IYGB - MMS PAPER I - QUESTION 1

a) X = NO OF SALES (JANE)

I) $X \sim B(10, 0.1)$

$P(X=0) = \binom{10}{0} 0.1^0 \times 0.9^{10}$

$= 0.3487$

II) $X \sim B(20, 0.1)$

$P(X>4) = P(X \geq 5)$

$= 1 - P(X \leq 4)$

$= 1 - 0.9568$

$= 0.0432$

b) Y = NO OF SALES (AMBER)

I) $Y \sim B(20, 0.15)$

$P(Y=2) = \binom{20}{2}(0.15)^2(0.85)^{18}$

$= 0.229338\ldots$

$X \sim B(20, 0.1)$

$P(X=2) = \binom{20}{2}(0.1)^2(0.9)^{18}$

$= 0.28517\ldots$

$\therefore$ REQUIRED PROBABILITY $= 0.229338 \times 0.28517\ldots \simeq 0.0654$

II) THIS CAN HAPPEN SEVERAL WAYS

$P(X+Y=4) = P(X=0) \times P(Y=4) = 0.121576 \times 0.182121 = 0.02214$

$P(X=1) \times P(Y=3) = 0.27017 \times 0.24282 = 0.06561$

$P(X=2) \times P(Y=2) = \ldots$ above $\ldots = 0.0654$

$P(X=3) \times P(Y=1) = 0.190120 \times 0.13679 = 0.026008$

$P(X=4) \times P(Y=0) = 0.08978 \times 0.038759 = 0.003480$

0.1826

IYGB - MMS PAPER I - QUESTION 1

c) $Y \sim B(n, 0.15)$

$\Rightarrow P(Y \geq 1) > 0.99$

$\Rightarrow 1 - P(Y=0) > 0.99$

$\Rightarrow -P(Y=0) > -0.01$

$\Rightarrow P(Y=0) < 0.01$

$\Rightarrow \cancel{\binom{n}{0}(0.15)^0}(0.85)^n < 0.01$

$\Rightarrow 0.85^n < 0.01$

USING LOGS OR TRIAL & IMPROVEMENT

$\Rightarrow \log(0.85^n) < \log(0.01)$

$\Rightarrow n \log(0.85) < \log(0.01)$

$\Rightarrow n > \dfrac{\log(0.01)}{\log(0.85)}$ ← THIS IS NEGATIVE

$\Rightarrow n > 28.336\ldots$

$\therefore n = 29$

MUST SHOW

$0.85^{28} = 0.011 > 0.01$

$0.85^{29} = 0.009 < 0.01$

IYGB - MMS PAPER J - QUESTION 2

X = NUMBER OF VEGETARIAN ORDERS

$X \sim B(80, 0.3)$

MEAN $= E(X) = np = 80 \times 0.3 = 24$

VARIANCE $= Var(X) = np(1-p) = 24 \times 0.7 = 16.8 > 5$

APPROXIMATE BY $Y \sim N(24, 16.8)$

$\Rightarrow P(X > 30)$

$= P(X \geqslant 31)$

$= P(Y > 30.5)$

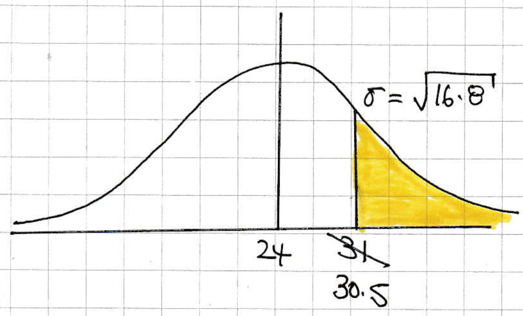
$= 1 - P(Y < 30.5)$

$= 1 - P\left(Z < \frac{30.5 - 24}{\sqrt{16.8}}\right)$

$= 1 - \Phi(1.585837...)$

$= 1 - 0.9436118...$

$= \underline{0.0564}$



IYGB – MMS PAPER I – QUESTION 3

a) RECONSTRUCT THE TABLE

DIAMETERS (mm)	MIDPOINTS (x)	$y = 50(x-0.09)$	FREQUENCY (f)
$0.02 < d \leqslant 0.04$	0.03	-3	25 (25)
$0.04 < d \leqslant 0.06$	0.05	-2	76 (101)
$0.06 < d \leqslant 0.08$	0.07	-1	111 (212)
$0.08 < d \leqslant 0.10$	0.09	0	255 (467)
$0.10 < d \leqslant 0.12$	0.11	1	33 (500)

