IYGB GCE

Mathematics FP3

Advanced Level

Practice Paper V

Difficulty Rating: 4.0200/2.0202

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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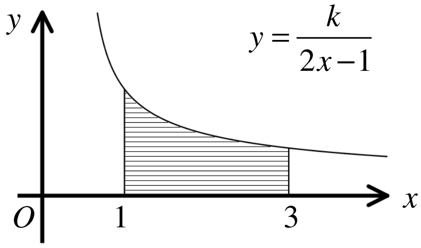
Question 1

Determine the solution interval of the following inequality.

$$\frac{x-7}{x} \le \frac{5}{x(x-3)}.$$ **(6)**

Question 2



The figure above shows part of the curve C with equation

$$y=\frac{k}{2x-1},$$

where k is a positive constant.

When Simpson's rule with 4 equally spaced strips is used, the area bounded by C, the x axis and the vertical straight lines with equations $x=1$ and $x=3$, is approximated to 30 square units.

a) Determine the value of k. **(5)**

b) By considering suitable graph transformation, find an approximate value of

$$\int_{0.5}^{1.5} \frac{k}{12x-3}\, dx.$$ **(4)**

Question 3

If $p \in (0,\infty)$, show that

$$\lim_{x \to 0^+} \left[x^p \ln x \right] = 0, \quad x \in (0,\infty). \qquad (6)$$

Question 4

The system of equations below has a unique solution.

$$\begin{aligned} 5x + \ \ y + 6z &= 9 \\ 3x + 6y + 2z &= 8 \\ 4x + 2y - 9z &= 75 \end{aligned}$$

a) Show that $z = -5$ and find the values of x and y. **(5)**

The straight line L and the plane Π have respective vector equations

$$\mathbf{r}_1 = \begin{pmatrix} -29 \\ -9 \\ 46 \end{pmatrix} + t \begin{pmatrix} -6 \\ -2 \\ 9 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} -38 \\ -17 \\ -29 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix},$$

where t, λ and μ are scalar parameters.

b) Show that L is perpendicular to Π. **(3)**

c) Show further that L meets Π at the point with coordinates $(1,1,1)$. **(5)**

Question 5

Use a suitable substitution to solve the following differential equation.

$$\frac{dy}{dx} + \sqrt{y+1} = y + 1, \quad y > -1, \quad y(0) = 3.$$

Given the answer in the form $y = f(x)$. **(11)**

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Question 6

The curve with equation $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = \frac{e^{x+y}}{3x+y+k}, \quad y(0)=0,$$

where k is a positive constant.

Using, in the standard notation, the approximation

$$\left(\frac{dy}{dx}\right)_r \approx \frac{y_{r+1}-y_r}{h},$$

with $h = 0.1$, the value of y at $x = 0.1$ was estimated to 0.025.

Use the approximation formula given above to find, correct to 3 significant figures, the value of y at $x = 0.2$. **(6)**

Question 7

Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to find the value of

$$\int_0^{\frac{\pi}{2}} \frac{1}{5+3\sin x+4\cos x}\,dx.$$

All relevant results used in this evaluation must be carefully derived. **(12)**

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Question 8

The general point $P\left(\frac{p}{2}, \frac{1}{2p}\right)$, $p \neq 0$, where p is a parameter, lies on the rectangular hyperbola, with Cartesian equation

$$4xy = 1.$$

The normal to the hyperbola at P meets the hyperbola again at the point Q.

Show that the Cartesian form of the locus of the midpoint of PQ, as p varies, is given by

$$\left(y^2 - x^2\right)^2 + 16x^3y^3 = 0.$$ **(12)**