IYGB – FP2 PAPER M – QUESTION 1

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 12(x + e^x)$$

START WITH THE AUXILIARY EQUATION

$$\lambda^2 + 5\lambda + 6 = 0$$
$$(\lambda + 2)(\lambda + 3) = 0$$
$$\lambda = \begin{cases} -2 \\ -3 \end{cases}$$

$\therefore$ COMPLEMENTARY FUNCTION: $y = Ae^{-2x} + Be^{-3x}$

FOR PARTICULAR INTEGRAL WE TRY $y = Px + Q + Re^x$

$$\frac{dy}{dx} = P + Re^x$$
$$\frac{d^2y}{dx^2} = Re^x$$

SUB INTO THE O.D.E

$$(Re^x) + 5(P + Re^x) + 6(Px + Q + Re^x) \equiv 12x + 12e^x$$
$$6Px + (5P + 6Q) + e^x(R + 5R + 6R) \equiv 12x + 12e^x$$

$\therefore$ $P = 2$ $\quad R = 1$ $\quad$ & $\quad 5P + 6Q = 0$

$$10 + 6Q = 0$$
$$Q = -\frac{5}{3}$$

HENCE THE GENERAL SOLUTION IS

$$y = Ae^{-2x} + Be^{-3x} + e^x + 2x - \frac{5}{3}$$

IYGB – FP2 PAPER M – QUESTION 2

$$\int_e^{\infty} \frac{1-\ln x}{x^2}\,dx = \ldots \text{ INTEGRATION BY PARTS}$$

$1-\ln x$	$-\frac{1}{x}$
$-\frac{1}{x}$	$\frac{1}{x^2}$

$$= \left[-\frac{1}{x}(1-\ln x)\right]_e^{\infty} - \int_e^{\infty} \frac{1}{x^2}\,dx$$

$$= \left[-\frac{1}{x} + \frac{1}{x}\ln x\right]_e^{\infty} - \left[-\frac{1}{x}\right]_e^{\infty}$$

$$= \left[\cancel{-\frac{1}{x}} + \frac{1}{x}\ln x + \cancel{\frac{1}{x}}\right]_e^{\infty}$$

$$= \left[\frac{\ln x}{x}\right]_e^{\infty}$$

$$= \lim_{k\to\infty}\left[\left[\frac{\ln x}{x}\right]_e^{k}\right]$$

$$= \lim_{k\to\infty}\left[\frac{\ln k}{k} - \frac{\ln e}{e}\right]$$

$$= 0 - \frac{1}{e}$$

$$= -\frac{1}{e}$$

$\frac{\ln k}{k} \to 0$ AS $k \to \infty$

AS $\frac{1}{k} \to 0$ FASTER THAN $\ln k \to \infty$

IYGB - FP2 PAPER M - QUESTION 3

a)
$$f(r) = r^2(r+1)^2 - (r-1)^2 r^2$$
$$= r^2\left[(r+1)^2 - (r-1)^2\right]$$
$$= r^2(r+1+r-1)(r+1-r+1)$$
$$= r^2 \times 2r \times 2$$
$$= 4r^3$$

b) USING PART (a)

$$\boxed{4r^3 \equiv r^2(r+1)^2 - (r-1)^2 r^2}$$

IF $r=1$	4×1^3	$=$	$1^2\times\cancel{2^2}$	$-$	$0^2\times1^2$
IF $r=2$	4×2^3	$=$	$\cancel{2^2\times3^2}$	$-$	$\cancel{1^2\times2^2}$
IF $r=3$	4×3^3	$=$	$\cancel{3^2\times4^2}$	$-$	$\cancel{2^2\times3^2}$
IF $r=4$	4×4^3	$=$	$\cancel{4^2\times5^2}$	$-$	$\cancel{3^2\times4^2}$
$\vdots$	$\vdots$		$\vdots$		$\vdots$
IF $r=20$	4×20^3	$=$	$20^2\times21^2$	$-$	$\cancel{19^2\times20^2}$

