IYGB - MMS PAPER J - QUESTION 1

MAX TEMPERATURE °C	10	12	14	16	18	20	22	24
AMOUNT OF POLLUTANT (in mg / litre)	513	475	525	530	516	520	507	521

a) FROM CALCULATOR IN STAT MODE

$r = 0.320$

b) UNCHANGED AT 0.320, AS THE P.M.C.C IS INDEPENDENT OF SCALING (OR CHANGE OF ORIGIN)

c) SETTING HYPOTHESES

- $H_0 : \rho = 0$
- $H_1 : \rho > 0$

WHERE ρ IS THE P.M.C.C OF ALL PAIRINGS OF TEMPERATURES & AMOUNT OF POLLUTANT (POPULATION)

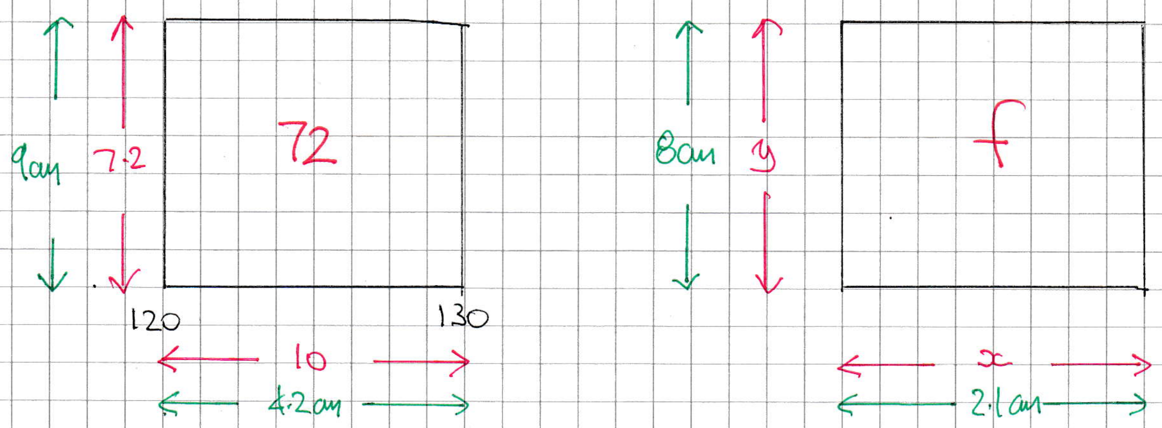
THE CRITICAL VALUE AT 10% SIGNIFICANCE & $n = 8$ IS 0.5067

AS $0.320 < 0.5067$ IT APPEARS THERE IS NO POSITIVE CORRELATION BETWEEN THE MAX DAILY TEMPERATURE & THE AMOUNT OF POLLUTANT

INSUFFICIENT EVIDENCE TO REJECT H_0

IYGB - MMS PAPER J - QUESTION 2

DRAWING TWO HISTOGRAM RECTANGLES (NOT TO SCALE)



BY RATIO/PROPORTION CONSIDERATIONS

- $\frac{9}{7.2} = \frac{8}{y}$

  $9y = 57.6$

  $y = 6.4$

- $\frac{10}{4.2} = \frac{x}{2.1}$

  $4.2x = 21$

  $x = 5$

- $f = xy$

  $f = 5 \times 6.4$

  $f = 32$

ALTERNATIVE APPROACH

- AREA OF 1^{ST} RECTANGLE $= 9\text{ cm} \times 4.2\text{ cm} = 37.8\text{ cm}^2$
- AREA OF 2^{ND} RECTANGLE $= 8\text{ cm} \times 2.1\text{ cm} = 16.8\text{ cm}^2$

$$\times\frac{4}{9}\left(\begin{array}{ccc} 37.8\text{ cm}^2 & : & 72 \\ 16.8\text{ cm}^2 & : & 32 \end{array}\right)\times\frac{4}{9}$$

IYGB - MMS PAPER J - QUESTION 3

a) LOOKING AT THE STEM & LEAF DIAGRAM

0	5	(1)
1	9 9	(2)
2	1 6 8	(3)
3	3 4 5 7	(4)
4	2 3 4 4 8 9 9	(7)
5	0 0 0 0 0	(5)

