IYGB GCE

Mathematics M456

Advanced Level

Practice Paper B

Difficulty Rating: 3.26

Time: 3 hours 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Advanced Level Further Mechanics Syllabi, assessed between 1993 and 2005 and with minor omissions between 2006 and 2017.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 18 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

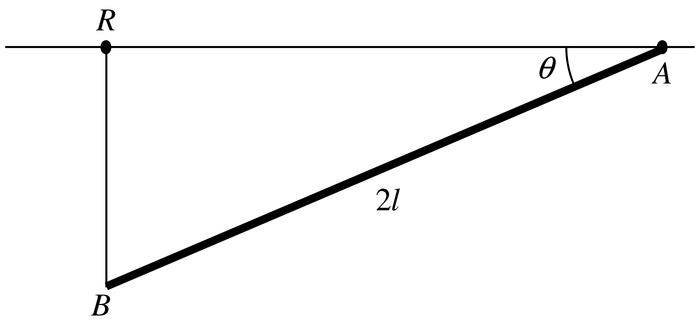
A particle of mass 0.5 kg is moving in a straight line on a smooth horizontal surface.

The particle is acted on by a horizontal force of magnitude $(4t-9)$ N, where t represents the time, in seconds, measured from a certain instant.

At time $t=1$, the particle has speed 6 ms^{-1}.

Find the value of t when the particle has a speed of 18 ms^{-1}. **(6)**

Question 2



A small light smooth ring is attached to the end A of a uniform rod AB, of mass m and length $2l$. The ring is threaded on a smooth horizontal wire. Another small light smooth ring R is threaded on the same wire and a light elastic spring has one end attached to R and the other end attached to B. The system rest is equilibrium with B vertically below R, as shown in the figure above.

The angle BAR is denoted by θ, $0 \le \theta \le \frac{\pi}{2}$.

Find the two positions of equilibrium of the system and determine their stability. **(10)**

Question 3

A rocket is moving vertically upwards relative to the surface of the earth. The motion takes place close to the surface of the earth and it is assumed that g is the constant gravitational acceleration.

At time t the mass of the rocket is $M(1-kt)$, where M and k are positive constants, and the rocket is moving upwards with speed v.

The rocket expels fuel vertically downwards with speed u relative to the rocket.

Given further that when $t=0$, $v=0$ determine an expression for v at time t, in terms of u, g and k. **(10)**

Question 4

A particle moves with constant speed u on the curve with intrinsic equation

$$s = a\tan\psi, \quad 0 \le \psi < \frac{\pi}{2},$$

where a is a positive constant, s is measured from the origin O, and ψ is the angle the tangent to the curve makes with the x axis.

Show that the magnitude of the normal component of the acceleration of the particle, t seconds after starting from the point where $\psi = 0$, is given by

$$\frac{au^2}{a^2+u^2t^2}.$$ **(9)**

Question 5

In this question take $\boldsymbol{g = 10 \text{ ms}^{-2}}$**.**

A particle of mass M kg is released from rest from a height H m, and allowed to fall down through still air all the way to the ground.

Let v ms^{-1} be the velocity of the particle t s after it was released.

The motion of the particle is subject to air resistance of magnitude $\frac{mv^2}{60}$.

Given that the particle reaches the ground with speed 14 ms^{-1}, find the value of H. **(9)**

Question 6

In a plane polar coordinate system (r,θ), the base unit vectors are defined as $\hat{\mathbf{r}}$ in the direction of r increasing, and $\hat{\boldsymbol{\theta}}$ perpendicular to $\hat{\mathbf{r}}$, in the direction of θ increasing.

a) Given that the position vector $\mathbf{r}$ of a particle P is given by $\mathbf{r} = r\hat{\mathbf{r}}$, derive expressions for the velocity and acceleration of P in plane polar coordinates.

You may assume standard differentiation results for $\hat{\mathbf{r}}$ *and* $\hat{\boldsymbol{\theta}}$. **(6)**

b) If $r^2 \frac{d\theta}{dt}$ is constant state what can be deduced about the force acting on P. **(1)**

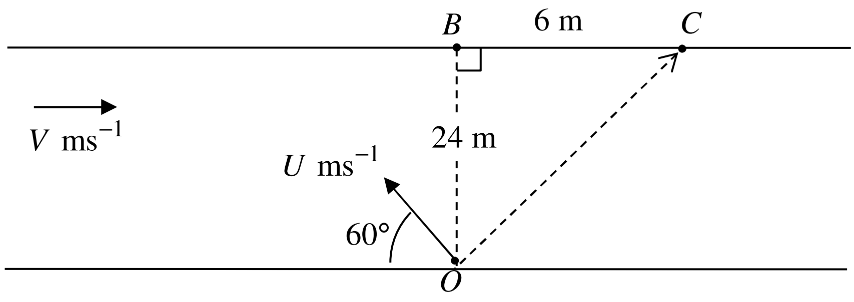
P is moving on the curve with polar equation

$$r = 2 + \cos\theta, \quad 0 \le \theta < 2\pi,$$

with **constant** angular speed $\sqrt{5}$ rad s^{-1}.

c) Find the speed and the magnitude of the acceleration of P, when $\theta = \frac{\pi}{2}$. **(7)**

Question 7



A river flows with constant speed of V ms^{-1}, throughout its width. The banks of the river are modelled as parallel lines of constant width of 24 m.