ADDING
$$\sum_{r=1}^{20} 4r^3 = 20^2\times21^2$$
$$\Rightarrow 4\sum_{r=1}^{20} r^3 = 20^2\times21^2$$
$$\Rightarrow \sum_{r=1}^{20} r^3 = \frac{20^2\times21^2}{4}$$
$$\Rightarrow \sum_{r=1}^{20} r^3 = 44100$$

IYGB - FP2 PAPER M - QUESTION 4

LET $\cos\theta + i\sin\theta \equiv C + iS$

$$\Longrightarrow (\cos\theta + i\sin\theta)^4 = (C+iS)^4$$

$$\Longrightarrow \cos4\theta + i\sin4\theta = C^4 + 4iC^3S^1 - 6C^2S^2 - 4iCS^3 + S^4$$

NOTE THE PATTERN

+ + − − + + ...

Re Im Re Im Re Im ....

$$\begin{array}{ccccccccc} &&&&1&&&& \\ &&&1&&1&&& \\ &&1&&2&&1&& \\ &1&&3&&3&&1& \\ 1&&4&&6&&4&&1 \end{array}$$

EQUATE REAL & IMAGINARY PARTS

$$\cos4\theta = C^4 - 6C^2S^2 + S^4$$

$$\sin4\theta = 4C^3S - 4CS^3$$

FORMING THE $\tan4\theta$

$$\Rightarrow \tan4\theta = \frac{\sin4\theta}{\cos4\theta} = \frac{4C^3S - 4CS^3}{C^4 - 6C^2S^2 + S^4}$$

$$\Rightarrow \tan4\theta = \frac{\frac{4C^3S}{C^4} - \frac{4CS^3}{C^4}}{\frac{C^4}{C^4} - \frac{6C^2S^2}{C^4} + \frac{S^4}{C^4}}$$

$$\Longrightarrow \tan4\theta = \frac{4T - 4T^3}{1 - 6T^2 + T^4}$$

$$\therefore \tan4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$$

IYGB - FP2 PAPER M - QUESTION 5

a) USING STANDARD EXPANSIONS

- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + O(x^4)$

$e^{2x} = 1 + (2x) + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + O(x^4)$

$e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + O(x^4)$

- $\sin x = x - \frac{x^3}{3!} + O(x^5)$

$\sin 3x = (3x) - \frac{(3x)^3}{3!} + O(x^5)$

$\sin 3x = 3x - \frac{9}{2}x^3 + O(x^5)$

COMBINING RESULTS

$\Longrightarrow y = e^{2x}\sin 3x = \left[1 + 2x + 2x^2 + \frac{4}{3}x^3 + O(x^4)\right]\left[3x - \frac{9}{2}x^3 + O(x^5)\right]$

$$\Longrightarrow y = \begin{array}{llll} 3x & & -\frac{9}{2}x^3 & & + O(x^5) \\ & 6x^2 & & -9x^4 & + O(x^6) \\ & & 6x^3 & & + O(x^5) \\ & & & 4x^4 & + O(x^6) \end{array}$$

$\Longrightarrow y = 3x + 6x^2 + \frac{3}{2}x^3 - 5x^4 + O(x^5)$

IYGB - FP2 PAPER N1 - QUESTION 5

b) USING PART (a)

$$\int_0^{0.1} e^{2x}\sin 3x \, dx \simeq \int_0^{0.1} 3x + 6x^2 + \frac{3}{2}x^3 - 5x^4 \, dx$$

$$\simeq \left[\frac{3}{2}x^2 + 2x^3 + \frac{3}{8}x^4 - x^5\right]_0^{0.1}$$

$$\simeq \left(\frac{3}{200} + \frac{1}{500} + \frac{3}{80000} - \frac{1}{100000}\right) - (0)$$

$$\simeq 0.0170275\ldots$$

IYGB, FP2 PAPER M, QUESTION 6

a) SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\left.\begin{array}{l} r = 1+\sin2\theta \\ r = 1.5 \end{array}\right\} \Rightarrow 1+\sin2\theta = 1.5$$