TOTAL OF 23 OBSERVATION

$Q_2 = \frac{1}{2}(23+1) = 12^{TH}$ OBS

$\therefore Q_2 = 43$

b) FIND THE QUARTILES

$Q_1 = \frac{1}{4}(23+1) = 6^{TH}$ OBS $\qquad \therefore Q_1 = 28$

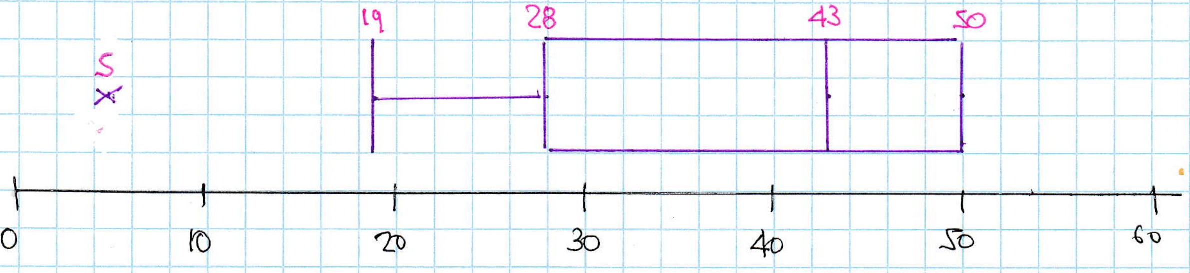
$Q_3 = \frac{3}{4}(23+1) = 18^{TH}$ OBS $\qquad \therefore Q_3 = 50$

$\therefore IQR = Q_3 - Q_1 = 50 - 28 = 22$

c) LOWER BOUND $= Q_1 - IQR = 28 - 22 = 6$

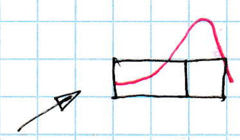
$\therefore$ ONLY ONE EMPLOYEE WITH A SCORE OF 5 WILL UNDERGO RETRAINING

d)



e) LOOKING AT THE BOX PLOT

$Q_3 - Q_2 < Q_2 - Q_1 \implies$ NEGATIVE SKEW



IYGB - MMS PAPER J - QUESTION 4

$P(A) = P(B)$ • $P(A' \cap B') = \frac{17}{24}$ • $P(A' \cup B') = \frac{19}{24}$

- USING $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Longrightarrow P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$$

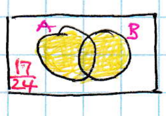
$$\Longrightarrow \frac{19}{24} = P(A') + P(A') - \frac{17}{24}$$

$$\Longrightarrow 2P(A') = \frac{36}{24}$$

$$\Longrightarrow P(A') = \frac{18}{24} = \frac{3}{4}$$

$$\Longrightarrow P(A) = \frac{1}{4} \quad \& \quad P(B) = \frac{1}{4}$$

- AS $P(A' \cap B') = \frac{17}{24} \Longrightarrow P(A \cup B) = \frac{7}{24}$



$$\Longrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Longrightarrow \frac{7}{24} = \frac{1}{4} + \frac{1}{4} - P(A \cap B)$$

$$\Longrightarrow P(A \cap B) = \frac{5}{24}$$

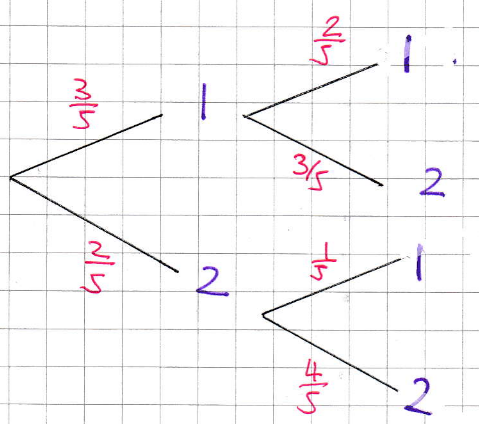
IYGB - MMS PAPER J - QUESTION 5

a) INITIAL CONFIGURATION

$\boxed{1 \cdot 1 \cdot 1 \cdot 2 \cdot 2}$ X

$\boxed{1 \cdot 2 \cdot 2 \cdot 2}$ Y

DRAWING A TREE DIAGRAM



P(1 pound coin, both times) $= \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

b) LOOKING AT THE INITIAL CONFIGURATION

P(both bags with £7 afterwards) = P(same coin both trips)