CALCULATE SUMMARY STATISTICS IN y

$$\sum fy = -305 \qquad \sum fy^2 = 673 \qquad \sum f = 500$$

CALCULATE THE MEAN & STANDARD DEVIATION IN y

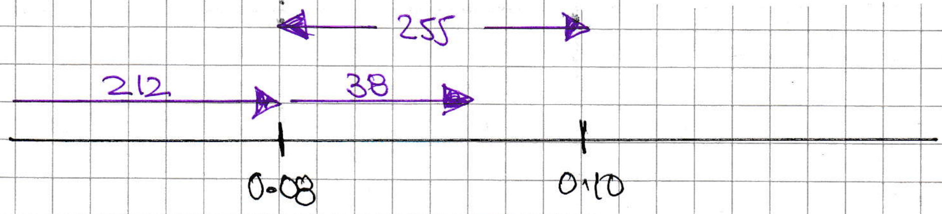
- $\bar{y} = \dfrac{\sum fy}{\sum f} = \dfrac{-305}{500} = -0.61$
- $\sigma_y = \sqrt{\dfrac{\sum fy^2}{\sum f} - \bar{y}^2} = \sqrt{\dfrac{673}{500} - (-0.61)^2} \simeq 0.9868637191\ldots$

UNCODE BACK INTO x

- $\bar{x} = \bar{y} \div 50 + 0.09 = -0.61 \div 50 + 0.09 = 0.0778$
- $\sigma_x = \sigma_y \div 50 = 0.986863\ldots \div 50 \simeq 0.0197$

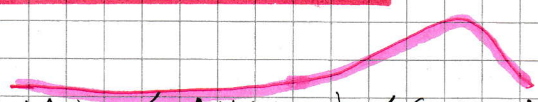
IYGB – MMS PAPER I – QUESTION 3

b) Q_2 IS $\frac{1}{2} \times 500 = 250$ OBS, WHICH LIES IN $0.08 < d \leqslant 0.10$



$$\Longrightarrow Q_2 = 0.08 + \frac{38}{255} \times 0.02 \approx 0.0830$$

c) USING THE AVERAGES



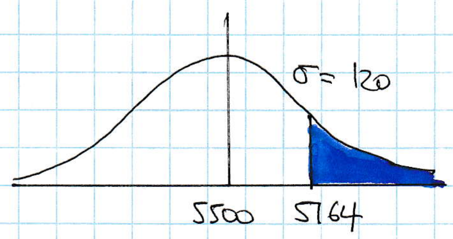
MEAN < MEDIAN < (MODE) $\Longrightarrow$ POSITIVE SKEW

0.0778   0.0830

IYGB - MMS PAPER I - QUESTION 4

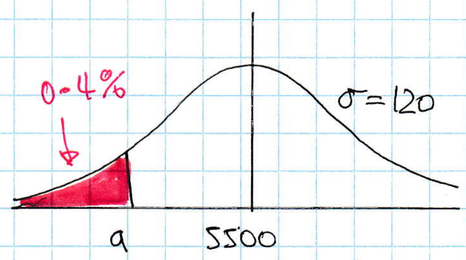
a) T = LIFETIME OF A LIGHT-BULB

$T \sim N(5500, 120^2)$



$$P(T > 5764) = 1 - P(T < 5764)$$
$$= 1 - P\left(Z < \frac{5764 - 5500}{120}\right)$$
$$= 1 - \Phi(2.2)$$
$$= 1 - 0.9861$$
$$= \underline{0.0139}$$

b) DRAWING A NEW NORMAL CURVE



"NOT ACHIEVED BY 0.4%"

