IYGB - FS1 PAPER P, QUESTION 1

a) X = NUMBER OF BACTERIA PER 250 ml

$X \sim Po(0.75)$

$\frac{1}{4} \times 3$

I) $P(X=2) = \frac{e^{-0.75} \times 0.75^2}{2} = 0.1329$

II) $P(X \geqslant 2) = 1 - P(X \leqslant 1) = 1 - 0.82664\ldots = 0.1734$

b) ADJUSTING THE RATE TO 2 LITRES — $3 \times 2 = 6$

Y = NUMBER OF BACTERIA PER 2 LITRES

$Y \sim P(6)$

$$P(6 \leqslant Y < 10) = P(6 \leqslant Y \leqslant 9) = P(Y \leqslant 9) - P(Y \leqslant 5)$$

$$= 0.91608\ldots - 0.44568\ldots$$

$$= 0.4704$$

IYGB - FS2 PAPER P - QUESTION 2

X = WEIGHT OF A MINI CAKE

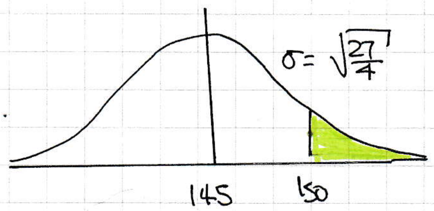
$X \sim N(145, 9^2)$

THE SAMPLING DISTRIBUTION OF THE MEAN WILL ALSO BE NORMAL

$$\bar{X}_{12} \sim N\left(145, \frac{9^2}{12}\right)$$

$$\bar{X}_{12} \sim N\left(145, \frac{27}{4}\right)$$

USING A STANDARD DIAGRAM



$P(\bar{X}_{12} > 150)$

$= 1 - P(\bar{X}_{12} < 150)$

$= 1 - P\left(Z < \frac{150-145}{\sqrt{27/4}}\right)$

$= 1 - \Phi(1.9245)$

... tables or calculator

$= 1 - 0.9729$

$= 0.0271$

IYGB – FS1 PAPER P – QUESTION 3

a) $\boxed{\begin{array}{l} X = \text{NUMBER OF LETTERS DELIVERED INCORRECTLY} \\ X \sim B(200, 0.02) \end{array}}$

$$P(X=6) = \binom{200}{6}(0.02)^6(0.98)^{194} \simeq 0.1047$$

b) As n is large & p is small approximate by $X \sim Po(\underset{np}{4})$

$$P(X>8) = P(X \geqslant 9)$$
$$= 1 - P(X \leqslant 8)$$
$$= 1 - 0.9786365\ldots$$
$$= 0.0214$$

IYGB – FS1 PAPER P – QUESTION 4

a) WRITE THE FORMULA INTO A TABLE FORM

x	-2	-1	0	1	~~2~~
$P(X=x)$	$16k$	$9k$	$4k$	k	~~0~~

$$16k + 9k + 4k + k = 1$$
$$30k = 1$$
$$k = \frac{1}{30}$$

b) USING THE STANDARD FORMULAE

I) $E(X) = \sum x P(X=x)$

$$E(X) = (-2 \times 16k) + (-1 \times 9k) + (0 \times 4k) + (1 \times k)$$
$$E(X) = -32k - 9k + k$$
$$E(X) = -40k$$
$$E(X) = -\frac{4}{3}$$

II) $E(X^2) = \sum x^2 P(X=x)$

$$E(X^2) = (-2)^2 \times 16k + (-1)^2 \times 9k + 0^2 \times 4k + 1^2 \times k$$
$$E(X^2) = 64k + 9k + k$$
$$E(X^2) = 74k$$
$$E(X^2) = \frac{37}{15} \simeq 2.4666\ldots$$

IYGB - ES1 PAPER P - QUESTION 4

c) I) USING THE STANDARD TRANSFORMATION EQUATIONS

$$E(1-15X) = E(-15X+1)$$
$$= -15E(X) + 1$$
$$= -15 \times \left(-\tfrac{4}{3}\right) + 1$$
$$= 21$$

II) NEED THE VARIANCE OF X FIRST

$$\text{Var}(X) = E(X^2) - (E(X))^2$$
$$\text{Var}(X) = \frac{37}{15} - \left(-\frac{4}{3}\right)^2$$
$$\text{Var}(X) = \frac{37}{15} - \frac{16}{9}$$
$$\text{Var}(X) = \frac{31}{45}$$

HENCE WE CAN NOW TRANSFORM

$$\text{Var}(1-15X) = \text{Var}(-15X+1)$$
$$= (-15)^2\,\text{Var}(X)$$
$$= 225 \times \frac{31}{45}$$
$$= 155$$