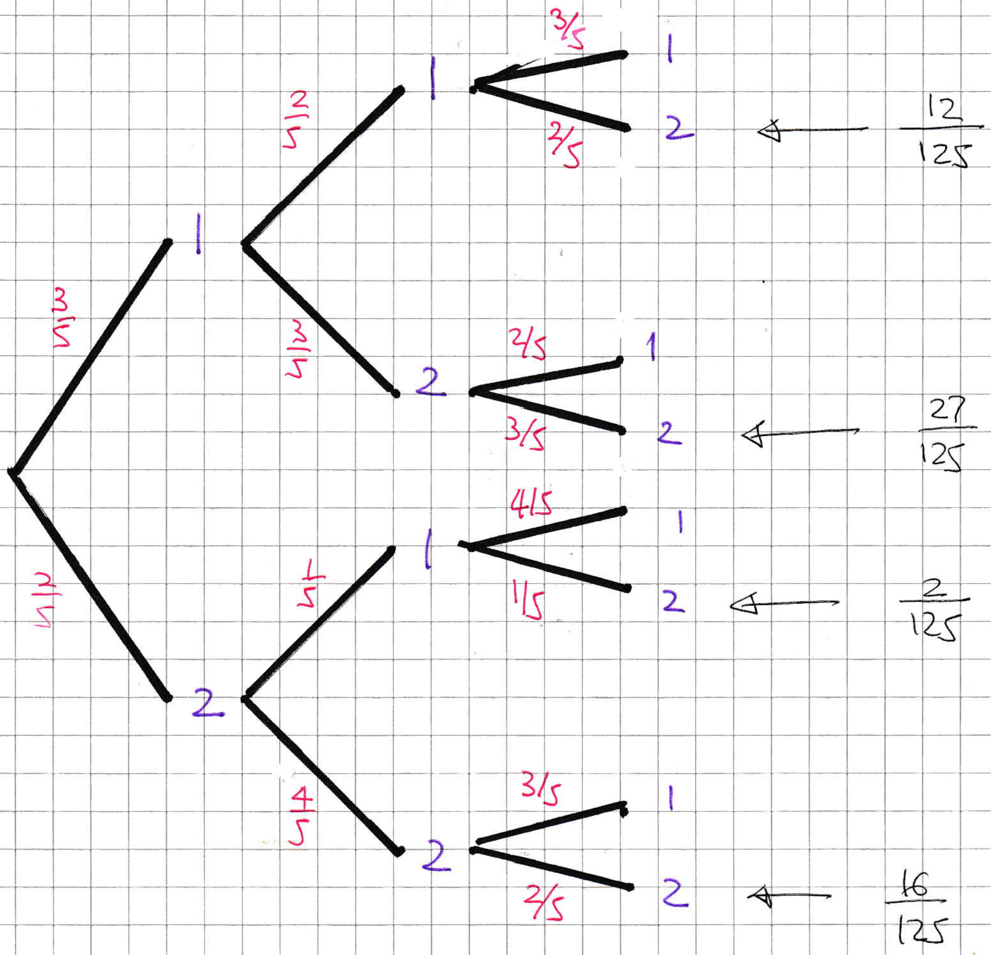
$$= \left(\frac{3}{5} \times \frac{2}{5}\right) + \left(\frac{2}{5} \times \frac{4}{5}\right)$$

$$= \frac{6}{25} + \frac{8}{25}$$

$$= \frac{14}{25}$$

IYGB – MMS PAPER J – QUESTION 5

c) EXTENDING THE TREE DIAGRAM



ADDING $\frac{57}{125}$

IYGB - MMS PAPER J - QUESTION 6

a) LOOKING AT THE TABLE, THE PROBABILITIES MUST BE POSITIVE OR ZERO

$\therefore \quad 0.4 - a \geq 0 \qquad 2a \geq 0 \qquad 0.6 - a \geq 0$

$\quad a \leq 0.4 \qquad a \geq 0 \qquad a \leq 0.6$

$\therefore \quad 0 \leq a \leq 0.4$

b) COLLECTING ALL THE OUTCOMES FOR $X_1 + X_2 = 6$

$$\left.\begin{array}{lll} 2,4 \Rightarrow (0.4-a)(0.6-a) & = & 0.24 - a + a^2 \\ 4,2 \Rightarrow (0.6-a)(0.4-a) & = & 0.24 - a + a^2 \\ 3,3 \Rightarrow 2a \times 2a & = & 4a^2 \end{array}\right\} \text{ADDING}$$

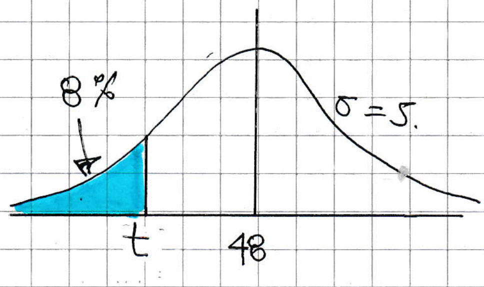
$\therefore \quad P(X_1 + X_2 = 6) = 6a^2 - 2a + 0.48$