$\Updownarrow$

"ACHIEVED BY 99.6%)

$$\Rightarrow P(T < a) = 0.4\%$$
$$\Rightarrow P(T > a) = 99.6\%$$
$$\Rightarrow P\left(Z > \frac{a - 5500}{120}\right) = 0.9960$$

$\downarrow$ INVERTING

$$\Rightarrow \frac{a - 5500}{120} = -\Phi^{-1}(0.9960)$$
$$\Rightarrow \frac{a - 5500}{120} = -2.65$$
$$\Rightarrow a - 5500 = -318$$
$$\Rightarrow a = \underline{5182}$$

IYGB - MMS PAPER I - QUESTION 4

c) SETTING A BINOMIAL DISTRIBUTION

- Y = A BULB WITH LIFETIME EXCEEDING 5764 HOURS
- $Y \sim B(30, 0.0139)$

$$P(Y=2) = \binom{30}{2}(0.0139)^2(0.9861)^{28} \approx \underline{0.0568}$$

IYGB - MMS PAPER J - QUESTION 5

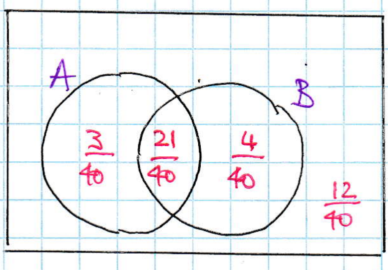
$P(A) = \frac{3}{5}$ • $P(B) = \frac{5}{8}$ • $P(A \cup B) = \frac{7}{10}$

a) USING $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Longrightarrow \frac{7}{10} = \frac{3}{5} + \frac{5}{8} - P(A \cap B)$$

$$\Longrightarrow P(A \cap B) = \frac{21}{40}$$

$\Longrightarrow$



b) TO CHECK FOR INDEPENDENCE

$$P(A) \times P(B) = \frac{3}{5} \times \frac{5}{8} = \frac{15}{40} \neq \frac{21}{40} = P(A \cap B)$$

$\therefore$ EVENTS ARE NOT INDEPENDENT

c) COLLECTING ALL INFORMATION FOR A & C

$P(A) = \frac{3}{5}$ $P(A \cup C) = \frac{7}{10}$ $P(C|A) = \frac{1}{3}$

$$\Longrightarrow P(C|A) = \frac{P(C \cap A)}{P(A)}$$

$$\Longrightarrow \frac{1}{3} = \frac{P(C \cap A)}{\frac{3}{5}}$$

$$\Longrightarrow P(C \cap A) = \frac{1}{5}$$

IYGB - MMS PAPER I - QUESTION 5

USING $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$$\Longrightarrow \frac{7}{10} = \frac{3}{5} + P(C) - \frac{1}{5}$$

$$\Longrightarrow P(C) = 0.3$$

THUS $P(A|C) = \dfrac{P(A \cap C)}{P(C)}$

$$\Longrightarrow P(A|C) = \frac{1/5}{0.3}$$

$$\Longrightarrow P(A|C) = \frac{2}{3}$$

II) AS $P(A \cap C) = \frac{1}{5}$

$$P[(A \cap C)'] = 1 - \frac{1}{5} = \frac{4}{5}$$

IYGB, MMS PAPER I - QUESTION 6

a) $\boxed{\begin{array}{l} X = \text{NUMBER OF TILES WITH MINOR FAULTS} \\ X \sim B(20, 0.1) \end{array}}$

$H_0: p = 0.1$

$H_1: p > 0.1$, WHERE p IS IS THE PROPORTION OF ALL FAULTY TILES PRODUCED BY THE MACHINE

CRITICAL REGION REQUIRED AT 5% SIGNIFICANCE (ONE TAILED)