IYGB – FS1 PAPER P – QUESTION 5

X = NUMBER OF CUSTOMERS WALKING IN EVERY 10 MINUTES

$X \sim Po(8)$

a) $H_0: \lambda = 8$

$H_1: \lambda \neq 8$

WHERE λ IS RATE OF CUSTOMERS WALKING IN PER 10 MINUTES, IN GENERAL

CRITICAL REGION REQUIRED AT 5%, TWO TAILED, IE 2.5% EACH TAIL

LOOKING AT TABLES

$P(X \leq 2) = 0.0138 = 1.38\% < 2.5\%$

$P(X \leq 3) = 0.0424 = 4.24\% > 2.5\%$

⋮

$P(X \geq 14) = 1 - P(X \leq 13) = 1 - 0.9658 = 0.0342 = 3.42\% > 2.5\%$

$P(X \geq 15) = 1 - P(X \leq 14) = 1 - 0.9827 = 0.0173 = 1.73\% < 2.5\%$

∴ CRITICAL REGION

$$\{0, 1, 2\} \cup \{15, 16, 17, \ldots\}$$

b) TOTAL SIGNIFICANCE $= 1.38\% + 1.73\% = 3.11\%$

c) 14 IS NOT IN THE CRITICAL REGION

THERE IS NO SIGNIFICANT EVIDENCE THAT THE MEAN RATE HAS CHANGED

NO SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS1 PAPER P - QUESTION 6

IF MODELLED BY POISSON, DETERMINE THE MEAN

$$\text{MEAN} = \frac{(0\times5)+(1\times38)+(2\times32)+(3\times17)+(4\times7)+(5\times1)}{100} = 1.86$$

FORMING A TABLE

x_i	OBSERVED $= O_i$	EXPECTED $= E_i = P(X=i)\times100$	$\frac{(O_i-E_i)^2}{E_i}$
0	5	$e^{-1.86}\times100 = 15.567\ldots$	$7.173\ldots$
1	38	$e^{-1.86}\times1.86\times100 = 28.955\ldots$	$2.825\ldots$
2	32	$\frac{e^{-1.86}\times1.86^2}{2}\times100 = 26.928\ldots$	$0.955\ldots$
3	17	$\frac{e^{-1.86}\times1.86^3}{3!}\times100 = 16.695\ldots$	$0.006\ldots$
4	7	$\frac{e^{-1.86}\times1.86^4}{4!}\times100 =$	
5	1	$\frac{e^{-1.86}\times1.86^5}{5!}\times100 =$	
6	0	$\ldots\ldots =$	
4, 5, 6 combined	8	11.849	1.251

LESS THAN 5

H_0 : DATA CAN BE MODELLED BY $Po(1.86)$

H_1 : DATA CANNOT BE MODELLED BY $Po(1.86)$

- $\nu = 5 - 1 - 1 = 3$
- $\chi^2_3(1\%) = 11.345$
- $\sum_{i=1}^{5} \frac{(O_i-E_i)^2}{E_i} = 12.210$

AS $12.210 > 11.345$ THERE IS SIGNIFICANT EVIDENCE THAT THIS DATA CANNOT BE MODELLED BY A POISSON DISTRIBUTION — SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS1 PAPER P - QUESTION 7

WORKING AS FOLLOWS

- LET p BE THE PROBABILITY OF OBTAINING 4 HEADS
- LET X BE THE NUMBER OF TOSSES UNTIL WE OBTAIN A HEAD

$$\Rightarrow P(X=2) = (1-p)p$$
$$\Rightarrow P(X=4) = (1-p)^3 p$$
$$\Rightarrow P(X=6) = (1-p)^5 p \ldots$$

$$\Rightarrow p(1-p) + p(1-p)^3 + p(1-p)^5 + p(1-p)^7 + \ldots = \frac{2}{5}$$
$$\Rightarrow p\underbrace{\left[(1-p) + (1-p)^3 + (1-p)^5 + (1-p)^7 + \ldots\right]}_{\text{THIS IS A G.P WITH } a = 1-p,\ r = (1-p)^2} = \frac{2}{5}$$

USING THE SUM TO INFINITY FORMULA $S_\infty = \frac{a}{1-r}$

$$\Rightarrow p\left(\frac{1-p}{1-(1-p)^2}\right) = \frac{2}{5}$$
$$\Rightarrow \frac{p-p^2}{1-(1-2p+p^2)} = \frac{2}{5}$$
$$\Rightarrow \frac{p-p^2}{2p-p^2} = \frac{2}{5}$$