A boat starts at O and travels upstream with constant speed U ms^{-1} relative to the water, in a direction of 60° to the river bank, as shown in the figure above. The point B is on the opposite river bank so that OB is perpendicular to both river banks.

The point C is on the same bank as B, so that $|BC| = 6$ m, downstream.

To a stationary observer on one of the banks of the river the boat sails on a straight line from O to C.

Show that $V = \frac{1}{9}\left(3 + 4\sqrt{3}\right)$. **(9)**

Question 8

The unit vectors **i**, **j** and **k** are oriented in the positive x direction, positive y direction and positive z direction, respectively.

Three forces

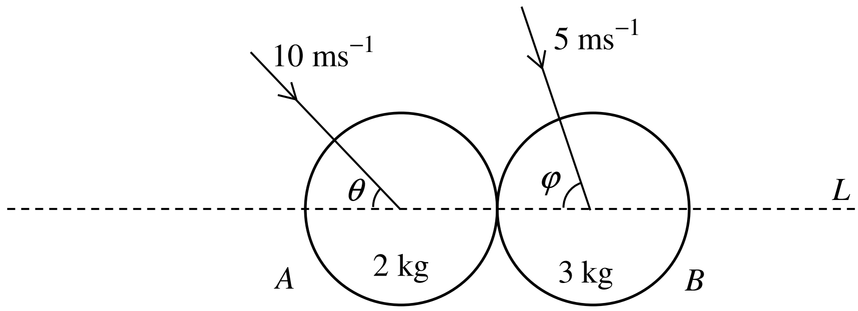
$$\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j})\,\text{N}, \quad \mathbf{F}_2 = (4\mathbf{i} - \mathbf{j} + 2\mathbf{k})\,\text{N} \quad \text{and} \quad \mathbf{F}_2 = (3\mathbf{j} - 4\mathbf{k})\,\text{N}$$

are acting at the points $A_1(-1,1,0)$, $A_2(2,0,5)$ and $A_3(-6,2,1)$, respectively.

a) Show that the system reduces to a single force **F**. **(6)**

b) Find an equation of the line of action of **F**. **(4)**

Question 9



Two smooth spheres, A and B, of equal radius and respective masses 2 kg and 3 kg, are moving on a smooth horizontal plane when they collide obliquely.

The speeds of A and B before their collision are 10 ms^{-1} and 5 ms^{-1} respectively.

When A and B collide the straight line through their centres is denoted by L.

The direction of the speed of A before the collision is at an acute angle θ to L, where $\tan\theta = \frac{3}{4}$.

The direction of the speed of B before the collision is at an acute angle φ to L, where $\tan\varphi = \frac{4}{3}$, as shown in the figure above.

The coefficient of restitution between A and B is $\frac{1}{2}$.

Calculate the speed of A and the speed of B, after the collision. **(10)**

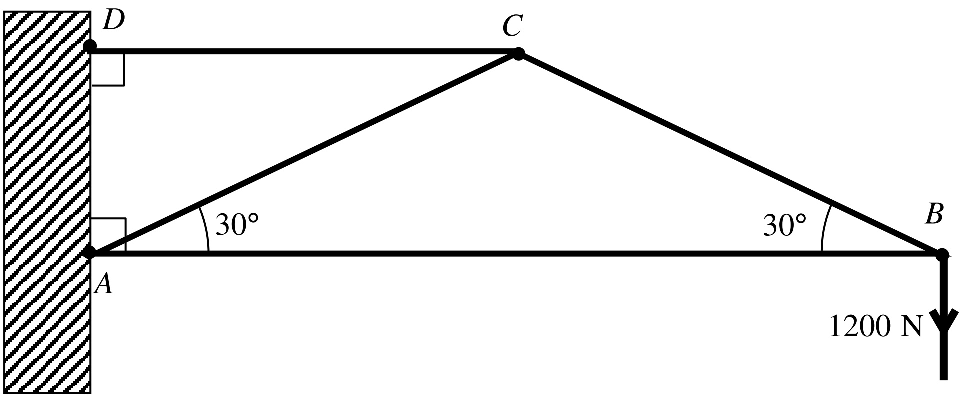
Question 10

A uniform square lamina $ABCD$ has side length a and mass m. The lamina is free to rotate in a vertical plane about a fixed horizontal axis, which is perpendicular to the plane of the lamina and passes through O, the centre of the lamina. Two particles, each of mass m, are attached to the vertices A and B. The system is released from rest with AB vertical.

Find, in terms of a and g, the angular velocity of the system when AB is horizontal.