IYGB - MMS PAPER J - QUESTION 7

a) PUTTING THE INFORMATION IN A DIAGRAM

T = TIME TO COMPLETE EXAM

$T \sim N(48, 5^2)$



$$\Rightarrow P(T < t) = 8\%$$

$$\Rightarrow P(T > t) = 92\%$$

$$\Rightarrow P\left(z > \frac{t-48}{5}\right) = 0.92$$

INVERTING

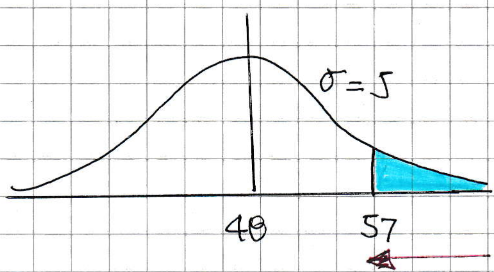
$$\Rightarrow \frac{t-48}{5} = -\Phi(0.92)$$

$$\Rightarrow \frac{t-48}{5} = -1.405$$

$$\Rightarrow t - 48 = -7.025$$

$$\Rightarrow t = 40.975 \approx 41$$

b) USING A NEW DIAGRAM



$P(T > 57)$

$= 1 - P(T < 57)$

$= 1 - P\left(z < \frac{57-48}{5}\right)$

$= 1 - \Phi(1.8)$

$= 1 - 0.9641$

$= 0.0359$

IYGB - MMS PAPER J - QUESTION 2

c) SETTING UP A BINOMIAL DISTRIBUTION

X = NUMBER OF STUDENTS WHICH TOOK OVER 57 MINUTES

$X \sim B(20, 0.0359)$

$$P(X>2) = P(X \geqslant 3) = 1 - P(X=0,1,2)$$

$$= 1 - \left[\binom{20}{0}(0.0359)^0(0.9641)^{20} + \binom{20}{1}(0.0359)^1(0.9641)^{19} + \binom{20}{2}(0.0359)^2(0.9641)^{18}\right]$$

$$= 1 - \left[0.481329 + 0.358463 + 0.126806\right]$$

$$= 0.0334$$

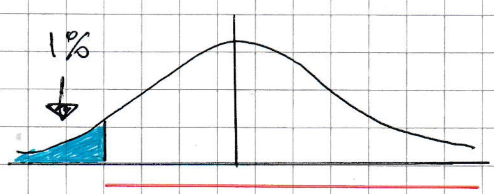
d) SETTING UP HYPOTHESES

- $H_0 : \mu = 48$
- $H_1 : \mu < 48$, WHERE μ REPRESENTS THE MEAN FINISHING TIME OF ALL TOP SET STUDENTS

GIVEN FURTHER

$\bar{x}_6 = 44$ $\quad n=6 \quad \sigma = 5 \quad$ 1% SIGNIFICANCE

OBTAIN THE CRITICAL VALUE & THE TEST STATISTIC



$$\Phi^{-1}(0.99) = -2.3263$$

$$Z\text{-STATISTIC} = \frac{\bar{x}_6 - \mu}{\sigma/\sqrt{n}} = \frac{44-48}{5/\sqrt{6}} = -1.9596$$

AS $-1.9596 > -2.3263$ THERE IS NO SIGNIFICANT EVIDENCE AT 1% SIGNIFICANCE TO SUPPORT THE CLAIM

NO SUFFICIENT EVIDENCE TO REJECT H_0

IYGB – MMS PAPER J – QUESTION 8

a) $X \sim B\left(6, \frac{1}{3}\right)$

$$P(X=2) = \binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4 = \frac{80}{243} \simeq 0.3292$$

b) WE REQUIRE $P(X_1 + X_2 < 2)$

$$P(X=0) \times P(X=0) = \left[\binom{6}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6\right]^2 \simeq 0.007707\ldots$$

$$\left.\begin{array}{l} P(X=0) \times P(X=1) \\ P(X=1) \times P(X=0) \end{array}\right\} = \left[\binom{6}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6\right] \times \left[\binom{6}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5\right] \times 2 \text{ WAYS}$$

$$\simeq 0.046244\ldots$$

$\therefore$ REQUIRED PROBABILITY IS $0.007707 + 0.046244 \simeq 0.0540$

c) $\boxed{\begin{array}{l} Y = \text{AN OBSERVATION OF 2 FROM } X \\ Y \sim B\left(8, \frac{80}{243}\right) \end{array}}$

$$P(Y=4) = \binom{8}{4}\left(\frac{80}{243}\right)^4\left(\frac{163}{243}\right) \simeq 0.1665$$

IYGB - MMS PAPER J - QUESTION 9

a) AN ENTIRE COLLECTION OF ITEMS (DATA) OF SOME STATISTICAL INTEREST IS CALLED A POPULATION //

b) A SURVEY WHICH TAKES INTO ACCOUNT A POPULATION IS CALLED A CENSUS //

c)
- WHEN THE POPULATION IS RELATIVELY SMALL
- WHEN BETTER ACCURACY IN REQUIRED
- WHEN "TIME CONSUMPTION" IS NOT A CONCERN
- WHEN RESOURCES IN DATA ANALYSING ARE PLENTIFUL
- WHEN RESPONSES IN QUESTIONS ARE LIKELY TO BE LOW

