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IYGB GCE

Mathematics M456

Advanced Level

Practice Paper D

Difficulty Rating: 3.53

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Advanced Level Further Mechanics Syllabi, assessed between 1993 and 2005 and with minor omissions between 2006 and 2017.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 15 questions in this question paper.
The total mark for this paper is 175.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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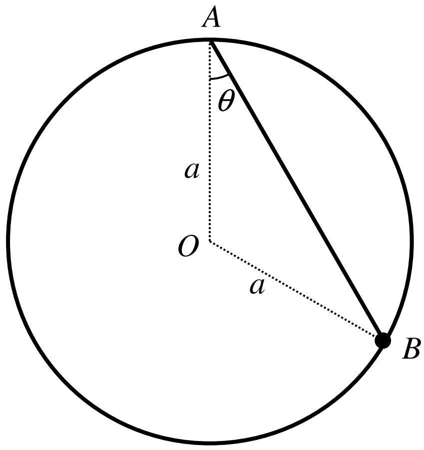
Question 1



The figure above shows a small bead B of mass m, threaded on a smooth circular wire of radius a, which is fixed in a vertical plane. The centre of the circle is at O, and the highest point of the circle is at A.

A light elastic string of natural length a and modulus of elasticity $3mg$ has one end attached to A and the other end attached to B.

The acute angle OAB is denoted by θ.

a) Given that the string is taut, show that the potential energy of the system is

$$2mga\left(2\cos^2\theta - 3\cos\theta\right) + \text{constant} . \qquad \textbf{(7)}$$

b) Hence find the three positions of equilibrium of the system and determine their stability. **(6)**

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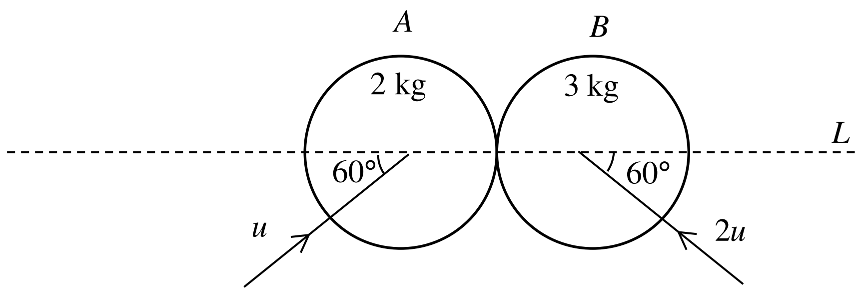
Question 2



Two smooth spheres, A and B, of equal radius and respective masses 2 kg and 3 kg, are moving on a smooth horizontal plane when they collide obliquely.

When A and B collide the straight line through their centres is denoted by L.

The speeds of A and B before their collision are u and $2u$, respectively and the direction of both speeds is at an angle $60°$ to L, as shown in the figure above.

Given that after the collision B is moving at right angles to L, find the value of the coefficient of restitution between A and B. **(7)**

Question 3

A pulley is modelled as a uniform circular disc of mass $4m$ and radius a.

The pulley is free to rotate about a fixed smooth horizontal axis through its centre and perpendicular to its plane. A light inextensible string passes over the pulley and two particles A and B, of respective mass $2m$ and $5m$ are attached to the two ends of the string.

The particles are released from rest with the string taut.

Assuming further that there is no slipping between the string and the pulley find, in terms of g, the acceleration of the particles. **(8)**

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Question 4

A spacecraft is moving in deep space in a straight line with speed $2u$.

At time $t=0$, the mass of the spacecraft is M and at that instant the engines of the spacecraft are fired in a direction opposite to that of the motion of the spacecraft.

Fuel is ejected at a constant mass rate k with speed u relative to the spacecraft.

At time t, the mass of the spacecraft is m and its speed is v.

a) Use the impulse momentum principle to show that

$$\frac{dv}{dt} = \frac{uk}{M-kt}.$$ **(7)**

b) Hence determine, in terms of u, the speed of the spacecraft when the mass of the spacecraft is $\frac{1}{3}$ of its initial mass. **(7)**

Question 5

At time $t=0$, a particle is on the initial line of a standard polar coordinate system (r,θ), and moving on a path with polar equation

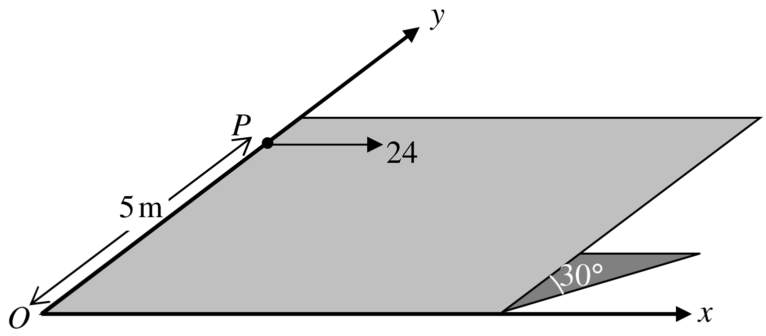
$$r = \frac{1}{4}e^{k\theta}, \theta \geq 0,$$

where k is a constant.