$$\Rightarrow \sin2\theta = 0.5$$

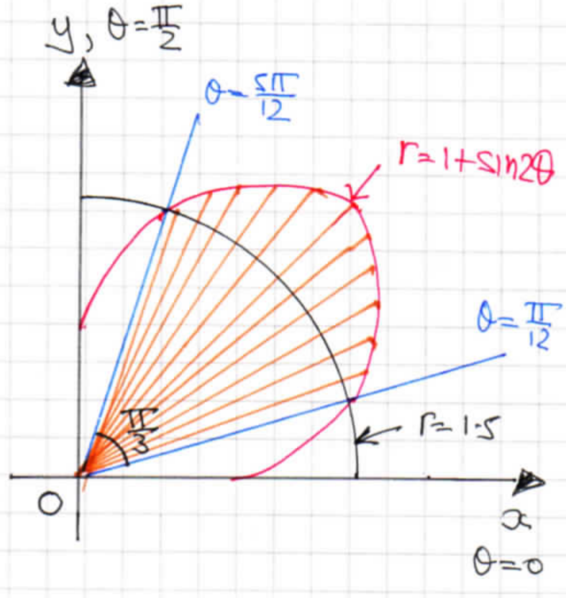
$$\Rightarrow 2\theta = \left< \begin{array}{l} \frac{\pi}{6}, \dots \\ \frac{5\pi}{6}, \dots \end{array}\right.$$

$$\Rightarrow \theta = \left< \begin{array}{l} \frac{\pi}{12} \\ \frac{5\pi}{12} \end{array}\right.$$

$\therefore \quad (r,\theta) = \left(1.5, \frac{\pi}{12}\right)$ OR $(r,\theta) = \left(1.5, \frac{5\pi}{12}\right)$

b) AREA OF $\frac{1}{6}$ OF A CIRCLE OF RADIUS 1.5.

$\left(\frac{\pi}{3} = \frac{1}{6} \text{ OF A CIRCLE}\right)$



$$\text{AREA} = \frac{1}{6} \times \pi r^2$$

$$= \frac{1}{6} \times \pi \times \left(\frac{3}{2}\right)^2$$

$$= \frac{3}{8}\pi$$

AREA OF POLAR SECTOR DEFINED BY $r = 1+\sin2\theta$

$$\text{AREA} = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta$$

$$\text{AREA} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{1}{2}(1+\sin2\theta)^2 \, d\theta$$

IYGB - FP2 PAPER M - QUESTION 6

$$\text{AREA} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{1}{2}\left(1+2\sin 2\theta+\sin^2 2\theta\right)\,d\theta$$

$$\text{AREA} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{1}{2}\left[1+2\sin 2\theta+\frac{1}{2}-\frac{1}{2}\cos 4\theta\right]\,d\theta$$

$$\text{AREA} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{3}{4}+\sin 2\theta-\frac{1}{4}\cos 4\theta\,d\theta$$

$$\text{AREA} = \left[\frac{3}{4}\theta-\frac{1}{2}\cos 2\theta-\frac{1}{16}\sin 4\theta\right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$$

$$\text{AREA} = \left[\frac{5}{16}\pi-\frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)-\frac{1}{16}\left(-\frac{\sqrt{3}}{2}\right)\right]-\left[\frac{1}{16}\pi-\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)-\frac{1}{16}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$\text{AREA} = \frac{5}{16}\pi+\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{32}-\frac{1}{16}\pi+\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{32}$$

$$\text{AREA} = \frac{1}{4}\pi+\frac{9}{16}\sqrt{3}$$

HENCE THE REQUIRED AREA CAN BE FOUND

$$\text{REQUIRED AREA} = \left(\frac{1}{4}\pi+\frac{9}{16}\sqrt{3}\right)-\frac{3}{8}\pi$$

$$= \frac{9}{16}\sqrt{3}-\frac{1}{8}\pi$$

$$= \frac{1}{16}\left(9\sqrt{3}-2\pi\right)$$

IYGB - FP2 PAPER M - QUESTION 7

- START BY USING THE "COMPOUND ANGLE" IDENTITIES IN HYPERBOLIC

$$
\begin{aligned}
5\cosh x + 3\sinh x &\equiv R\cosh(x+\alpha) \\
&\equiv R\cosh x\cosh\alpha + R\sinh x\sinh\alpha \\
&\equiv (R\cosh\alpha)\cosh x + (R\sinh\alpha)\sinh x
\end{aligned}
$$