ETC //

IYGB - MMS PAPER J - QUESTION 10

a)
b) LOOKING AT THE JOURNEY A TO B

$u = 24 \text{ ms}^{-1}$
$a = ?$
$s = 476 \text{ m}$
$t = ?$
$v = 10 \text{ ms}^{-1}$

"$v^2 = u^2 + 2as$"

$10^2 = 24^2 + 2a(476)$

$100 = 576 + 952a$

$-952a = 476$

$a = -0.5$

I.E DECELERATION 0.5 ms^{-2}

"$v = u + at$"

$10 = 24 + (-0.5)t$

$0.5t = 14$

$t = 28 \text{ s}$

c)
d) LOOKING AT THE JOURNEY FROM B TO C

$u = 10 \text{ ms}^{-1}$
$a = ?$
$s = 855 \text{ m}$
$t = 45 \text{ s}$
$v = ?$

$s = ut + \frac{1}{2}at^2$

$855 = 10 \times 45 + \frac{1}{2} \times a \times 45^2$

$855 = 450 + \frac{2025}{2}a$

$405 = 1012.5a$

$a = 0.4 \text{ ms}^{-2}$

"$v = u + at$"

$v = 10 + 0.4 \times 45$

$v = 28 \text{ ms}^{-1}$

e) TO FIND THE AVERAGE SPEED FOR THE ENTIRE JOURNEY

$$\text{AVERAGE SPEED} = \frac{\text{TOTAL DISTANCE}}{\text{TOTAL TIME}}$$

$$= \frac{476 + 855}{28 + 45}$$

$$= 18.23 \text{ ms}^{-1}$$

IYGB - MMS PAPER J - QUESTION 11

a) $\underline{a} = (2t-4)\underline{i} + 3\underline{j}$

$\underline{a}_4 = (2\times4-4)\underline{i} + 3\underline{j}$

$\underline{a}_4 = 4\underline{i} + 3\underline{j}$

$|\underline{a}_4| = \sqrt{4^2+3^2}$

$|\underline{a}_4| = 5 \text{ ms}^{-2}$

USING $F = ma$

$F = 0.2 \times 5$

$F = 1\text{ N}$

b) INTEGRATE THE ACCELERATION VECTOR TO OBTAIN VELOCITY VECTOR

$\Rightarrow \underline{v} = \int (2t-4)\underline{i} + 3\underline{j} \, dt$

$\Rightarrow \underline{v} = (t^2-4t+A)\underline{i} + (3t+B)\underline{j}$

WHEN $t=0$ $\underline{v} = 3\underline{i} - 9\underline{j}$

$\therefore 3\underline{i} - 9\underline{j} = A\underline{i} + B\underline{j}$

$A = 3$

$B = -9$

$\therefore \underline{v} = (t^2-4t+3)\underline{i} + (3t-9)\underline{j}$

$\underline{v} = (t-3)(t-1)\underline{i} + 3(t-3)\underline{j}$

$\therefore$ BY INSPECTION $\underline{v} = 0$ WHEN $t = 3$

IYGB - MMS PAPER J - QUESTION 11

c) INTEGRATE AGAIN TO OBTAIN THE POSITION VECTOR

$$\underline{r} = \int (t^2-4t+3)\underline{i} + (3t-9)\underline{j} \, dt$$

$$\underline{r} = \left(\tfrac{1}{3}t^3-2t^2+3t+C\right)\underline{i} + \left(\tfrac{3}{2}t^2-9t+D\right)\underline{j}$$

WHEN $t=0$ $\underline{r} = -18\underline{i}-24\underline{j}$

$$\Rightarrow -18\underline{i}-24\underline{j} = C\underline{i}+D\underline{j}$$

$$C=-18$$
$$D=-24$$

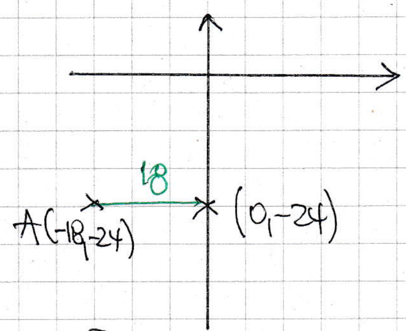
$$\therefore \underline{r} = \left(\tfrac{1}{3}t^3-2t^2+3t-18\right)\underline{i} + \left(\tfrac{3}{2}t^2-9t-24\right)\underline{j}$$

WHEN $t=6$

$$\underline{r} = (72-72+18-18)\underline{i} + (54-54-24)\underline{j} = -24\underline{j}$$

