IYGB - FS1 PAPER M - QUESTION 1

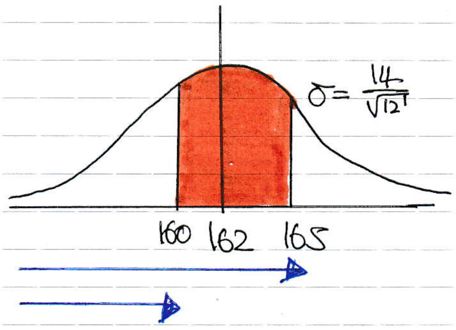
X = MASS OF TOMATOES

$X \sim N(162, 14^2)$

BY THE SAMPLING DISTRIBUTION OF THE MEAN THEOREM

$$\bar{X}_{12} \sim N\left(162, \frac{14^2}{12}\right)$$

LOOKING AT THE DIAGRAM



$$P(160 < \bar{X}_{12} < 165)$$

$$= P(\bar{X}_{12} < 165) - P(\bar{X}_{12} < 160)$$

$$= P(\bar{X}_{12} < 165) - [1 - P(\bar{X}_{12} > 160)]$$

$$= P(\bar{X}_{12} < 165) + P(\bar{X}_{12} > 160) - 1$$

$$= P\left(Z < \frac{165-162}{\frac{14}{\sqrt{12}}}\right) + P\left(Z > \frac{160-162}{\frac{14}{\sqrt{12}}}\right) - 1$$

$$= \Phi(0.7423) + \Phi(-0.4949) - 1$$

$$= 0.77105... + 0.68966 - 1$$

$$\simeq 0.4607$$

IYGB - FS1 PAPER M - QUESTION 2

a) $G_X(t) = k + \frac{1}{5}(t^2 + t^3 + t^5)$

$G_X(1) = 1 \implies 1 = k + \frac{1}{5}(1+1+1)$

$1 = k + \frac{3}{5}$

$k = \frac{2}{5}$

b) WRITE THE PROBABILITY MASS FUNCTION IN TABLE FORM

$\implies G_X(t) = \sum P(X=x)t^x = \frac{2}{5}t^0 + \frac{1}{5}t^2 + \frac{1}{5}t^3 + \frac{1}{5}t^5$

$\therefore$

x	0	2	3	5
$P(X=x)$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\therefore P(X>4) = P(X=5) = \frac{1}{5}$

c) DIFFERENTIATE THE P.G.F TWICE

- $G_X(t) = \frac{2}{5} + \frac{1}{5}(t^2 + t^3 + t^5)$
- $G'_X(t) = \frac{1}{5}(2t + 3t^2 + 5t^4)$
- $G''_X(t) = \frac{1}{5}(2 + 6t + 20t^3)$

- $E(X) = G'_X(1) = \frac{1}{5}(2\times1 + 3\times1^2 + 5\times1^4) = \frac{1}{5}(2+3+5) = 2$

- $\text{Var}(X) = G''_X(1) + G'_X(1) - [G'_X(1)]^2$

$= \frac{1}{5}(2 + 6\times1 + 20\times1^3) + 2 - 2^2$

$= \frac{28}{5} + 2 - 4$

$= \frac{18}{5}$

$= 3.6$

IYGB – FS1 PAPER M – QUESTION 3

USING STANDARD RESULTS FOR $X \sim NB(r, p)$

$$E(X) = \frac{r}{p} \qquad \text{Var}(X) = \frac{r(1-p)}{p^2}$$

$$12 = \frac{r}{p} \qquad 4 = \frac{r(1-p)}{p^2}$$

COMBINE EQUATIONS

$$\Rightarrow 4 = \frac{r}{p}\left(\frac{1-p}{p}\right)$$

$$\Rightarrow 4 = 12\left(\frac{1-p}{p}\right)$$

$$\Rightarrow \frac{1}{3} = \frac{1-p}{p}$$

$$\Rightarrow p = 3 - 3p$$

$$\Rightarrow 4p = 3$$

$$\Rightarrow p = \frac{3}{4}$$

USING $12 = \frac{r}{p}$

$$\Rightarrow 12 = \frac{r}{3/4}$$

$$\Rightarrow r = 9$$

FINALLY WE HAVE

$$X \sim NB(9, 0.75)$$

$$\Rightarrow P(X=12) = \underbrace{\binom{11}{8} 0.75^8 \, 0.25^3}_{\text{8 SUCCESSES IN THE FIRST 11 TRIALS}} \times \underbrace{0.75}_{\text{SUCCESS IN THE 12TH}}$$

$$= \binom{11}{8} 0.75^9 \times 0.25^3$$

$$= 0.1936$$

IYGB – FS1 PAPER M – QUESTION 4

a) $X \sim B(25, 0.25)$ / DETERMINE THE CRITICAL REGION

$H_0 : p = 0.25$

$H_1 : p > 0.25$, p THE PROPORTION IN THE ENTIRE POPULATION

LOOKING AT THE BINOMIAL TABLES

$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9287 = 0.0713 > 0.05$

$P(X \geq 11) = 1 - P(X \leq 10) = 1 - 0.9703 = 0.0297 < 0.05$

$$C.R = \{11, 12, 13, \ldots, 25\}$$

SIZE OF TEST = P(TYPE I ERROR) = ACTUAL SIGNIFICANCE = 0.0297

b) POWER OF A TEST = 1 − P(TYPE II ERROR)

