-ORDINATES OF P

$$y = 4(-1)^3 + 7(-1)^2 + (-1) + 11 = -4 + 7 - 1 + 11 = 13$$

IE $P(-1, 13)$

EQUATION OF TANGENT

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 13 = -1(x + 1)$$

$$\Rightarrow y - 13 = -x - 1$$

$$\Rightarrow x + y = 12 \quad //$$

b) SOLVING SIMULTANEOUSLY THE EQUATION OF THE TANGENT AND THE EQUATION OF THE CURVE

$$\left.\begin{array}{l} y = 4x^3 + 7x^2 + x + 11 \\ y = 12 - x \end{array}\right\} \Rightarrow 4x^3 + 7x^2 + x + 11 = 12 - x$$

$$\Rightarrow 4x^3 + 7x^2 + 2x - 1 = 0$$

$$\Rightarrow (x+1)^2(4x-1) = 0$$

THIS IS THE POINT OF TANGENCY P, APPEARING AS REPEATED ROOT

$$\Rightarrow \text{Q HAS } x \text{ CO-ORDINATE } \frac{1}{4} \quad //$$

IYGB - MP1 PAPER A - QUESTION 8

$$f(x) \equiv 12x^2 + 4x - 161 \quad , \quad x \in \mathbb{R}$$

CALCULATE THE DISCRIMINANT

$$\Delta = b^2 - 4ac = 4^2 - 4 \times 12 \times (-161)$$
$$= 16 + 7728$$
$$= 7744$$

NOW $\sqrt{\Delta} = \sqrt{7744} = 88$

BY THE QUADRATIC FORMULA, THE EQUATION $f(x)=0$ HAS TWO REAL SOLUTIONS GIVEN BY

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-4 \pm 88}{2 \times 12} = \begin{cases} -\frac{23}{6} \\ \frac{7}{2} \end{cases}$$

THUS WE HAVE

$$x = -\frac{23}{6} \qquad \text{OR} \qquad x = \frac{7}{2}$$
$$6x = -23 \qquad\qquad 2x = 7$$
$$6x + 23 = 0 \qquad\qquad 2x - 7 = 0$$

$$\therefore f(x) = (6x+23)(2x-7)$$

IYGB – MP1 PAPER A – QUESTION 9

EXPAND AS A BINOMIAL

$$(2+x-x^2)^5 = [2+(x-x^2)]^5$$

$$= \binom{5}{0}2^5(x-x^2)^0 + \binom{5}{1}2^4(x-x^2)^1 + \binom{5}{2}2^3(x-x^2)^2 + \binom{5}{3}2^2(x-x^2)^3 + \dots$$

$$= 1\times32\times1 + 5\times16(x-x^2) + 10\times8(x^2-2x^3+\dots) + 10\times4(x^3-\dots)$$

$$= 32 + 80x - \cancel{80x^2}$$
$$\cancel{80x^2} - 160x^3 + \dots$$
$$40x^3 + \dots$$

$$= 32 + 80x - 120x^3$$

IYGB - MP1 PAPER A - QUESTION 10

a)

$$f(x) = x^4 - 4x$$

$$f(2+h) - f(2) = \left[(2+h)^4 - 4(2+h)\right] - \left[2^4 - 4\times 2\right]$$

$$= (2+h)^4 - \cancel{8} - 4h - 16 \cancel{+8}$$

$$= (2+h)^4 - 4h - 16$$

$$= (2+h)^2(2+h)^2 - 4h - 16$$

$$= (4+4h+h^2)(4+4h+h^2) - 4h - 16$$

$$= \begin{array}{l} 16 + 16h + 4h^2 \\ \quad 16h + 16h^2 + 4h^3 \\ \qquad + 4h^2 + 4h^3 + h^4 \quad -4h - 16 \\ \hline \cancel{16} + 32h + 24h^2 + 8h^3 + h^4 - 4h \cancel{-16} \end{array}$$

$$= h^4 + 8h^3 + 24h^2 + 28h$$

b)

$$\boxed{f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]}$$

$$f'(2) = \lim_{h \to 0}\left[\frac{f(2+h) - f(2)}{h}\right]$$

$$f'(2) = \lim_{h \to 0}\left[\frac{h^4 + 8h^3 + 24h^2 + 28h}{h}\right]$$

$$f'(2) = \lim_{h \to 0}\left[\cancel{h^3} + \cancel{8h^2} + \cancel{24h} + 28\right]$$

