IYGB - FS1 PAPER N - QUESTION 1

$X \sim NB(6, 0.4)$, ie $r=6$ $p=0.4$

a) $E(X) = \frac{r}{p} = \frac{6}{0.4} = 15$

b) $Var(X) = \frac{r(1-p)}{p^2} = \frac{6(1-0.4)}{0.4^2} = \frac{6 \times 0.6}{0.16} = 22.5$

c) $P(X=12) = \binom{11}{5} 0.4^5 0.6^6 \times 0.4 = \binom{11}{5} 0.4^6 0.6^6$

$= 462 \times 0.24^6$

$= 0.0883$

IYGB – FS1 PAPER N – QUESTION 2

x	0	1	2	3	4
$P(X=x)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	a	$\frac{1}{24}$

a) $\sum P(X=x) = 1$

$\frac{3}{8} + \frac{1}{3} + \frac{1}{4} + a + \frac{1}{24} = 1$

$1 + a = 1$

$a = 0$

b) $E(X) = \sum x P(X=x)$

$\Rightarrow E(X) = \left(0 \times \frac{3}{8}\right) + \left(1 \times \frac{1}{3}\right) + \left(2 \times \frac{1}{4}\right) + (3 \times 0) + \left(4 \times \frac{1}{24}\right)$

$\Rightarrow E(X) = 0 + \frac{1}{3} + \frac{1}{2} + 0 + \frac{1}{6}$

$\Rightarrow E(X) = 1$

c) $E(X^2) = \sum x^2 P(X=x)$

$\Rightarrow E(X^2) = \left(0^2 \times \frac{3}{8}\right) + \left(1^2 \times \frac{1}{3}\right) + \left(2^2 \times \frac{1}{4}\right) + (3^2 \times 0) + \left(4^2 \times \frac{1}{24}\right)$

$\Rightarrow E(X^2) = 0 + \frac{1}{3} + 1 + 0 + \frac{2}{3}$

$\Rightarrow E(X^2) = 2$

$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$

$\mathrm{Var}(X) = 2 - 1^2$

$\mathrm{Var}(X) = 1$

IYGB - FS1 PAPER N - QUESTION 3

a) $\boxed{X \sim Geo(0.2)}$

- TRIALS ARE INDEPENDENT OF ONE ANOTHER
- TRIALS HAVE CONSTANT PROBABILITY OF SUCCESS

b) I) $P(X=3) = 0.8^2 \times 0.2 = \frac{16}{125} = \underline{0.128}$

II) $P(X>8) = P(X \geqslant 9) = (0.8)^8 = \underline{0.1678}$

III) $P(5 \leqslant X < 13) = P(5 \leqslant X \leqslant 12)$

$$= P(X \leqslant 12) - P(X \leqslant 4)$$

$$= \left[1 - P(X \geqslant 13)\right] - \left[1 - P(X \geqslant 5)\right]$$

$$= P(X \geqslant 5) - P(X \geqslant 13)$$

$$= (0.8)^4 - (0.8)^{12}$$

$$= \underline{0.3409}$$

IYGB - FS1 - PAPER N - QUESTION 4

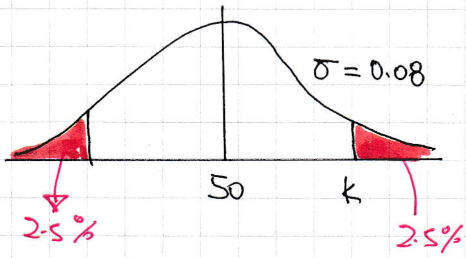
W = WEIGHT OF BAG OF CEMENT
$W \sim N(50, 0.4^2)$

$$\bar{X}_{25} \sim N\left(50, \frac{0.4^2}{25}\right) \quad \text{ie} \quad \bar{X}_{25} \sim N(50, 0.08^2)$$

a) ACTUAL SIGNIFICANCE FOR CONTINUOUS VARIABLES = SIGNIFICANCE LEVEL

$\therefore$ P(TYPE I ERROR) = 5%

b) LOOKING AT THE HYPOTHESES, WE HAVE TWO TAILED AT 2.5% (PER TAIL)



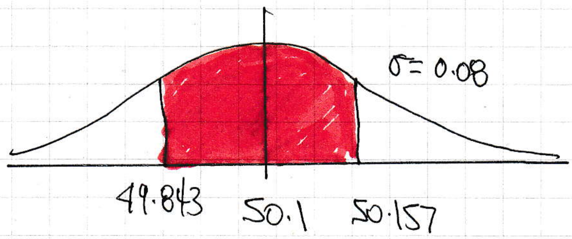
$$P(\bar{X} > k) = 0.025$$
$$P(X < k) = 0.975$$
$$P\left(Z < \frac{k-50}{0.08}\right) = 0.975$$
$$\frac{k-50}{0.08} = +\Phi^{-1}(0.975)$$
$$\frac{k-50}{0.08} = 1.96$$
$$k = 50.1568$$