Relative to the pole O, the particle has a constant angular velocity of 2 rad s^{-1}, throughout the motion.

Given that the initial magnitude of the acceleration of the particle is 1.04 ms^{-2}, determine the possible values of k. **(11)**

Question 6



The point O lies at the bottom end of a fixed smooth plane, inclined at 30° to the horizontal. A positive y axis is defined up the line of greatest slope of the plane and a positive x axis is defined perpendicular to the y axis through O, as shown in the figure above.

At particle P is projected along the plane with speed 24 ms^{-1} in a direction parallel to the x axis, from the point with coordinates $(0,5)$, relative to O.

P reaches the bottom of the plane at the point $(X,0)$, with speed V, after time T.

Determine in any order the value of X, V and T. **(10)**

Question 7

A ship B is travelling due east at a constant speed of 15 km h^{-1}.

At midnight, another ship A is 24 km away from B so that the bearing of B from A is 240°. Ship B is travelling at a constant speed of U km h^{-1} and sets on a course to intercept A.

a) Find the least possible value of U. **(3)**

b) Given that $U = 12$, determine the two possible bearings at which A can sail so it can intercept B.

Determine the actual distance covered by A in each of these two cases. **(10)**

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Question 8

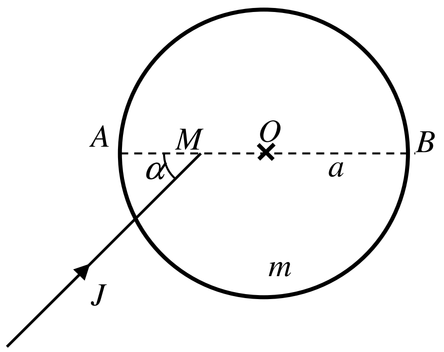
A simple pendulum consists of a small heavy bob attached at the end of a light inextensible string of length 24.5 cm. The other end of the string is attached to a fixed point O so that the bob is hanging in equilibrium vertically below O.

The bob is then displaced, so that the taut string makes an angle of $\frac{\pi}{9}$ with the downward vertical through O, and released from rest.

In the subsequent motion, the angle the taut string makes with the downward vertical through O, is denoted by θ.

Calculate the time it takes for the pendulum to travel from $\theta = \frac{\pi}{18}$ to $\theta = \frac{\pi}{36}$. **(11)**

Question 9



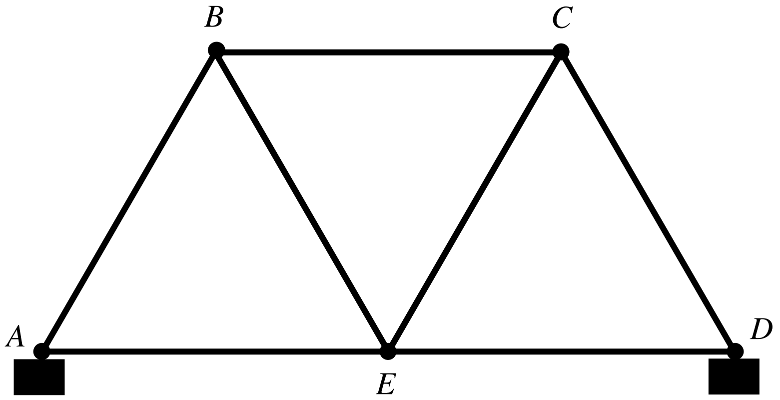
A uniform circular disc, of mass m and radius a, is lying flat on a smooth horizontal surface. The points A and B lie on the circumference of the disc, so that AB is a diameter and the point M is the midpoint of AO, where O is the centre of the disc.

The disc is initially at rest, until a horizontal impulse J is applied at M, at an angle α to AB, as shown in the figure above.

Show that the kinetic energy generated by the impulse is

$$\frac{J^2}{4m}\left(2+\sin^2\alpha\right).$$ **(10)**

Question 10



A rigid framework $ABCDE$ consists of seven identical light pin jointed rods as shown in the figure above.

The framework rests at two fixed supports at the points A and D.

When the framework supports a weight of W N at the midpoint AE, there is 600 N thrust on AB.

Determine the magnitude of each of the reaction forces at A and at C and the magnitude of each of the internal forces in the rods EA, EB, EC, ED, CB and CD, further classifying each of them as a tension or as a thrust.

(12)

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Question 11

A small raindrop of mass m kg, is released from rest from a rain cloud and is falling through still air under the action of its own weight.

The raindrop is subject to air resistance of magnitude kmv^2 N, where v ms^{-1} is the speed of the raindrop x m below the point of release, and k is a positive constant.

a) Solve the differential equation to show that

$$v^2 = \frac{g}{k}\left(1 - e^{-2kx}\right). \qquad \textbf{(8)}$$

The raindrop has a terminal velocity U.

b) Show further that the raindrop reaches a speed of $\frac{1}{2}U$, after falling through a distance of $\frac{U^2}{2g}\ln\left(\frac{4}{3}\right)$ metres. **(4)**

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Question 12

The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to each other.*

A particle, of mass 0.5 kg, passes through the point A whose position vector is $(12\mathbf{i}-15\mathbf{j}-2\mathbf{k})\text{ m}$, with speed $U\text{ ms}^{-1}$. The particle is moving due to the action of the following two constant forces, $\mathbf{F}_1$, and $\mathbf{F}_2$.

$$\mathbf{F}_1 = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}\text{N} \quad \text{and} \quad \mathbf{F}_2 = \begin{pmatrix} k-2 \\ 2k+3 \\ 3k-1 \end{pmatrix}\text{N},$$

where k is a scalar constant.