- HENCE WE HAVE

$$\left.\begin{aligned} R\cosh\alpha &= 5 \\ R\sinh\alpha &= 3 \end{aligned}\right\} \Longrightarrow \begin{aligned} R^2\cosh^2\alpha &= 25 \\ R^2\sinh^2\alpha &= 9 \end{aligned}$$

$\Longrightarrow R^2\cosh^2\alpha - R^2\sinh^2\alpha = 16$ ← SUBTRACT

$\Longrightarrow R^2(\cosh^2\alpha - \sinh^2\alpha) = 16$

$\Longrightarrow R^2 = 16$

$\Longrightarrow R = +4$

& $R\sinh\alpha = 3$

$\Longrightarrow 4\sinh\alpha = 3$

$\Longrightarrow \sinh\alpha = \frac{3}{4}$

$\Longrightarrow \alpha = \text{arsinh}\left(\frac{3}{4}\right) = \ln\left[\frac{3}{4} + \sqrt{\frac{9}{16}+1}\right]$

$\Longrightarrow \alpha = \ln\left[\frac{3}{4} + \sqrt{\frac{25}{16}}\right] = \ln\left[\frac{3}{4} + \frac{5}{4}\right]$

$\Longrightarrow \alpha = \ln 2$

- HENCE THE EQUATION BECOMES

$\Longrightarrow 5\cosh x + 3\sinh x = 12$

$\Longrightarrow 4\cosh(x+\ln 2) = 12$

$\Longrightarrow \cosh(x+\ln 2) = 3$

$\Longrightarrow x+\ln 2 = \pm\text{arcosh}\,3$

IYGB - FP2 PAPER M - QUESTION 7

$$\Rightarrow x + \ln 2 = \pm \ln\left[3 + \sqrt{3^2-1}\right]$$

$$\Rightarrow x + \ln 2 = \pm \ln\left[3 + 2\sqrt{2}\right]$$

$$\Rightarrow x + \ln 2 = \begin{cases} \ln(3+2\sqrt{2}) \\ -\ln(3+2\sqrt{2}) = \ln\left(\frac{1}{3+2\sqrt{2}}\right) \end{cases}$$

$$= \ln\left(\frac{3-2\sqrt{2}}{(3+2\sqrt{2})(3-2\sqrt{2})}\right)$$

$$= \ln\left(\frac{3-2\sqrt{2}}{9-8}\right)$$

$$= \ln(3-2\sqrt{2})$$

$$\Rightarrow x = \begin{cases} -\ln 2 + \ln(3+2\sqrt{2}) \\ -\ln 2 + \ln(3-2\sqrt{2}) \end{cases}$$

$$\Rightarrow x = \begin{cases} \ln\left(\frac{3+2\sqrt{2}}{2}\right) \\ \ln\left(\frac{3-2\sqrt{2}}{2}\right) \end{cases}$$

$$\Rightarrow x = \begin{cases} \ln\left(\frac{3}{2}+\sqrt{2}\right) \\ \ln\left(\frac{3}{2}-\sqrt{2}\right) \end{cases}$$

IYGB - FP2 PAPER N - QUESTION 8

a) STARTING AS SUGGESTED

$\Rightarrow y = \arctan x$

$\Rightarrow \tan y = x$

$\Rightarrow x = \tan y$

DIFFERENTIATE W.R.T y

$\Rightarrow \frac{dx}{dy} = \sec^2 y$

$\Rightarrow \frac{dx}{dy} = 1 + \tan^2 y$

$\Rightarrow \frac{dx}{dy} = 1 + x^2$

$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$ // AS REQUIRED

b) $f(x) = \arctan(x^{\frac{1}{2}})$

$\Rightarrow f'(x) = \frac{1}{1+(x^{\frac{1}{2}})^2} \times \frac{1}{2}x^{-\frac{1}{2}} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{1+x} = \frac{1}{2}x^{-\frac{1}{2}}(1+x)^{-1}$

