IYGB GCE

Mathematics FP3

Advanced Level

Practice Paper Q

Difficulty Rating: 3.1867/1.4218

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

I.Y.G.B www.madasmaths.com

Question 1

Find the set of values of x that satisfy the inequality

$$|x-1| > 6x-1 \qquad (5)$$

Question 2

The following three vectors are given

$$\mathbf{a} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$$
$$\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$$
$$\mathbf{c} = \mathbf{i} + 2\mathbf{j} + \lambda\mathbf{k}$$

where λ is a scalar constant.

a) If the three vectors given above are coplanar, find the value of λ. **(3)**

b) Express $\mathbf{a}$ in terms of $\mathbf{b}$ and $\mathbf{c}$. **(3)**

Question 3

$$f(x) = \cos x.$$

a) Find the first four terms in the Taylor expansion of $f(x)$, in ascending powers of $\left(x - \frac{\pi}{6}\right)$. **(4)**

b) Use the expansion of part **(a)** to show that

$$\cos\frac{\pi}{4} \approx \frac{\sqrt{3}}{2} - \frac{\pi}{24} - \frac{\sqrt{3}\pi^2}{576} + \frac{\pi^3}{20736}. \qquad (2)$$

Question 4

Find the value of the following limit

$$\lim_{x \to 0}\left[\frac{\cos 7x - 1}{x \sin x}\right]. \qquad (6)$$

Question 5

Relative to a fixed origin O, the plane Π_1 passes through the points A, B and C with position vectors $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $6\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, respectively.

a) Determine an equation of Π_1 in the form $\mathbf{r} \cdot \mathbf{n} = c$, where $\mathbf{n}$ is the normal to Π_1 and c is a scalar constant. **(6)**

b) Find, in exact surd form, the shortest distance of Π_1 from the origin O. **(3)**

The plane Π_2 passes through the point A and has normal $5\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$.

c) Calculate, to the nearest degree, the acute angle between Π_1 and Π_2. **(3)**

Question 6

The curve with equation $y = f(x)$, passes through the point $(1,1)$ and satisfies the following differential equation.

$$\frac{dy}{dx} = \ln(x + y + 1), \ x + y > -1.$$

Use the approximation

$$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h},$$

with $h = 0.1$, to find, correct to 3 decimal places, the value of y at $x = 1.2$. **(6)**

Question 7

Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to find the value of

$$\int_0^{\frac{2\pi}{3}} \frac{1}{5+4\cos x} \, dx .$$

(10)

Question 8

An ellipse E has Cartesian equation

$$\frac{x^2}{16} + \frac{y^2}{4} = 1 .$$

a) Show that an equation of the tangent to E at the point $A(4\cos\theta, 2\sin\theta)$ is given by

$$2y\sin\theta + x\cos\theta = 4 .$$

(4)

The point $B(4\cos\theta, 4\sin\theta)$ lies on the circle with Cartesian equation

$$x^2 + y^2 = 16 .$$

The tangent to the circle at the point B meets the tangent to the ellipse at the point A at the point P.

b) Determine the coordinates of P, in terms of θ. **(6)**

c) Describe mathematically the locus of P as θ varies. **(2)**

I.Y.G.B. www.madasmaths.com

Question 9

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx}\tan x - y\sec^4 x = 0.$$

The above differential equation is to be solved by a substitution.

a) If $t = \tan x$ show that …

i. … $\dfrac{dy}{dx} = \dfrac{dy}{dt}\sec^2 x$ **(3)**

ii. … $\dfrac{d^2y}{dx^2} = \dfrac{d^2y}{dt^2}\sec^4 x + 2\dfrac{dy}{dt}\sec^2 x\tan x$ **(3)**

b) Use the results obtained in part **(a)** to find a general solution of the differential equation in the form $y = f(x)$. **(6)**