[& BY SYMMETRY : 49.8432]

$\therefore$ CRITICAL REGION

$$\bar{X}_{25} < 49.843 \ \cup \ \bar{X} > 50.157$$

c) REDRAW THE DIAGRAM WITH $\mu = 50.1$



P(TYPE II ERROR)
= P(REJECT H_1 WITH H_1 TRUE)
$= P(49.8 < \bar{X}_{25} < 50.157)$

IYGB, FS1 PAPER QUESTION

$$= P(\bar{X}_{25} < 50.157) - P(\bar{X}_{25} < 49.843)$$

$$= P(\bar{X}_{25} < 50.157) - [1 - P(\bar{X}_{25} > 49.843)]$$

$$= P(\bar{X}_{25} < 50.157) + P(\bar{X}_{25} > 49.843) - 1$$

$$= P\left(Z < \frac{50.157 - 50.1}{0.08}\right) + P\left(Z > \frac{49.843 - 50.1}{0.08}\right) - 1$$

$$= \Phi(0.7125) + \Phi(-0.3125) - 1$$

TABLES OR CALCULATOR ...

$$= 0.7619 + 0.9993 - 1$$

$$= 0.7612$$

IYGB - FS1 PAPER N - QUESTION 5

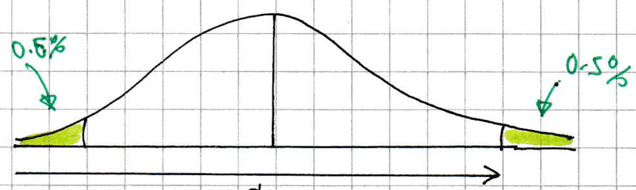
a)

$H_0 : \mu = 33$

$H_1 : \mu \neq 33$

WHERE μ IS THE MEAN AGE OF EVERY SINGLE CUSTOMERS OF THIS SHOP

- $\bar{x} = 35.6$
- $s = 8.2$
- $n = 64$
- 1% TWO TAILED



$\Phi^{-1}(0.995) = 2.5758$

$$Z \text{ STAT} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{35.6-33}{8.2/\sqrt{64}} = 2.5365\ldots$$

AS $2.5365 < 2.5758$ THE CLAIM OF THE MANAGER IS JUSTIFIED (NOT SIGNIFICANT) — INSUFFICIENT EVIDENCE TO REJECT H_0

b)

ASSUMPTIONS MADE

- SAMPLE IS RANDOM
- STANDARD DEVIATION OF THE SAMPLE = STANDARD DEVIATION OF THE POPULATION

ALTHOUGH IT IS NOT KNOWN IF THE AGES ARE NORMALLY DISTRIBUTED THE DISTRIBUTION OF THEIR MEAN FOR LARGE SAMPLES ($n > 30$) WILL BE APPROXIMATELY NORMAL, BY THE CENTRAL LIMIT THEOREM

IYGB - FS1 PAPER N - QUESTION 6

SETTING HYPOTHESES

H_0: Data could be modelled by $B(5, 0.2)$

H_1: Data could not be modelled by $B(5, 0.2)$

ANALYSING THE DATA TO OBTAIN EXPECTED FREQUENCIES & CONTRIBUTIONS

NUMBER x	FREQUENCY O_i	EXPECTED FREQUENCIES $E_i = P(X=x) \times 80$	CONTRIBUTIONS $\frac{(O_i - E_i)^2}{E_i}$
0	15	$0.32768 \times 80 = 26.2144$	4.797
1	36	$0.4096 \times 80 = 32.768$	0.319
2	17	$0.2048 \times 80 = 16.384$	
3	10	$0.0512 \times 80 = 4.096$	3.032
4	1	$0.0064 \times 80 = 0.512$	
5	1	$0.00032 \times 80 = 0.0256$	
	80		

Frequencies for $x = 2$ to 5: $17 + 10 + 1 + 1 = 29$; expected frequencies for $x = 2$ to 5: 21.0176

- $\nu = 3 - 1 = 2$
- $\chi^2_2(5\%) = 5.991$
- $\sum_{i=1}^{3} \frac{(O_i - E_i)^2}{E_i} = 8.148$

