IYGB

Mathematical Methods 2

Practice Paper B

Time: 3 hours

Candidates may use any non programmable, non graphical calculator which does not have the capability of storing data or manipulating algebraic expressions.

Information for Candidates

This practice paper follows the most common syllabi of Mathematical Methods used the United Kingdom Universities for Mathematics, Physics and Engineering degrees.

Booklets of *Mathematical formulae and statistical tables* may NOT be used.

Full marks may be obtained for answers to MOST questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 11 questions in this question paper.

The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

Non exact answers should be given to an appropriate degree of accuracy.

The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

The maximum number of marks on this paper is 100, which can be obtained by attempting all the questions on the paper.

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Topics Examined Under Mathematical Methods 2

- Vector Operators: Gradient, Divergence and Curl
- Index Summation Notation
- Double Integrals in Cartesian and Polar Coordinates
- Triple Integrals in Cartesian, Cylindrical and Spherical Coordinates
- Jacobians
- Volume Integrals
- Surface Integrals
- Line Integrals
- Multiple and Vector Integration in Parametric Form
- Applications of Multiple Integration and Vector Integration
  [Mass, Work, Flux, Pressure, Centre of Mass, Moment of Inertia etc.]
- Green's Theorem and Applications
- Divergence Theorem and Applications
- Stokes Theorem and Applications

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Question 1

Find the value of the following multiple integral

$$\int_0^{\pi}\int_0^{\cos\theta} r\sin\theta \, dr \, d\theta \, . \qquad (3)$$

Question 2

Evaluate the integral

$$\int_{(0,0)}^{(6,12)} \left(6x^2 - 2xy\right) ds \, ,$$

where s is the arclength along the straight line segment from $(0,0)$ to $(6,12)$. (7)

Question 3

Use index summation notation to prove the validity of the following vector identity

$$\nabla \cdot \left[\nabla f \wedge \nabla g\right] \equiv \mathbf{0} \, ,$$

where $f = f(x,y,z)$ and $g = g(x,y,z)$ are smooth scalar functions.

Any additional results used must be clearly stated. (7)

Question 4

A Cartesian position vector is denoted by $\mathbf{r}$ and $r = |\mathbf{r}|$.

Given that $f(r)$ is a differentiable function, show that

$$\nabla f(r) = \frac{\mathbf{r}}{r} f'(r) \, . \qquad (7)$$

Question 5

The surface S is the sphere with Cartesian equation

$$x^2 + y^2 + z^2 = 1$$

Use the Divergence Theorem to evaluate

$$\oiint_S \left(x^2 + y + z\right) dS \, . \qquad (7)$$

Question 6

The contour C is the boundary of a triangle with vertices at the points with Cartesian coordinates $(0,0)$, $(1,0)$ and $(1,2)$, traced in an anticlockwise direction.

Verify Green's Theorem on the plane for the line integral

$$\oint_C (3x+4y)\,dx + (5x-2y)\,dy \, . \qquad (10)$$

Question 7

Find the exact simplified value for the following integral.

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{0} \frac{\sqrt{16x^2+16y^2}}{x^2+y^2+1}\,dx\,dy \, . \qquad (10)$$

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Question 8

A hemispherical solid piece of glass, of radius a m, has small air bubbles within its volume.

The air bubble density $\rho(z)$, in m^{-3}, is given by

$$\rho(z) = kz,$$

where k is a positive constant, and z is a standard Cartesian coordinate, whose origin is at the centre of the flat face of the solid.

Given that the solid is contained in the part of space for which $z \geq 0$, determine the total number of air bubbles in the solid. **(10)**

Question 9

Evaluate the surface integral

$$\int_S z\,\mathbf{k} \cdot \mathbf{dS},$$

where S is the surface represented parametrically by

$$\mathbf{r}(\theta,\varphi) = \begin{bmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{bmatrix}, \ 0 \leq \theta \leq \tfrac{1}{2}\pi, \ 0 \leq \varphi \leq \tfrac{1}{2}\pi.$$

(11)

Question 10

The finite region R in the x-y plane is enclosed by the rectilinear triangle with vertices at $(0,0)$, $(0,1)$ and $(1,0)$.

Use a suitable coordinate transformation to find an exact value for

$$\int_R e^{\frac{x-2y}{x+2y}} \, dxdy .$$ **(12)**

Question 11

A bead is modelled as a sphere with a cylinder, whose axis is a diameter of the sphere, removed from the sphere.

If the respective equations of the sphere and the cylinder are

$$x^2 + y^2 + z^2 = a^2 \quad \text{and} \quad x^2 + y^2 = b^2 , \quad 0 < b < a .$$

Show that the total surface area of the bead is

$$4\pi(a+b)\sqrt{a^2 - b^2} .$$ **(16)**

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