"REJECT H_1 WHEN H_1 IS TRUE"

$P(X \leq 10)$ FROM $B(25, 0.45)$

... tables ...

$= 1 - 0.3843$

$= 0.6157$

IYGB - FS1 PAPER M - QUESTION 5

H_0 : THERE IS NO ASSOCIATION BETWEEN GENDER AND THE CLASS OF THE DEGREE ACHIEVED (INDEPENDENT EVENTS)

H_1 : THERE IS ASSOCIATION BETWEEN GENDER AND THE CLASS OF THE DEGREE ACHIEVED (DEPENDENT EVENTS)

- WRITE THE TABLE WITH ACTUAL DATA

	1ST CLASS	2ND UPPER	2ND LOWER (OR LESS	TOTAL
MALE	54 52.5 0.043	84 88.5 0.229	102 99 0.091	240
FEMALE	16 17.5 0.129	34 29.5 0.686	30 33 0.273	80
TOTAL	70	118	132	320

= ACTUAL DATA / OBSERVED FREQUENCY (O_i)

= EXPECTED FREQUENCY IF INDEPENDENT (E_i)

= CONTRIBUTION $\frac{(O_i - E_i)^2}{E_i}$

- COLLECTING THE REST OF THE INFORMATION

$$\nu = (c-1)(r-1) = (3-1)(2-1) = 2 \times 1 = 2$$

$$\sum_{i=1}^{6} \frac{(O_i - E_i)^2}{E_i} = 1.451$$

$$\chi_2^2(10\%) = 4.605$$

- AS $1.451 < 4.605$ THERE IS SIGNIFICANT EVIDENCE THAT THERE IS NO ASSOCIATION, SO DEAN IS JUSTIFIED — REJECT H_1

IYGB - FS1 PAPER M - QUESTION 6

- DETERMINE THE PROBABILITY SPACE DIAGRAM

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

- THE PROBABILITY DISTRIBUTION OF X IS GIVEN BY

x	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

- BY SYMMETRY $E(X) = 7$

- DETERMINE THE $E(X^2)$

$$E(X^2) = \left(2^2 \times \tfrac{1}{36}\right) + \left(3^2 \times \tfrac{2}{36}\right) + \left(4^2 \times \tfrac{3}{36}\right) + \dots + \left(12^2 \times \tfrac{1}{36}\right)$$

$$= \frac{(2^2 \times 1) + (3^2 \times 2) + (4^2 \times 3) + \dots + (12^2 \times 1)}{36}$$

$$= \frac{1974}{36} = \frac{329}{6}$$

- FINALLY THE VARIANCE

$$\mathrm{Var}(x) = E(x^2) - \left(E(x)\right)^2$$

$$= \frac{329}{6} - 7^2 = \frac{35}{6}$$

IYGB – FS1 PAPER M – QUESTION 7

a) • CUSTOMERS ARRIVE INDEPENDENTLY OF ONE ANOTHER
• CUSTOMERS ARRIVE AT A UNIFORM CONSTANT RATE PER UNIT TIME

b) CUSTOMERS ARE UNLIKELY TO ARRIVE "SINGLY", AS GARDEN CENTRES TEND TO BE VISITED BY COUPLES OR FAMILIES

c) X = NO OF CUSTOMER ARRIVALS PER 10 MINUTES

$X \sim Po(3)$

$$P(X=4) = \frac{e^{-3} \times 3^4}{4!} = 0.1680$$

d) Y = NO OF CUSTOMER ARRIVALS PER 20 MINUTES

$Y \sim Po(6)$

$$P(Y=8) = \frac{e^{-6} \times 6^8}{8!} = 0.1033$$

e) USING THE RESULT OF PART (c)

