IYGB GCE

Mathematics FP1

Advanced Level

Practice Paper J

Difficulty Rating: 3.5733/1.6484

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The matrix $\mathbf{A} : \mathbb{R}^2 \mapsto \mathbb{R}^2$ and the matrix $\mathbf{B} : \mathbb{R}^3 \mapsto \mathbb{R}^3$ are defined as

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° \\ 0 & \sin 45° & \cos 45° \end{pmatrix}.$$

Describe geometrically the transformations given by each of these matrices.

State in each case the equation of the line of invariant points. **(6)**

Question 2

Use standard results on summations to show that

$$\sum_{r=1}^{n} \left(3r^2 + r - 1\right) \equiv n^2 (n+2).$$ **(6)**

Question 3

$$z^3 + 2z^2 + k = 0,$$

The roots of the above cubic equation, where k is a non zero constant, are denoted by α, β and γ.

a) Show that …

i. … $\alpha^2 + \beta^2 + \gamma^2 = 4$. **(2)**

ii. … $\alpha^3 + \beta^3 + \gamma^3 = -8 - 3k$. **(4)**

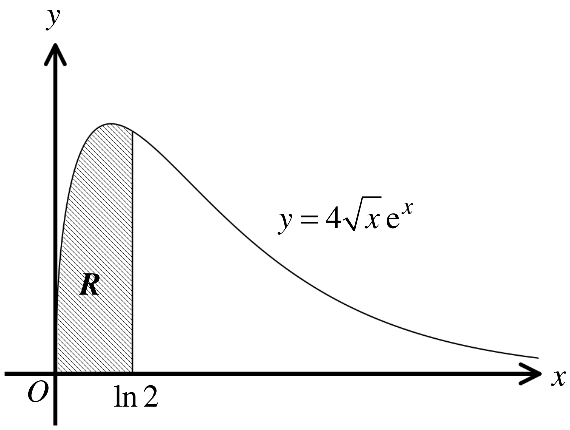
It is further given that $\alpha^4 + \beta^4 + \gamma^4 = 4$.

b) Show further that $k = -1$. **(4)**

c) Determine the value of

$$\alpha^5 + \beta^5 + \gamma^5.$$ **(2)**

Question 4



The figure above shows the graph of the curve C with equation

$$y = 4\sqrt{x}\,e^{x}, \ x \geq 0 .$$

The shaded region R bounded by the curve, the x axis and the vertical straight line with equation $x = \ln 2$, is rotated by 2π radians in the x axis, forming a solid of revolution S.

Find an exact value for the volume of S, giving the answer in the form $\pi(a + b\ln 2)$ where a and b are integers.

(7)

Question 5

Solve the following quadratic equation

$$z^2 - 6z + 10 + (z-6)i = 0, \quad z \in \mathbb{C} .$$

Give the answers in the form $a + bi$, $a \in \mathbb{R}$, $b \in \mathbb{R}$. **(10)**

Question 6

The position vectors and coordinates in this question are relative to a fixed origin O.

The points A, B and C have coordinates $(0,0,8)$, $(2,6,4)$ and $(8,8,0)$, respectively.

The point D is such so that $ABCD$ is a parallelogram.

The angle ABC is θ.

a) Determine the coordinates of D. **(2)**

b) Use the scalar product to find an exact value for $\cos\theta$ and hence show

$$\sin\theta = \tfrac{2}{7}\sqrt{6}.$$ **(5)**

c) Explain, with reference to the calculations of part **(b)**, why AC must be perpendicular to BD. **(1)**

d) Show that the area of the parallelogram is $16\sqrt{6}$. **(3)**

Question 7

Sketch on a single Argand diagram the locus of the points z which satisfy

$$|z-5-\mathrm{i}| = 2\sqrt{5} \quad \text{and} \quad \arg(z+1-\mathrm{i}) = \tfrac{1}{4}\pi,$$

and hence find the complex numbers which lie on both of these loci.

No credit will be given to solutions based on a scale drawing. **(10)**

Question 8

Prove by induction that

$$\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \frac{n+2}{2^n}, \quad n \geq 1, \ n \in \mathbb{N}.$$ **(7)**

Question 9

The 2×2 matrix $\mathbf{M}=\begin{pmatrix}-2 & 1\\ -9 & 4\end{pmatrix}$ is given.

Under the transformation represented by $\mathbf{M}$ a straight line passing through the origin remains invariant.

Determine the equation of this line. (6)