IYGB

Mathematical Methods 2

Practice Paper E

Time: 3 hours

Candidates may use any non programmable, non graphical calculator which does not have the capability of storing data or manipulating algebraic expressions.

Information for Candidates

This practice paper follows the most common syllabi of Mathematical Methods used the United Kingdom Universities for Mathematics, Physics and Engineering degrees.

Booklets of *Mathematical formulae and statistical tables* may NOT be used.

Full marks may be obtained for answers to MOST questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 9 questions in this question paper.

The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

Non exact answers should be given to an appropriate degree of accuracy.

The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

The maximum number of marks on this paper is 100, which can be obtained by attempting all the questions on the paper.

Topics Examined Under Mathematical Methods 2

- Vector Operators: Gradient, Divergence and Curl
- Index Summation Notation
- Double Integrals in Cartesian and Polar Coordinates
- Triple Integrals in Cartesian, Cylindrical and Spherical Coordinates
- Jacobians
- Volume Integrals
- Surface Integrals
- Line Integrals
- Multiple and Vector Integration in Parametric Form
- Applications of Multiple Integration and Vector Integration
  [Mass, Work, Flux, Pressure, Centre of Mass, Moment of Inertia etc.]
- Green's Theorem and Applications
- Divergence Theorem and Applications
- Stokes Theorem and Applications

Question 1

It is given that the vector function $\mathbf{F}$ satisfies

$$\mathbf{F} = \left(\sin x^3 - xy\right)\mathbf{i} + \left(x + y^3 \sin y\right)\mathbf{j}.$$

Evaluate the line integral

$$\oint_C \mathbf{F} \cdot d\mathbf{r} \, ,$$

where C is the ellipse with cartesian equation

$$2x^2 + 3y^2 = 2y \, . \qquad (9)$$

Question 2

Use index summation notation to prove the validity of the following vector identity.

$$(\mathbf{A} \wedge \mathbf{B}) \cdot (\mathbf{C} \wedge \mathbf{D}) \equiv (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}) \, . \qquad (7)$$

Question 3

Use plane **polar** coordinates, (r, θ) to determine the value of

$$\int_{y=0}^{\infty} \int_{x=y}^{\infty} \frac{e^{-x}}{x} \, dx dy \, . \qquad (9)$$

Question 4

A surface S has Cartesian equation

$$x^2 + y^2 + z^2 = 2x.$$

a) Describe fully the graph of S, and hence find a parameterization for its equation in terms of the parameters u and v. **(4)**

b) Use the parameterization of part **(a)** to find the area for the part of S, for which $\frac{3}{5} \le z \le \frac{4}{5}$. **(10)**

Question 5

The finite region V in the first octant, is bounded by the surfaces with equations

$$y = 4 - x^2, \quad z = x \quad \text{and} \quad z = 3.$$

Find the volume of the solid. **(10)**

Question 6

The finite R region is defined as

$$4z \le x^2 + y^2 + z^2 \le 16z.$$

Determine an exact simplified value for

$$\int_R \left(\frac{z}{8}\right)^3 \, dx\,dy\,dz.$$ **(14)**

Question 7

A smooth scalar field is denoted by $\varphi = \varphi(x, y, z)$ and a smooth vector field is denoted by $\mathbf{A} = \mathbf{A}(x, y, z)$.

a) Use the standard definitions of vector operators to show that

$$\nabla \cdot (\varphi \mathbf{A}) = \nabla \varphi \cdot \mathbf{A} + \varphi \nabla \cdot \mathbf{A} \,.$$ **(7)**

b) Given further that f and g are functions of x, y and z, whose second partial derivatives exist, deduce that

$$\nabla \cdot (f \nabla g - g \nabla f) = f \nabla^2 g - g \nabla^2 f \,.$$ **(4)**

Question 8

The finite region R satisfies the inequalities

$$1 \le x + y \le 2 \quad \text{and} \quad 0 \le y \le x \,.$$

Show clearly that

$$\iint_R \frac{y \ln(x+y)}{x^2} \, dx\, dy = (1 - \ln 2)(-1 + 2\ln 2) \,.$$ **(12)**

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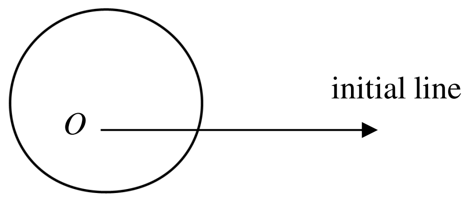
Question 9



The figure above shows the closed curve C with polar equation

$$r = 3 + \sin\theta, \; 0 \le \theta \le 2\pi .$$

The vector field $\mathbf{F}$ is given in Cartesian coordinates by

$$\mathbf{F}(x, y) = (x + y)\mathbf{i} + (-x + y)\mathbf{j} .$$

Evaluate the line integral

$$\oint_C \mathbf{F} \cdot d\mathbf{r} . \qquad (14)$$

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