INDEED ON THE y AXIS

$\therefore$ DISTANCE FROM A IS 18 m



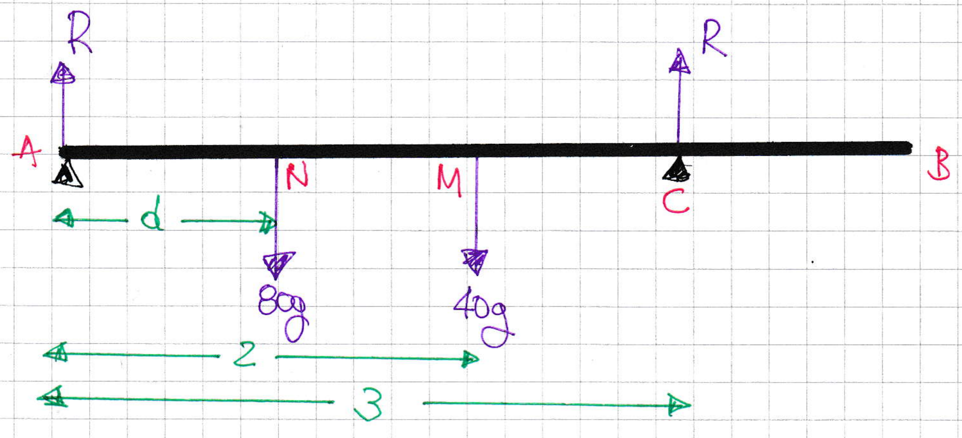
d) WHEN $y=0$, IE $\underline{j}$ COMPONENT ZERO IN $\underline{r}$

$$\Rightarrow \tfrac{3}{2}t^2-9t-24=0$$
$$\Rightarrow t^2-6t-16=0$$
$$\Rightarrow (t-8)(t+2)=0$$

$\therefore t = 8$ or $\cancel{-2}$

IYGB - MMS PAPER J - QUESTION 12

STARTING WITH A DIAGRAM



RESOLVING VERTICALLY

$R + R = 80g + 40g$

$2R = 120g$

$R = 60g$

TAKING MOMENTS ABOUT A

$\overset{\curvearrowright}{A}$: $80g \times d + 40g \times 2 = R \times 3$

$80gd + 80g = 180g$

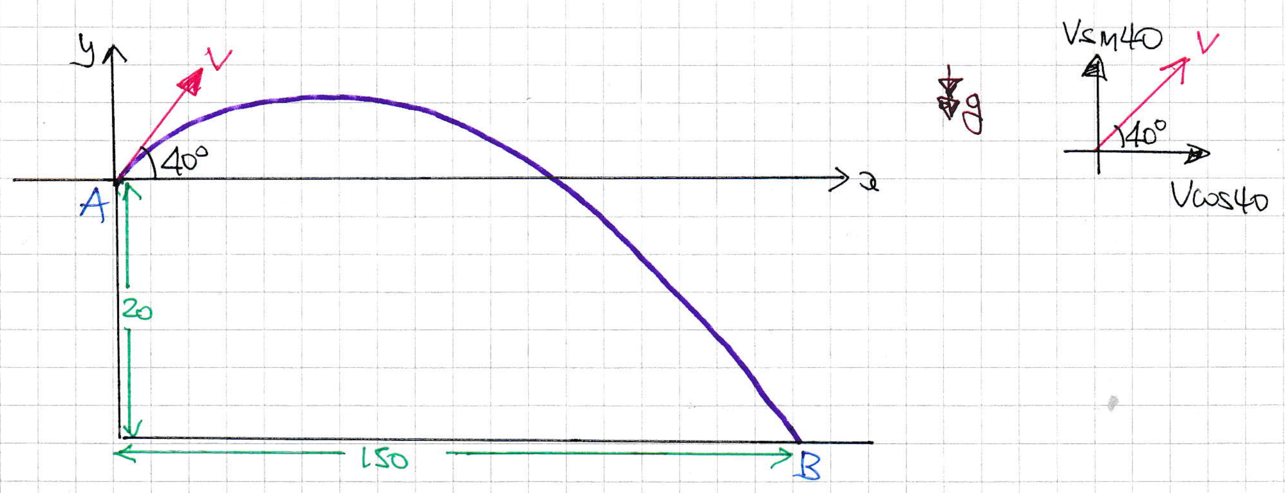
$80d + 80 = 180$

$80d = 100$

$d = 1.25 \text{ m}$

IYGB - MMS PAPER J - QUESTION 13

a)



WE CONSIDER THE LIMITING CASE, IE THE VALUE OF V, SO THAT IT LANDS ON B — LET THE FLIGHT TIME BE T

- VERTICALLY FROM A TO B

"$s = ut + \frac{1}{2}at^2$"