$P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - 0.8670 = 0.1330$

$= 13.3\% > 5\%$

$P(X \geqslant 5) = 1 - P(X \leqslant 4) = 1 - 0.9568 = 0.0432$

$= 4.32\% < 5\%$

$\therefore$ CRITICAL REGION IS $\{5, 6, 7, \ldots, 20\}$ //

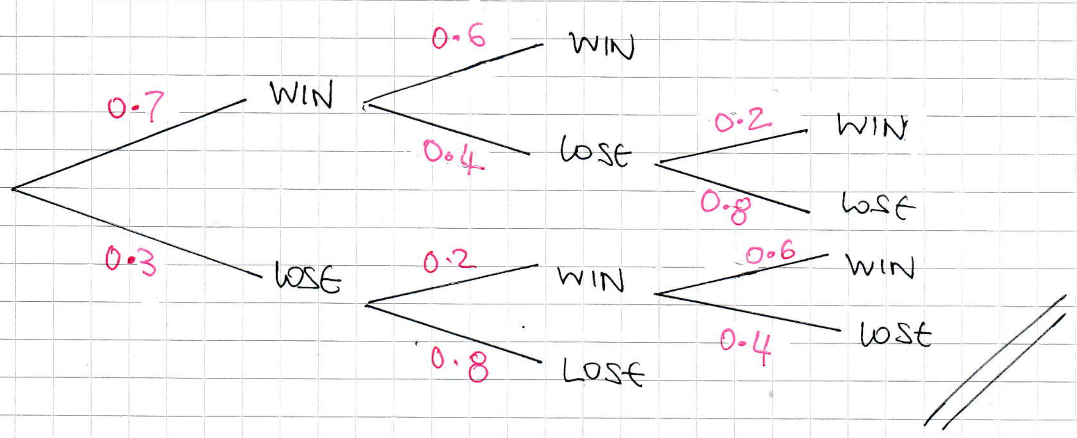
b) ACTUAL SIGNIFICANCE IS 4.32% //

c) 4 IS NOT IN THE CRITICAL REGION

THERE IS INSUFFICIENT EVIDENCE TO SUPPORT THE MANAGER'S BELIEF //

IYGB - MMS PAPER J - QUESTION 7

DRAWING THE TREE DIAGRAM FROM ARNIE'S POINT VIEW



a) $P(\text{ARNIE WINS}) = (0.7\times0.6)+(0.7\times0.4\times0.2)+(0.3\times0.2\times0.6)$

$= 0.42 + 0.056 + 0.036$

$= 0.512 \quad \left(= \frac{64}{125}\right)$

b) $P(\text{WINS IN } 2 \mid \text{WIN}) = \frac{P(\text{WIN IN } 2 \cap \text{WINS})}{P(\text{WIN})} = \frac{0.7\times0.6}{0.512}$

$= \frac{0.42}{0.512} = \frac{105}{128} \simeq 0.8203$

$P(\text{WINS} \mid \text{WINS FIRST GAME}) = \frac{P(\text{WINS} \cap \text{WINS FIRST GAME})}{P(\text{WINS FIRST GAME})}$

$= \frac{(0.7\times0.6)+(0.7\times0.4\times0.2)}{0.7}$

$= \frac{0.476}{0.7}$

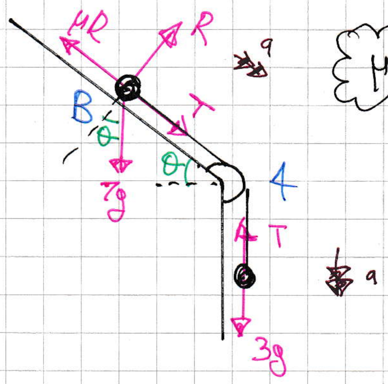
$= 0.68 \quad \left(= \frac{17}{25}\right)$

