IYGB GCE

Mathematics FP1

Advanced Level

Practice Paper U

Difficulty Rating: 4.18/1.5385

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

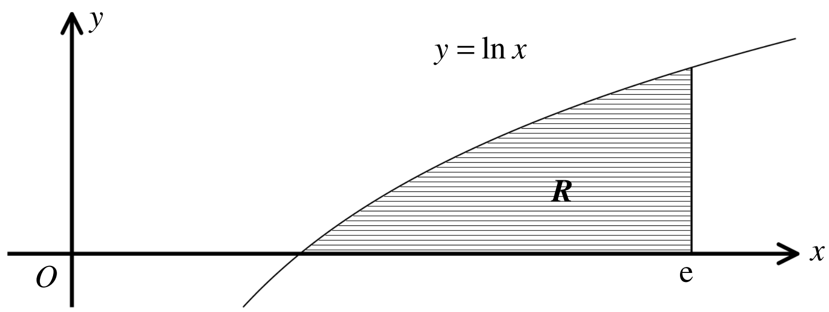
Question 1

Two vectors $\mathbf{a}$ and $\mathbf{b}$ are given below.

$$\mathbf{a} = (\sin\theta)\mathbf{i} + (2\cos 2\theta)\mathbf{j} + (\sin\theta)\mathbf{k} \qquad \text{and} \qquad \mathbf{a} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}\,.$$

Find the values of θ, $0 \le \theta < 2\pi$, for which $\mathbf{a}$ is perpendicular to $\mathbf{b}$. **(8)**

Question 2



The figure above shows the graph of

$$y = \ln x\,,\ x > 0\,.$$

The shaded region R is bounded by the curve, the line $x = \mathrm{e}$ and the x axis.

R is rotated by 2π radians about the y axis, forming a solid of revolution S.

Show that the volume of S is

$$\frac{1}{2}\pi\left(\mathrm{e}^2+1\right).$$ **(7)**

Question 3

The straight lines l_1 and l_2 have the following Cartesian equations

$$l_1: \quad \frac{x-8}{1} = \frac{y+1}{-1} = \frac{z-2}{1}.$$

$$l_2: \quad \frac{x-3}{-1} = \frac{y-4}{1} = \frac{z-1}{1}.$$

a) Show that l_1 and l_2 intersect at some point P, and find its coordinates. **(5)**

b) Find the exact value of $\cos\theta$, where θ is the acute angle formed by l_1 and l_2. **(3)**

The point $A(6,1,0)$ lies on l_1 and the point $B(4,3,0)$ lies on l_2.

c) By considering $|AP|$ and $|BP|$ show further that the angle bisector of $\measuredangle APB$ is parallel to the vector $\mathbf{k}$. **(6)**

Question 4

It is given that

$$\sum_{r=1}^{20}(r-10) = 200 \quad \text{and} \quad \sum_{r=1}^{20}(r-10)^2 = 2800.$$

Find the value of

$$\sum_{r=1}^{20} r^2.$$

(8)

Question 5

Prove by induction that

$$\sum_{r=1}^{n} \frac{r \times 2^r}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}, \quad n \geq 1, \ n \in \mathbb{N}. \qquad (9)$$

Question 6

The complex number z lies in the region R of an Argand diagram, defined by the inequalities

$$\frac{\pi}{3} \leq \arg(z-4) \leq \pi \quad \text{and} \quad 0 \leq \arg(z-12) \leq \frac{5\pi}{6}.$$

a) Sketch the region R, indicating clearly all the relevant details. **(4)**

The complex number w lies in R, so that $|w|$ is minimum.

b) Find $|w|$, further giving w in the form $u + \mathrm{i}v$, where u and v are real numbers. **(6)**

Question 7

The points $P(2,2,1)$ and $Q(6,-7,-1)$ lie on the plane Π with Cartesian equation

$$cx + 4y - 12z = k,$$

where c and k are constants.

a) Determine an equation of the straight line L, which is perpendicular to Π and passing through P. **(5)**

The points A and B are both located on L and each of these points is at a distance of 26 units from Π.

b) Show that the area of the triangle ABQ is approximately 261 square units. **(5)**

Question 8

$$z^3 - (4+2\mathrm{i})z^2 + (4+5\mathrm{i})z - (1+3\mathrm{i}) = 0, \quad z \in \mathbb{C}.$$

Given that one of the solutions of the above cubic equation is $z = 2 + \mathrm{i}$, find the other two solutions.

(10)

Question 9

The roots of the quadratic equation

$$x^2 - 3x + 4 = 0$$

are denoted by α and β.

Find the quadratic equation, with integer coefficients, whose roots are

$$\alpha^3 - \beta \quad \text{and} \quad \beta^3 - \alpha.$$

(12)

Question 10

A curve has equation

$$5x^2 - 16xy + 13y^2 = 25.$$

This curve is to be mapped onto another curve C, under the transformation defined by the 2×2 matrix $\mathbf{A}$, given below.

$$\mathbf{A} = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}.$$

Show that the equation of C is the circle with equation

$$x^2 + y^2 = 25.$$

(12)