DIFFERENTIATE AGAIN VIA THE PRODUCT RULE

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1+x)^{-1} + \frac{1}{2}x^{-\frac{1}{2}} \times (-1)(1+x)^{-2}$

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1+x)^{-1} - \frac{1}{2}x^{-\frac{1}{2}}(1+x)^{-2}$

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1+x)^{-2}\left[(1+x) + 2x\right]$

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1+x)^{-2}(3x+1)$

// AS REQUIRED

IYGB – FP2 PAPER M – QUESTION 9

WRITE THE O.D.E IN THE USUAL ORDER

$$\Longrightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{5}{(x^2+2)(4x^2+3)}$$

INTEGRATING FACTOR CAN BE FOUND

$$e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x$$

HENCE WE OBTAIN

$$\Longrightarrow \frac{d}{dx}(yx) = \frac{5x}{(x^2+2)(4x^2+3)}$$

$$\Longrightarrow yx = \int \frac{5x}{(x^2+2)(4x^2+3)}\,dx$$

PARTIAL FRACTIONS ARE NEEDED

$$\frac{5x}{(x^2+2)(4x^2+3)} \equiv \frac{Ax+B}{x^2+2} + \frac{Cx+D}{4x^2+3}$$

$$\boxed{5x \equiv (Ax+B)(4x^2+3) + (x^2+2)(Cx+D)}$$

$$5x \equiv 4Ax^3 + 4Bx^2 + 3Ax + 3B$$
$$Cx^3 + Dx^2 + 2Cx + 2D$$

$$5x \equiv (4A+C)x^3 + (4B+D)x^2 + (3A+2C)x + (3B+2D)$$

$$\left.\begin{array}{l} 4A+C=0 \\ 3A+2C=5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 8A+2C=0 \\ 3A+2C=5 \end{array}\right\} \Rightarrow \begin{array}{l} A=-1 \\ C=4 \end{array}$$

$$\left.\begin{array}{l} 4B+D=0 \\ 3B+2D=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 8B+2D=0 \\ 3B+2D=0 \end{array}\right\} \Rightarrow \begin{array}{l} B=0 \\ D=0 \end{array}$$

IYGB - FP2 PAPER M - QUESTION 9

CARRYING OUT THE REQUIRED INTEGRATION

$$\Longrightarrow yx = \int \frac{4x}{4x^2+3} - \frac{x}{x^2+2} \, dx$$

$$\Longrightarrow 2yx = \int \frac{8x}{4x^2+3} - \frac{2x}{x^2+2} \, dx$$

$$\Longrightarrow 2yx = \ln(4x^2+3) - \ln(x^2+2) + \ln A$$

$$\Longrightarrow 2yx = \ln\left[\frac{A(4x^2+3)}{x^2+2}\right]$$

APPLY CONDITION $x=1$, $y=\frac{1}{2}\ln\frac{7}{6}$

$$\Longrightarrow 2 \times \tfrac{1}{2}\ln\tfrac{7}{6} \times 1 = \ln\left(\frac{7A}{3}\right)$$

$$\Longrightarrow \ln\frac{7}{6} = \ln\frac{7A}{3}$$

$$\Longrightarrow \frac{7}{6} = \frac{7A}{3}$$

$$\Longrightarrow A = \frac{1}{2}$$

FINALLY WE HAVE

$$\Longrightarrow 2yx = \ln\left[\frac{4x^2+3}{2(x^2+2)}\right]$$

$$\Longrightarrow y = \frac{1}{2x}\ln\left[\frac{4x^2+3}{2x^2+4}\right]$$

AS REQUIRED