$$f'(2) = 28$$

IYGB - MP1 PAPER A - QUESTION 11

AS WE ARE NOT ALLOWED CALCULATORS, TO TAKE LOGS, OBSERVE THAT THE EXPRESSION IS ALL POWERS OF 2

$$\Longrightarrow \frac{1}{2} \times 4^{2x} = 64^{64}$$

$$\Longrightarrow 2^{-1} \times (2^2)^{2x} = (2^6)^{64}$$

$$(a^m)^n = a^{m \times n}$$

$$\Longrightarrow 2^{-1} \times 2^{4x} = 2^{6 \times 64}$$

$$\Longrightarrow 2^{4x-1} = 2^{384}$$

- $6 \times 64 = 6 \times (60+4) = 360 + 24 = 384$

$$\Longrightarrow 4x - 1 = 384$$

$$\Longrightarrow 4x = 385$$

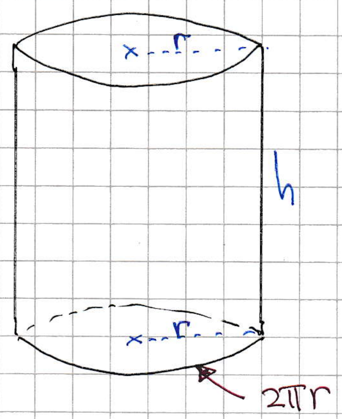
$$\Longrightarrow x = \frac{385}{4}$$

$$\Longrightarrow x = 96.25$$

- $\frac{385}{4} = \frac{360+24+1}{4} = \frac{360}{4} + \frac{24}{4} + \frac{1}{4} = 90 + 6 + \frac{1}{4} = 96.25$

IYGB - MP1 PAPER A - QUESTION 12

a)



CONSTRAINT ON THE VOLUME

$$V = 330$$
$$\pi r^2 h = 330$$
$$(\pi r h) r = 330$$
$$\pi r h = \frac{330}{r}$$
$$2\pi r h = \frac{660}{r}$$

$$A = \pi r^2 \times 2 + (2\pi r \times h)$$
$$A = 2\pi r^2 + 2\pi r h$$
$$A = 2\pi r^2 + \frac{660}{r}$$

// AS REQUIRED

b) DIFFERENTIATE & SOLVE FOR ZERO

$$A = 2\pi r^2 + 660 r^{-1}$$
$$\frac{dA}{dr} = 4\pi r - 660 r^{-2}$$
$$\frac{dA}{dr} = 4\pi r - \frac{660}{r^2}$$

FOR MIN/MAX $\frac{dA}{dr} = 0$

$$0 = 4\pi r - \frac{660}{r^2}$$
$$\frac{660}{r^2} = 4\pi r$$
$$660 = 4\pi r^3$$
$$r^3 = \frac{165}{\pi}$$
$$r = 3.745 \text{ cm}$$

//

IYGB – MP1 PAPER A – QUESTION 12

c) USING THE SECOND DERIVATIVE

$$\frac{dA}{dr} = 4\pi r - 660r^{-2}$$

$$\frac{d^2A}{dr^2} = 4\pi + 1320r^{-3}$$

$$\left.\frac{d^2A}{dr^2}\right|_{r=3.745\ldots} = 12\pi \simeq 37.7 > 0$$

INDEED $r = 3.745$ MINIMIZES A

d) FINALLY USING

$$A = 2\pi r^2 + \frac{660}{r}$$

$$A_{MIN} = 2\pi(3.745..)^2 + \frac{660}{3.745}$$

$$A_{MIN} \approx 264 \text{ cm}^2$$

IYGB - MP1 PAPER A - QUESTION 13

PROCEED AS FOLLOWS

$$\left\{\begin{array}{l}\log_2(y-1) = 1 + \log_2 x \\ 2\log_3 y = 2 + \log_3 x\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\log_2(y-1) = \log_2 2 + \log_2 x \\ \log_3 y^2 = 2\log_3 3 + \log_3 x\end{array}\right\}$$

$$\Rightarrow \left\{\begin{array}{l}\log_2(y-1) = \log_2(2x) \\ \log_3 y^2 = \log_3 9 + \log_3 x\end{array}\right\}$$

$$\Rightarrow \left\{\begin{array}{l}\log_2(y-1) = \log_2(2x) \\ \log_3 y^2 = \log_3(9x)\end{array}\right\}$$

EXTRACTING THE LOGARITHMS IN EACH EQUATION

$$\left.\begin{array}{l}y-1 = 2x \\ y^2 = 9x\end{array}\right\}$$ DIVIDE THE EQUATIONS SIDE BY SIDE

$$\Rightarrow \frac{y-1}{y^2} = \frac{2x}{9x}$$

$$\Rightarrow 2y^2 = 9y - 9$$

$$\Rightarrow 2y^2 - 9y + 9 = 0$$

$$\Rightarrow (2y-3)(y-3) = 0$$

$$\Rightarrow y = \left\langle\begin{array}{l}3 \\ \frac{3}{2}\end{array}\right.$$

$$\Rightarrow x = \left\langle\begin{array}{l}\frac{3-1}{2} = 1 \\ \frac{\frac{3}{2}-1}{2} = \frac{1}{4}\end{array}\right.$$

$\therefore$ $(1, 3)$ OR $\left(\frac{1}{4}, \frac{3}{2}\right)$

(BOTH ARE FINE)