(8)

Question 11



A light rigid framework consists of 4 light pin jointed rods, AB, AC, BC and CD, where $|AC| = |BC|$, $\measuredangle CAB = \measuredangle ABC = 30°$ and $\measuredangle ADC = \measuredangle BAD = 90°$, as shown in the figure above.

The framework is freely hinged at the points A and D and a weight of 1200 N is supported at B as shown the figure above.

Find the magnitude of the reaction forces acting on the framework at A and D, and the magnitudes of all the internal forces acting on each of the four rods, classifying them as tension or thrust.

(12)

Question 12

A uniform circular disc, of radius $3a$ and mass $2m$, is free to rotate about a smooth fixed horizontal axis L, which is perpendicular to the plane of the disc and is passing through the point A, which lies at the circumference of the disc. The disc is held with its centre O at the same horizontal level as A, and released from rest.

Show that the horizontal component of the force exerted on L has magnitude

$$\frac{2}{3} mg\sqrt{1+48\sin^2\theta},$$

where θ is the angle that AO makes with the horizontal.

(16)

Question 13

A particle P, of mass 2 kg, is attached to one end of a light elastic spring of natural length 0.55 m and stiffness 8 Nm^{-1}.

The other end of the spring is attached to a fixed point O, so that P is hanging in equilibrium vertically below O.

At time $t=0$, P is pulled vertically downwards, so that $OP = 1.5$ m ,and released from rest.

The motion of P takes place in a medium which provides resistance of magnitude $10|v|$ N, where $|v| \text{ ms}^{-1}$ is the speed of P at time t s.

If x denotes the distance of the particle from O at time t, express x in terms of t.

(13)

Question 14

The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to each other.*

A particle P moves from the point A, with position vector $(-10\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ m to the point B, with position vector $(8\mathbf{i} + 11\mathbf{j} + 9\mathbf{k})$ m, under the action of the following three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.

- $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$ N.
- $\mathbf{F}_2 = (7\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$ N
- $\mathbf{F}_3 = [(2k-1)\mathbf{i} + (2k+2)\mathbf{j} + (3-2k)\mathbf{k}]$ N, where k is a scalar constant.

Determine the work done by the three forces in moving the particle from A to B. **(7)**

Question 15

Relative to a fixed origin O the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are oriented horizontally and vertically upwards, respectively.

The gravitational acceleration constant g is taken to be $-10\mathbf{j}\ \text{ms}^{-2}$ in this question.

A particle is projected with velocity $(u\mathbf{i}+v\mathbf{j})\text{ms}^{-1}$, where u and v are positive constants, from a point P with position vector $105\mathbf{j}$ m.

The particle moves freely under gravity passing through the point Q with position vector $210\mathbf{i}$ m.

a) Show clearly that

$$u^2+2uv=2100.$$ **(6)**

b) Given that when $t=2$ the particle is moving parallel to $\mathbf{i}$, determine the time it takes the particle to travel from P to Q. **(6)**

The particle passes through the point R with a speed of $10\sqrt{29}\ \text{ms}^{-1}$.

c) Show R is 80 m below the level of P. **(4)**

Question 16

A thin uniform shell in the shape of a right circular cylinder of radius a and height h has both its circular ends removed.

The resulting open cylindrical shell has mass M.

Show by integration that the moment of inertia of this shell about a diameter coplanar with one of its removed circular ends, is given by

$$\frac{1}{6}M\left(3a^2+2h^2\right).$$ **(12)**

[*In this proof, you may assume standard results for the moment of inertia of uniform circular hoops.*]

Question 17

The point O lies on the foot of a fixed plane which is inclined at an angle of $45°$ to the horizontal. A particle is projected from O, up the line of greatest slope of the plane, with speed of u.

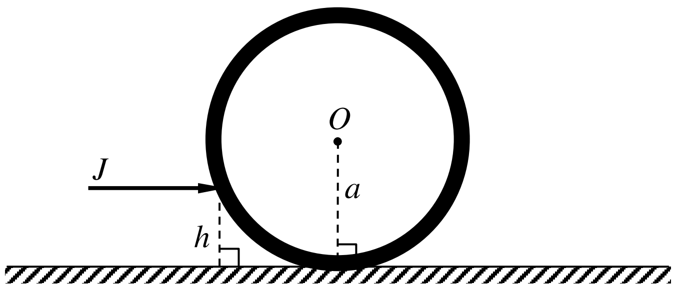
The gravitational acceleration g is assumed constant and air resistance is ignored.

Given that the particle achieves the greatest range up the plane, determine the angle of projection. **(12)**

Question 18

A uniform solid sphere, of mass m and radius a, lies at rest on a rough horizontal surface when it is set in motion by a horizontal impulse of magnitude J.

The impulse is applied at a height of $\frac{1}{2}a$ above the horizontal surface, in a vertical plane through the centre of the sphere O, as shown in the figure below.



(17)

Determine the speed of O as a fraction of its original speed, when the sphere first begins to roll along the surface.