IYGB, MMS PAPER I - QUESTION 8

LOOKING AT THE EQUATION OF MOTION OF EACH PARTICLE SEPARATELY

$(A): 3g - T = 3a$

$(B): T + 7g\sin\theta - \mu R = 7a$



$\{\mu = 0.6\}$

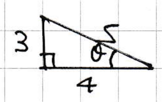
ADDING THE EQUATIONS

$$3g + 7g\sin\theta - \mu R = 10a$$

$$3g + 7g\sin\theta - \mu(7g\cos\theta) = 10a$$

EQUILIBRIUM PERPENDICULAR TO THE PLANE, SO $R = 7g\cos\theta$

$\tan\theta = \frac{3}{4}$



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$$3g + 7g \times \frac{3}{5} - 0.6 \times 7g \times \frac{4}{5} = 10a$$

$$10a = 37.632$$

$$a = 3.7632$$

$$a \simeq 3.76 \text{ ms}^{-2}$$

FINALLY THE TENSION

$$3g - T = 3a$$

$$3g - 3a = T$$

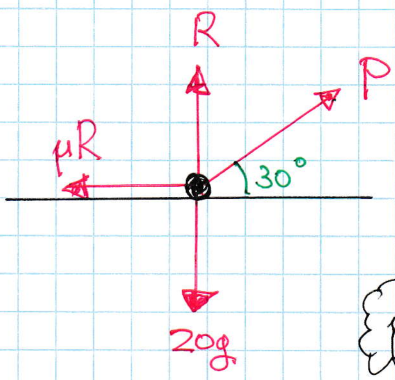
$$T = 3 \times 9.8 - 3 \times 3.7632$$

$$T = 18.1104$$

$$T \simeq 18.1 \text{ N}$$

IYGB - MMS PAPER I - QUESTION 9

STARTING WITH A DIAGRAM IN ORDER TO FORM TWO EQUATIONS



$\mu = 0.25$

$(\updownarrow)$: $R + P\sin 30 = 20g$

$(\leftrightarrow)$: $\mu R = P\cos 30°$

SOLVE THE FIRST EQUATION FOR R

$$R = 20g - P\sin 30$$

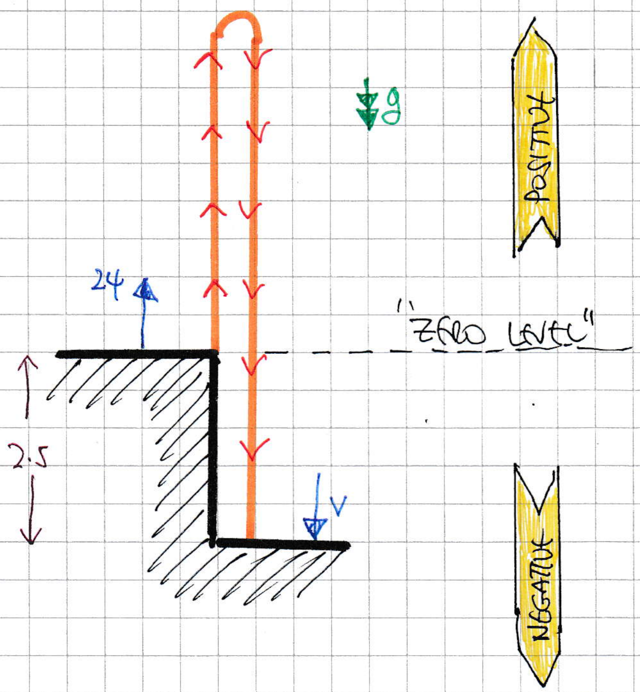
SUBSTITUTE INTO THE SECOND EQUATION

$$\Rightarrow \mu(20g - P\sin 30) = P\cos 30$$
$$\Rightarrow 20\mu g - \mu P\sin 30 = P\cos 30$$
$$\Rightarrow 20\mu g = P\cos 30 + \mu P\sin 30$$
$$\Rightarrow 20\mu g = P(\cos 30 + \mu\sin 30)$$
$$\Rightarrow P = \frac{20\mu g}{\cos 30 + \mu\sin 30}$$
$$\Rightarrow P = \frac{20 \times 0.25 \times 9.8}{\frac{\sqrt{3}}{2} + \frac{1}{4} \times \frac{1}{2}}$$
$$\Rightarrow P = 49.44373\ldots$$