$-20 = (V\sin 40)T + \frac{1}{2}(-9.8)T^2$

$-20 = VT\sin 40 - 4.9T^2$

- HORIZONTALLY FROM A TO B

"x = SPEED × TIME"

$150 = V\cos 40 \times T$

$150 = VT\cos 40$

SOLVING SIMULTANEOUSLY

$VT = \frac{150}{\cos 40} \implies -20 = \left(\frac{150}{\cos 40}\right)\sin 40 - 4.9T^2$

$\implies -20 = 150\tan 40 - 4.9T^2$

$\implies 4.9T^2 = 150\tan 40 + 20$

$\implies T^2 = 29.768\ldots$

$\implies T = 5.456038\ldots$

$\therefore V = \frac{150}{T\cos 40} = 35.888\ldots$

$\therefore V_{MIN} = 35.89 \text{ ms}^{-1}$

IYGB - MMS PAPER J QUESTION 13

b) IF $V = 50$ – LOOKING AT THE VELOCITY COMPONENTS

VERTICALLY "$v = u + at$"

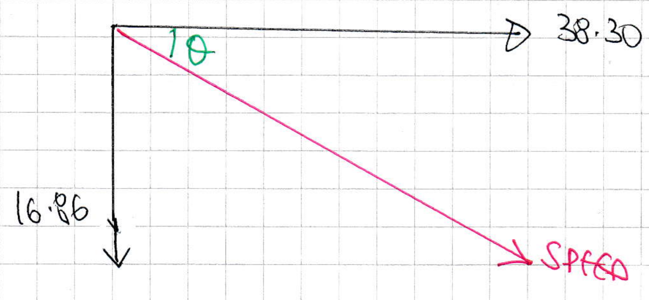
$v = 50\sin 40 - 9.8 \times 5$

$v = -16.86\ldots$

HORIZONTALLY UNCHANGED

$50\cos 40 \simeq 38.30\ldots$

LOOKING AT THE SPEEDS WHEN $t = 5$



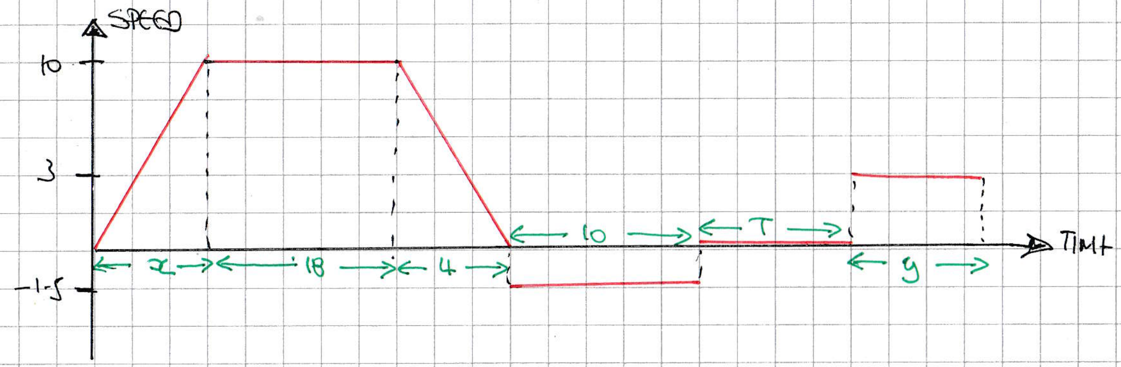
- SPEED $= \sqrt{(16.86\ldots)^2 + (38.30\ldots)^2} \simeq 41.85 \text{ ms}^{-1}$

- $\tan\theta = \dfrac{16.86\ldots}{38.30\ldots}$

  $\theta = 23.8°$

  BELOW THE HORIZONTAL

IYGB - MMS PAPER J - QUESTION 14



a) ACCELERATION = GRADIENT

$$1.25 = \frac{10}{x}$$

$$x = 8$$

IT 8 SECONDS

b) DECELERATION = GRADIENT

$$a = \frac{10}{4}$$

$$a = 2.5 \text{ ms}^{-2}$$

c) THE RECTANGLES MUST HAVE THE SAME AREA

[SAME DISTANCE BACK, AND THEN BACK TO BIKE]

$$(10 \times 1.5) = 3y$$

$$y = 5$$

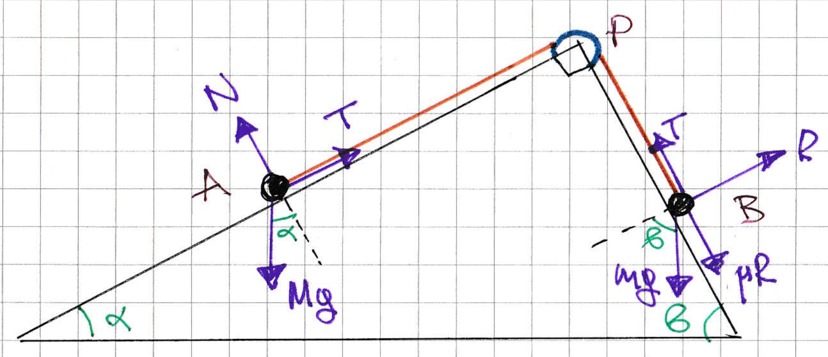
$$\therefore \quad x + 18 + 4 + 10 + T + y = 90$$