AS $p \neq 0$ WE MAY DIVIDE IT THROUGH IN THE L.H.S

$$\Rightarrow \frac{1-p}{2-p} = \frac{2}{5}$$
$$\Rightarrow 5-5p = 4-2p$$
$$\Rightarrow 1 = 3p$$
$$\Rightarrow p = \frac{1}{3}$$

IYGB - FS1 PAPER P - QUESTION 8

a) NO REAL NEED TO WRITE THE PROBABILITY DISTRIBUTION IN A TABLE

$$G_X(t) = \sum P(X=x)t^x$$

$$= \frac{1^2}{k}t^1 + \frac{2^2}{k}t^2 + \frac{3^2}{k}t^3 + \frac{4^2}{k}t^4 + \frac{5^2}{k}t^5$$

$$= \frac{1}{k}\left[t + 4t^2 + 9t^3 + 16t^4 + 25t^5\right]$$

$$\therefore G_X(t) = \frac{1}{k}\sum_{r=1}^{5}\left(r^2 t^r\right)$$

b) EASIER TO DIFFERENTIATE IN SIGMA NOTATION — NOTE THAT ALL THE DIFFERENTIATIONS ARE WITH RESPECT TO t

$$\Rightarrow \frac{d}{dt}\left(G_X(t)\right) = G_X'(t) = \frac{1}{k}\sum_{r=1}^{5}\left[r^2\left(rt^{r-1}\right)\right] = \frac{1}{k}\sum_{r=1}^{5}\left(r^3 t^{r-1}\right)$$

$$\Rightarrow \frac{d^2}{dt^2}\left(G_X(t)\right) = \frac{d}{dt}\left[\frac{1}{k}\sum_{r=1}^{5}\left(r^3 t^{r-1}\right)\right] = \frac{1}{k}\sum_{r=1}^{5}\left[r^3(r-1)t^{r-2}\right]$$

AS THE FIRST TERM IS ZERO ($r=1$) WE MAY START FROM $r=2$

$$\therefore G_X''(t) = \frac{1}{k}\sum_{r=2}^{5}\left[r^3(r-1)t^{r-2}\right]$$

c) USING THE FACT $G_X(1) = 1$

$$\Rightarrow G_X(t) = \frac{1}{k}\sum_{r=1}^{5}\left(r^2 t^r\right) = \frac{1}{k}\left(1^2 \times t^1 + 2^2 \times t^2 + 3^2 \times t^3 + 4^2 \times t^4 + 5^2 \times t^5\right)$$

$$\Rightarrow 1 = \frac{1}{k}\left(1^2 + 2^2 + 3^2 + 4^2 + 5^2\right)$$

$$\Rightarrow 1 = \frac{1}{k}(1 + 4 + 9 + 16 + 25)$$

$$\Rightarrow k = 55$$

EASIER TO HAVE USED THE EXPANDED FORM FROM ABOVE

$$G_X(t) = \frac{1}{k}\left(t + 4t^2 + 9t^3 + 16t^4 + 25t^5\right)$$

IYGB - FS1 PAPER P - QUESTION 8

d) USING ALL THE PREVIOUS RESULTS WITH $k=55$

- $$G'_X(t) = \frac{1}{k}\sum_{r=1}^{5}\left(r^3 t^{r-1}\right)$$

$$G'_X(1) = \frac{1}{k}\sum_{r=1}^{5} r^3$$

$1^{r-1}=1$

$$G'_X(1) = \frac{1}{55}(1+8+27+64+125)$$

$$\boxed{G'_X(1) = \frac{45}{11}}$$

- $$G''_X(t) = \frac{1}{k}\sum_{r=2}^{5}\left[r^3(r-1)t^{r-2}\right]$$

$$G''_X(t) = \frac{1}{k}\sum_{r=2}^{5}\left[r^3(r-1)\right]$$

$1^{r-2}=1$

$$G''_X(1) = \frac{1}{55}\left[2^3\times1 + 3^3\times2 + 4^3\times3 + 5^3\times4\right]$$

$$G''_X(1) = \frac{1}{55}(8+54+192+500)$$

$$\boxed{G''_X(1) = \frac{754}{11}}$$

FINALLY WE HAVE

$$\text{Var}(X) = G''_X(1) + G'_X(1) - \left[G'_X(1)\right]^2$$

$$= \frac{754}{11} + \frac{45}{11} - \left(\frac{45}{11}\right)^2$$

$$= \frac{644}{605} \simeq 1.06$$