HENCE THE REQUIRED FORCE IS 49.4 N

IYGB - MMS PAPER J - QUESTION 10

a) LOOKING AT THE DIAGRAM & CONSIDERING THE ENTIRE JOURNEY



u	=	$+24\ \text{ms}^{-1}$
a	=	$-9.8\ \text{ms}^{-2}$
s	=	-2.5
t	=	?
v	=	?

$$s = ut + \tfrac{1}{2}at^2$$

$$-2.5 = 24T + \tfrac{1}{2}(-9.8)T^2$$

$$-2.5 = 24T - 4.9T^2$$

$$-25 = 240T - 49T^2$$

$$49T^2 - 240T - 25 = 0$$

QUADRATIC FORMULA

$$T = \frac{240 \pm \sqrt{(-240)^2 - 4 \times 49(-25)}}{2 \times 49}$$

$$T = \begin{cases} 5 \\ \cancel{-\frac{5}{49}} \end{cases}$$

b) FINALLY USING $v = u + at$

$$v = 24 + (-9.8) \times 5$$

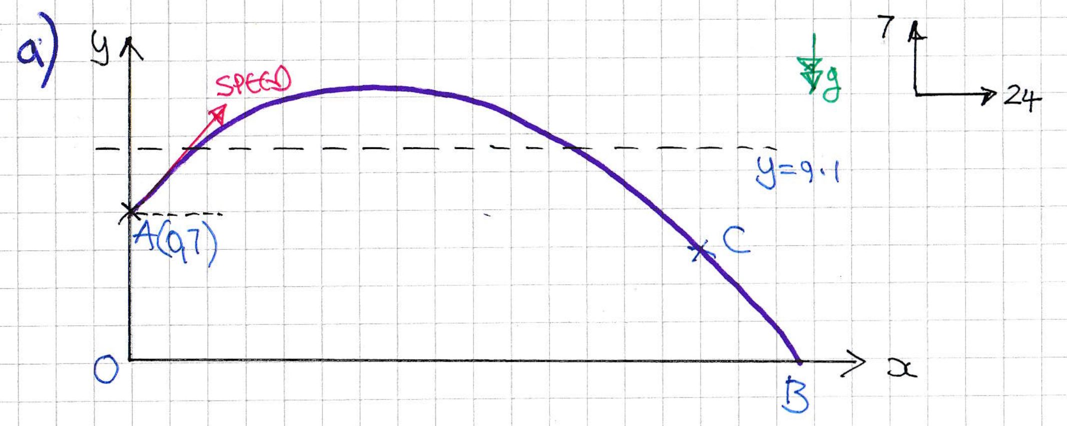
$$v = 24 - 49$$

$$v = -25 \quad \text{(MINUS DENOTES DOWNWARDS)}$$

$\therefore$ SPEED $v = 25$

IYGB - MMS PAPER I - QUESTION 11

a)



LOOKING AT THE VERTICAL MOTION

$u =$	$7\ ms^{-1}$
$a =$	$-9.8\ ms^{-2}$
$s =$	$9.1\ m$
$t =$	$?$
$v =$	

$\Rightarrow s = s_0 + ut + \frac{1}{2}at^2$

$\Rightarrow 9.1 = 7 + 7t + \frac{1}{2}(-9.8)t^2$

$\Rightarrow 4.9t^2 - 7t + 2.1 = 0$

$\Rightarrow 49t^2 - 70t + 21 = 0$

$\Rightarrow 7t^2 - 10t + 3 = 0$

$\Rightarrow (t-1)(7t-3) = 0$

$t = \begin{cases} 1 \\ \frac{3}{7} \end{cases}$

$\therefore$ REQUIRED TIME $= 1 - \frac{3}{7} = \frac{4}{7}$

b) USING SPEED $= \frac{\text{DISTANCE}}{\text{TIME}}$ IN THE HORIZONTAL DIRECTION