$$8 + 18 + 4 + 10 + T + 5 = 90$$

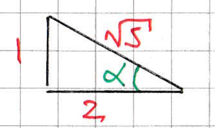
$$T = 45$$

IYGB - MMS PAPER J - QUESTION 15

PUTTING ALL THE INFORMATION IN A DETAILED DIAGRAM



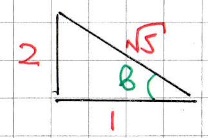
$\tan\alpha = \frac{1}{2}$



$\sin\alpha = \frac{1}{\sqrt{5}}$

$\cos\alpha = \frac{2}{\sqrt{5}}$

$\tan\beta = 2$



$\sin\beta = \frac{2}{\sqrt{5}}$

$\cos\beta = \frac{1}{\sqrt{5}}$

LOOKING AT THE FORCES PARALLEL TO THE PLANES

$$\left.\begin{array}{l} (A)\ Mg\sin\alpha = T \\ (B)\ mg\sin\beta + \mu R = T \end{array}\right\} \Rightarrow Mg\sin\alpha = mg\sin\beta + \mu R$$

$$\Rightarrow M\cancel{g}\sin\alpha = m\cancel{g}\sin\beta + \mu(m\cancel{g}\cos\beta)$$

$$\Rightarrow M\sin\alpha = m\sin\beta + \mu m\cos\beta$$

$$\Rightarrow M\left(\frac{1}{\sqrt{5}}\right) = m\left(\frac{2}{\sqrt{5}}\right) + \mu m\left(\frac{1}{\sqrt{5}}\right)$$

$\times\sqrt{5}$

$$\Rightarrow M = 2m + \mu m$$

$$\Rightarrow M = m(2+\mu)$$

AS REQUIRED

IYGB - MMS PAPER J - QUESTION 16

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$$(16\underline{i} - 2\underline{j}) = (18\underline{i} - 5\underline{j}) + \underline{v} \times 0.5$$
$$-2\underline{i} + 3\underline{j} = \tfrac{1}{2}\underline{v}$$
$$\underline{v} = -4\underline{i} + 6\underline{j}$$

SPEED $= |\underline{v}| = |-4\underline{i} + 6\underline{j}| = \sqrt{(-4)^2 + 6^2} = \sqrt{16+36} = \sqrt{52}$

$\simeq 7.21 \text{ km h}^{-1}$

b) USING AGAIN $\underline{r} = \underline{r}_0 + \underline{v}t$

$$\underline{r} = (18\underline{i} - 5\underline{j}) + (-4\underline{i} + 6\underline{j})t$$
$$\underline{r} = (18 - 4t)\underline{i} + (6t - 5)\underline{j}$$

If $f(t) = 18 - 4t$
$g(t) = 6t - 5$

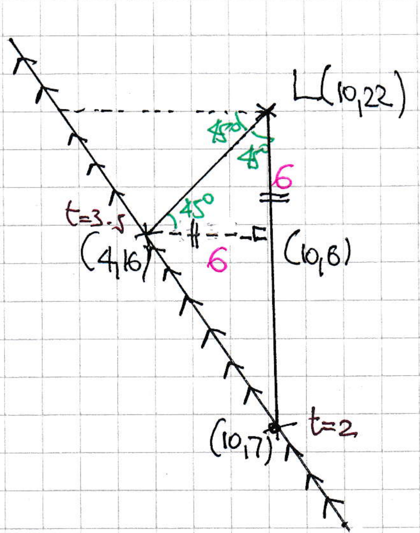
c) WHEN $t = 2$ (AT 14:00)

$$\underline{r} = (18 - 4\times2)\underline{i} + (6\times2 - 5)\underline{j}$$
$$\underline{r} = 10\underline{i} + 7\underline{j}$$

WHEN $t = \frac{7}{2}$ (AT 15:30)

$$\underline{r} = (18 - 4\times\tfrac{7}{2})\underline{i} + (6\times\tfrac{7}{2} - 5)\underline{j}$$
$$\underline{r} = 4\underline{i} + 16\underline{j}$$

LOOKING AT A DIAGRAM.



BY INSPECTION $L(10,22)$

I.E $10\underline{i} + 22\underline{j}$