IYGB GCE

Mathematics FP3

Advanced Level

Practice Paper L

Difficulty Rating: 3.4000/1.5385

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Solve the following inequality.

$$\frac{(2x-1)(x+3)}{(x-3)(x-2)} < 2. \qquad (7)$$

Question 2

$$\frac{dy}{dx} = \mathrm{e}^x - y^2, \quad y(0) = 0.$$

a) Use, in the standard notation, the approximation

$$y_{n+1} \approx h\,y'_n + y_n,$$

with $h = 0.1$, to find the approximate value of y at $x = 0.1$. **(3)**

b) Use the answer of part **(a)** and the approximation

$$y'_n \approx \frac{y_{n+1} - y_{n-1}}{2h},$$

with $h = 0.1$, to find, correct to 4 decimal places, the approximate value of y at $x = 0.3$. **(4)**

c) By differentiating the differential equation given, determine the first four **non zero** terms in the infinite series expansion of y in ascending powers of x, and use it to find, correct to 4 decimal places, another approximation for the value of y at $x = 0.3$. **(8)**

Question 3

Use standard expansions of functions to find the value of the following limit.

$$\lim_{x \to 0} \left[\frac{\ln(1-x)}{\sin^2 x} + \operatorname{cosec} x \right]. \qquad (8)$$

Question 4

A tetrahedron has its four vertices at the points $A(-3,6,4)$, $B(0,11,0)$, $C(4,1,28)$ and $D(7,k,24)$, where k is a constant.

a) Calculate the area of the triangle ABC. **(5)**

b) Find the volume of the tetrahedron $ABCD$, in terms of k. **(4)**

The volume of the tetrahedron is 150 cubic units.

c) Determine the possible values of k. **(2)**

Question 5

The point $P(a\sec\theta, b\tan\theta)$ lies on the hyperbola with equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

where a and b are positive constants.

a) Show that an equation of the normal at P is given by

$$by + ax\sin\theta = \left(a^2 + b^2\right)\tan\theta \ . \qquad \textbf{(4)}$$

The normal to the hyperbola meets the coordinate axes at the points A and B.

b) Show that, as θ varies, the Cartesian locus of the midpoint of AB is given by

$$4\left(a^2x^2 - b^2y^2\right) = \left(a^2 + b^2\right)^2. \qquad \textbf{(6)}$$

Question 6

$$\frac{1}{y}\frac{dy}{dx} = 1 + 2xy^2, \quad y > 0.$$

a) Show that the substitution $z = \dfrac{1}{y^2}$ transforms the above differential equation into the new differential equation

$$\frac{dz}{dx} + 2z = -4x. \qquad \mathbf{(4)}$$

b) Hence find the general solution of the original differential equation, giving the answer in the form $y^2 = f(x)$. **(8)**

Question 7

$$\sec x \equiv \frac{1 + \tan^2\left(\frac{x}{2}\right)}{1 - \tan^2\left(\frac{x}{2}\right)}.$$

a) Prove the validity of the above trigonometric identity. **(3)**

b) Express $\dfrac{2}{1-t^2}$ into partial fractions. **(1)**

c) Hence use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\int \sec x \, dx = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C. \qquad \mathbf{(8)}$$