AS $8.148 > 5.991$ IT APPEARS THAT THE DATA COULD NOT BE MODELLED BY $B(5, 0.2)$

SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS1 PAPER N - QUESTION 7

a) THE PROBABILITY MASS FUNCTION FOR $Po(\lambda)$ IS $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$$\Longrightarrow G_X(t) = \sum_{x=0}^{\infty}\left[P(X=x)t^x\right]$$

$$\Longrightarrow G_X(t) = \sum_{x=0}^{\infty}\left(\frac{e^{-\lambda}\lambda^x}{x!}\right)t^x$$

$$\Longrightarrow G_X(t) = \sum_{x=0}^{\infty}\frac{e^{-\lambda}(\lambda t)^x}{x!}$$

$$\Longrightarrow G_X(t) = e^{-\lambda}\sum_{x=0}^{\infty}\frac{(\lambda t)^x}{x!}$$

NOW THE EXPONENTIAL FUNCTION POWER SERIES IS

$$e^y = \frac{y^0}{0!} + \frac{y^1}{1!} + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots = \sum_{r=0}^{\infty}\frac{y^r}{r!}$$

THUS WE CONCLUDE THAT

$$\Longrightarrow G_X(t) = e^{-\lambda}\sum_{x=0}^{\infty}\frac{(\lambda t)^x}{x!} = e^{-\lambda}e^{\lambda t} = e^{\lambda t-\lambda}$$

$$\Longrightarrow G_X(t) = e^{\lambda(t-1)}$$

b) OBTAIN THE FIRST TWO DERIVATIVES OF $G_X(t)$ WITH RESPECT TO t

- $G_X(t) = e^{\lambda t-\lambda}$
- $G'_X(t) = \lambda e^{\lambda t-\lambda} \Longrightarrow G'_X(1) = \lambda e^0 = \lambda$
- $G''_X(t) = \lambda^2 e^{\lambda t-\lambda} \Longrightarrow G''_X(1) = \lambda^2 e^0 = \lambda^2$

$$E(X) = G'_X(1) = \lambda$$

$$\mathrm{Var}(X) = G''_X(1) + G'_X(1) - [G'(1)]^2$$
$$= \lambda^2 + \lambda - \lambda^2$$
$$= \lambda$$

IYGB - FS1 PAPER N - QUESTION 7

c) FIRSTLY $E(X) = \lambda$ & $E(Y) = \mu$

LET $W = X + Y$

$$G_W(t) = G_X(t)\,G_Y(t)$$

$$G_W(t) = e^{\lambda(t-1)}\,e^{\mu(t-1)}$$

$$G_W(t) = e^{\lambda(t-1)+\mu(t-1)}$$

$$G_W(t) = e^{(\lambda+\mu)(t-1)}$$

$$G_W(t) = e^{(\lambda+\mu)t-(\lambda+\mu)}$$

DIFFERENTIATE WITH RESPECT TO t

$$G'_W(t) = (\lambda+\mu)\,e^{(\lambda+\mu)t-(\lambda+\mu)}$$

$$G'_W(1) = (\lambda+\mu)\,e^{(\lambda+\mu)-(\lambda+\mu)}$$

$$G'_W(1) = (\lambda+\mu)\cancel{e^0}$$

$$G'_W(1) = \lambda + \mu$$

$$E(W) = E(X) + E(Y)$$

AS REQUIRED

IYGB - FS1 PAPER N - QUESTION 8

a) START BY DEFINING VARIABLES AND DISTRIBUTIONS

"FISH CATCHING" RATE $\Rightarrow$ 2.5 fish per hour

ADJUSTING THE RATE TO 3 HOURS

X = NO OF FISH CAUGHT PER 3 HOURS

$X \sim Po(7.5)$

- $P(X>9) = P(X \geqslant 10) = 1 - P(X \leqslant 9) = \dots$ tables....

$= 1 - 0.7764 = 0.2236$

Y = NO OF DAYS (OUT OF 5), WHERE MORE THAN 9 FISH IS CAUGHT

$Y \sim B(5, 0.2236)$

- $P(Y=2) = \binom{5}{2}(0.2236)^2(0.7764)^3 = 0.234$ //

b) ADJUSTING THE RATE TO 4 HOURS — $4 \times 2.5 = 12.5$

W = NO OF FISH CAUGHT PER 4 HOURS

$W \sim Po(10)$

- $H_0 : \lambda = 2.5 \; (\mu = 10)$
- $H_1 : \lambda \neq 2.5 \; (\mu = 10)$

- TESTING AT 5% SIGNIFICANCE ON THE BASIS THAT $W = 16$
- $P(W \geqslant 16) = 1 - P(W \leqslant 15)$

$= 1 - 0.9513$

$= 0.0487 > 2.5\%$

THERE IS NO SIGNIFICANT EVIDENCE TO SUGGEST THAT THE "FISH CATCHING" RATE IS DIFFERENT — NOT SUFFICIENT EVIDENCE TO REJECT H_0 //