$24 = \frac{48}{T}$

$T = 2$ ← IE IT TAKES 2 SECONDS TO REACH C

USING $s = s_0 + ut + \frac{1}{2}at^2$

$s = 7 + 7\times2 + \frac{1}{2}(-9.8)\times2^2$

$s = 7 + 14 - 19.6$

$s = 1.4$

$\therefore \lambda = 1.4$

IYGB - MMS PAPER J - QUESTION 12

DIFFERENTIATING TO OBTAIN THE VELOCITY

$$x = \frac{1}{3}t(t^2 - 3t - 24)$$

$$x = \frac{1}{3}(t^3 - 3t^2 - 24t)$$

$$v = \frac{dx}{dt} = \frac{1}{3}(3t^2 - 6t - 24)$$

$$v = t^2 - 2t - 8$$

INSTANTANEOUSLY AT REST $\Rightarrow$ $v=0$

$$\Rightarrow 0 = t^2 - 2t - 8$$

$$\Rightarrow (t+2)(t-4) = 0$$

$$\Rightarrow t = \cancel{-2},\ 4$$

THUS DISPLACEMENT WHEN $t=4$ CAN NOW BE FOUND

$$x(4) = \frac{1}{3} \times 4 \times (4^2 - 3\times 4 - 24) = -\frac{80}{3}$$

$(\simeq -26.7)$

IYGB - MMS PAPER I - QUESTION 13

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$ WITH $t=0$ AT 10:00 AM

$$\Rightarrow -5\underline{i} + 3.75\underline{j} = -2\underline{i} + 3\underline{j} + \underline{v} \times \frac{3}{4}$$

$$\Rightarrow -20\underline{i} + 15\underline{j} = -8\underline{i} + 12\underline{j} + 3\underline{v}$$

$$\Rightarrow -12\underline{i} + 3\underline{j} = 3\underline{v}$$

$$\Rightarrow \underline{v} = -4\underline{i} + \underline{j}$$

USING THE SAME EQUATION AGAIN

$$\Rightarrow \underline{b} = \underline{b}_0 + \underline{v}t$$

$$\Rightarrow \underline{b} = -2\underline{i} + 3\underline{j} + (-4\underline{i} + \underline{j}) \times t$$

$$\Rightarrow \underline{b} = (-2-4t)\underline{i} + (3+t)\underline{j}$$

b) FIND THE POSITION OF THE DRIFTING BOAT AT 11:45, IE $t = 1.75$

$$\underline{b} = (-2 - 4 \times 1.75)\underline{i} + (3 + 1.75)\underline{j}$$

$$\underline{b} = -9\underline{i} + 4.75\underline{j}$$

NOW USING $\underline{r} = \underline{r}_0 + \underline{V}T$, WHERE T IS MEASURED FROM 11:30

$$-9\underline{i} + 4.75\underline{j} = 2\underline{i} + \underline{j} + \underline{V} \times \frac{1}{4}$$

$$-36\underline{i} + 19\underline{j} = 8\underline{i} + 4\underline{j} + \underline{V}$$

$$\underline{V} = -44\underline{i} + 15\underline{j}$$

IYGB – MMS PAPER J – QUESTION 14

a) SKETCHING THE SPEED TIME GRAPH FROM THE INFO GIVEN

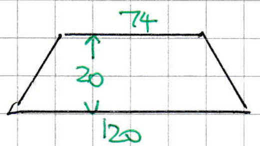
b) DECELERATION = GRADIENT

$$a = \frac{\Delta v}{\Delta t} \implies 1.25 = \frac{20}{\Delta t}$$

$$\implies \Delta t = 16$$

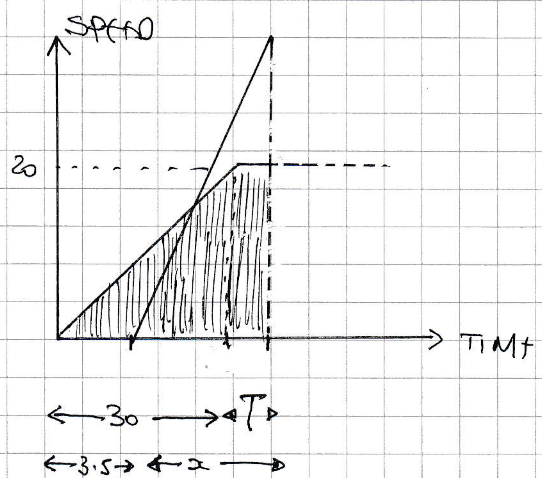
$\therefore$ TOTAL TIME IS 120

DISTANCE = AREA

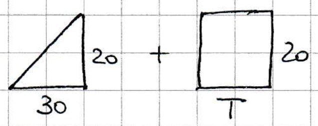


$$= \frac{1}{2}(74+120)\times 20 = 1940\text{ m}$$

c) LOOKING AT THE DIAGRAM



BOTH AREAS ARE $\frac{1940}{2} = 970$.