Determine the value of U, given further that it passes through the point B, whose position vector is $(-8\mathbf{i}+5\mathbf{j}+2\mathbf{k})\text{ m}$, with speed 29 ms^{-1}. **(9)**

Question 13

Four identical rods, each of mass m and length $2a$ are joined together to form a square rigid framework $ABCD$.

A fifth rod AC, of mass $3m$, is added to the framework for extra support.

The 5 rod framework is free to rotate about a smooth fixed horizontal axis L, which passes through A, so that the rotation of the framework takes place in a vertical plane.

The framework is held so that D is vertically above A and released from rest.

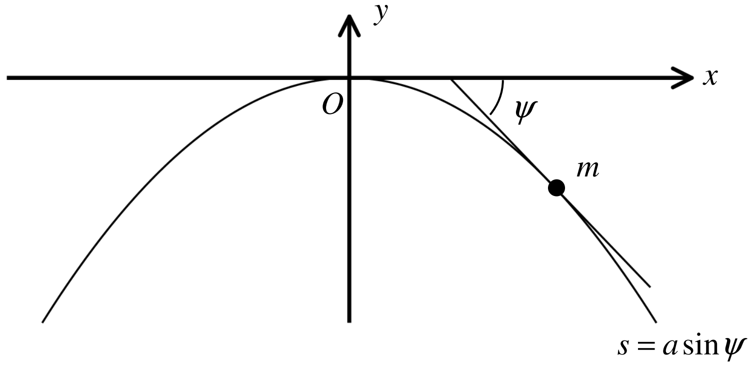
On the subsequent rotation, when B is vertically below A, a stationary particle of mass M adheres to B.

Given that the angular speed of the framework, after the particle has adhered to it, is

$$\frac{2}{9}\sqrt{\frac{21g}{a}},$$

determine M in terms of m. **(17)**

Question 14



A bead of mass m is made to slide along a smooth wire, which is fixed in a vertical plane, and is bent into the shape cycloidal arch with intrinsic equation

$$s = a\sin\psi ,$$

where a is a positive constant.

The arclength s is measured from the origin O, and the angle ψ is the angle the tangent to the cycloid makes with the positive x axis as shown in the figure above.

The bead passes through the highest point of the cycloidal arch O, with speed $\sqrt{\frac{1}{2}ag}$.

When the particle has travelled a distance s, its speed is v and the normal reaction from the wire to the bead is R.

a) Show, with a detailed method, that …

i. … $v^2 = \frac{g}{2a}\left[2s^2 + a^2\right]$. **(7)**

ii. … $R = \frac{mg}{2\cos\psi}\left[1 - 4\sin^2\psi\right]$. **(7)**

b) Find the distance travelled by the bead by the time $R = 0$. **(2)**

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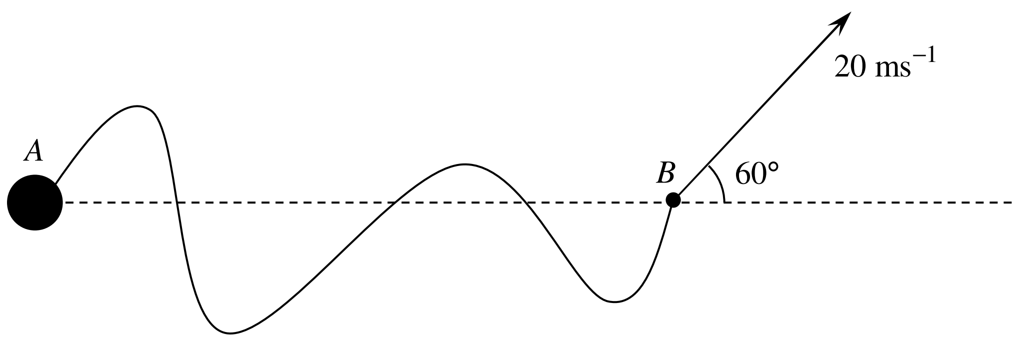
Question 15

Two particles, A and B, of mass 4 kg and 1 kg respectively, are each attached to the ends of a light inextensible string of length $\sqrt{3}a$ m. The particles are placed on a smooth horizontal surface so that $|AB| = a$ m.

A is projected along the surface with speed 20 ms^{-1} in a direction which makes an angle of $60°$ with the straight line joining the initial positions of A and B, as shown in the figure below.



When the string becomes taut, determine the magnitude of the impulsive tension in the string and show further that the speed of B is $4\sqrt{7} \text{ ms}^{-1}$. **(12)**