$$P(X=4) \times P(X=4) = \left(\frac{e^{-3} \times 3^4}{4!}\right)^2 = 0.0282$$

f) EITHER

V = NO OF CUSTOMERS PER MINUTE

$V \sim Po(0.3)$

$$P(V=0) = \frac{e^{-0.3} \times 0.3^0}{0!} = e^{-0.3}$$

∴ REQUIRED PROBABILITY

$$(e^{-0.3})^7 = e^{-2.1} \simeq 0.122$$

OR

W = NO OF CUSTOMERS PER 7 MINUTES

$W \sim Po(2.1)$

$$P(W=0) = \frac{e^{-2.1} \times 2.1^0}{0!}$$

$\simeq 0.1225$

IYGB - FS1 PAPER M - QUESTION 7

g) MODEL AS FOLLOWS, USING VARIABLE X, AS PREVIOUSLY DEFINED

$$P(X=4) = \frac{e^{-3} \times 3^4}{4!}$$

REQUIRED PROBABILITY $= \left(1 - \frac{e^{-3} \times 3^4}{4!}\right)^6 = 0.3316$

h) USING X AGAIN

$$P(X=0) = \frac{e^{-3} \times 0^3}{0!} = e^{-3}$$

REQUIRED PROBABILITY IS $(1-e^{-3})^5 \times e^{-3} \times 6$ WAYS

OR DEFINE A BINOMIAL.

U = NO OF INTERVALS WITHOUT CUSTOMERS

$U \sim B(6, e^{-3})$

$$P(U=1) = \binom{6}{1} \times (e^{-3})^1 \times (1-e^{-3})^5 \approx 0.2314$$

i) G = NO OF GERMINATING SEEDS

G' = NO OF NON GERMINATING SEEDS

$G \sim B(125, 0.98)$ AND $G' \sim B(\overset{n}{125}, \overset{p}{0.02})$

$\downarrow$ APPROX BY POISSON

$G' \sim Po(\overset{np}{2.5})$

IYGB - FS1 PAPER M - QUESTION 7

- TO TEST THE HYPOTHESIS WITH G $\quad P(G \leq 117)$ (117 GERMINATED)
- TO TEST THE HYPOTHESIS WITH G' $\quad P(G' \geq 8)$ (8 DIDN'T GERMINATE)

$$\left.\begin{array}{l} H_0 : p = 0.02 \\ H_1 : p > 0.02 \end{array}\right\}$$ WHERE p IS THE PROPORTION OF NON GERMINATING SEEDS IN THE ENTIRE POPULATION

- $P(G' \geq 8) = 1 - P(G' \leq 7)$

$$= 1 - 0.9958$$

$$= 0.0042$$

$$= 0.42\%$$

$$< 1\%$$

THERE IS SIGNIFICANT EVIDENCE THAT THE GARDEN CENTRE OVERSTATES THE GERMINATION PROPORTION OF ITS SEEDS - SUFFICIENT EVIDENCE TO REJECT H_0

NOTE IT IS FINE TO PUT HYPOTHESES AS

$$\left.\begin{array}{l} p = 0.98 \\ p < 0.98 \end{array}\right\}$$ WHERE p IS THE PROPORTION OF GERMINATING SEEDS

& THE REST IDENTICAL AS ABOVE

IYGB - FS1 PAPER M - QUESTION 8

- LET THE PROBABILITY OF SUCCESS OF DEAN BE p, $0 < p < 1$

- LET MARKUS GO FIRST

$$P(X=1) = 0.2$$
$$P(X=2) = 0.8(1-p) \times 0.2$$
$$P(X=3) = 0.8(1-p)\,0.8(1-p) \times 0.2$$
$$P(X=4) = 0.8(1-p)\,0.8(1-p)\,0.8(1-p) \times 0.2$$

ETC

- NOW FORM AN EXPRESSION FOR A MARKUS WIN, KNOWN TO BE $\frac{5}{13}$

$$\Rightarrow P(X=1) + P(X=2) + P(X=3) + P(X=4) + \dots = \frac{5}{13}$$
$$\Rightarrow 0.2 + 0.8(1-p) \times 0.2 + [0.8(1-p)]^2 \times 0.2 + [0.8(1-p)]^3 \times 0.2 + \dots = \frac{5}{13}$$
$$\Rightarrow 0.2\Big[\underbrace{1 + 0.8(1-p) + [0.8(1-p)]^2 + [0.8(1-p)]^3 + \dots}\Big] = \frac{5}{13}$$

THIS IS A G.P WITH $a = 1$, $r = 0.8(1-p)$

$$S_\infty = \frac{a}{1-r}$$

$$\Rightarrow \frac{1}{5} \times \frac{1}{1 - 0.8(1-p)} = \frac{5}{13}$$
$$\Rightarrow \frac{1}{1 - 0.8 + 0.8p} = \frac{25}{13}$$
$$\Rightarrow \frac{1}{0.2 + 0.8p} = \frac{25}{13}$$
$$\Rightarrow 0.8p + 0.2 = \frac{13}{25}$$
$$\Rightarrow 0.8p = \frac{8}{25}$$

$$\therefore p = 0.4$$