$$+ \quad = 970$$

$$300 + 20T = 970$$

$$20T = 670$$

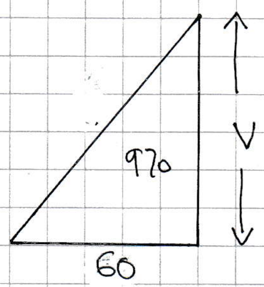
$$T = 33.5$$

IYGB - MMS PAPER J - QUESTION 14

BY INSPECTION $x = 60$

$(30 + T = 3.5 + x)$

FINALLY THE ACCELERATION CAN BE FOUND



$$\frac{1}{2} \times 60 \times V = 970$$

$$30V = 970$$

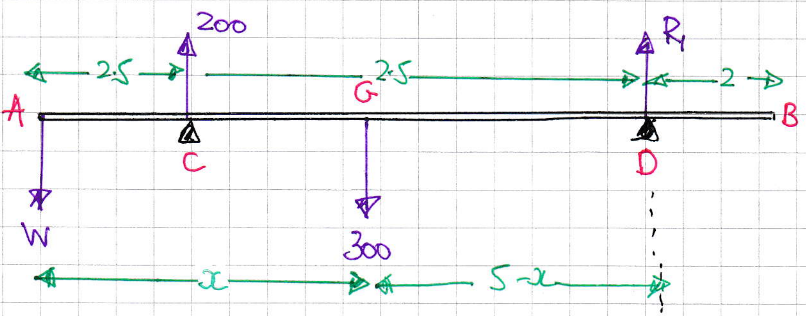
$$V = \frac{97}{3}$$

$$\therefore \; a = \frac{\Delta v}{\Delta t} = \frac{97/3}{60} = \frac{97}{180}$$

$\therefore$ ACCELERATION 0.539 ms^{-2}

IYGB - MMS PAPER J - QUESTION 15

a)



TAKING MOMENTS ABOUT D

$$D: \quad 300(5-x) + W \times 5 = 200 \times 2.5$$

$$1500 - 300x + 5W = 500$$

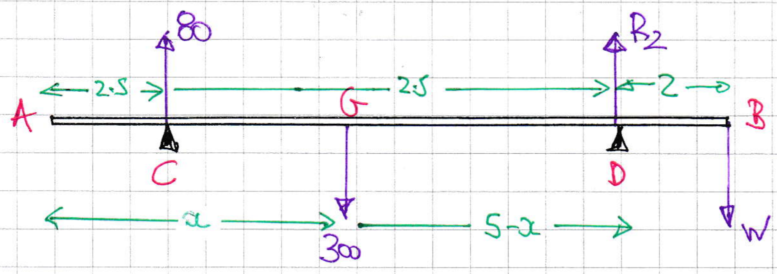
$$1000 = 300x - 5W$$

$$300x - 5W = 1000$$

$$60x - W = 200$$

AS REQUIRED

b) REDRAWING THE DIAGRAM



TAKING MOMENTS ABOUT D AGAIN

$$D: \quad 80 \times 2.5 + W \times 2 = 300(5-x)$$

$$200 + 2W = 1500 - 300x$$

$$300x + 2W = 1300$$

$$150x + W = 650$$

ADDING

$$60x - W = 200$$

$$150x + W = 650$$

$$210x = 850$$

$$x = \frac{85}{21} \simeq 4.05 \text{ m}$$

& $W = 650 - 150x$

$$W \simeq 42.9 \text{ N}$$