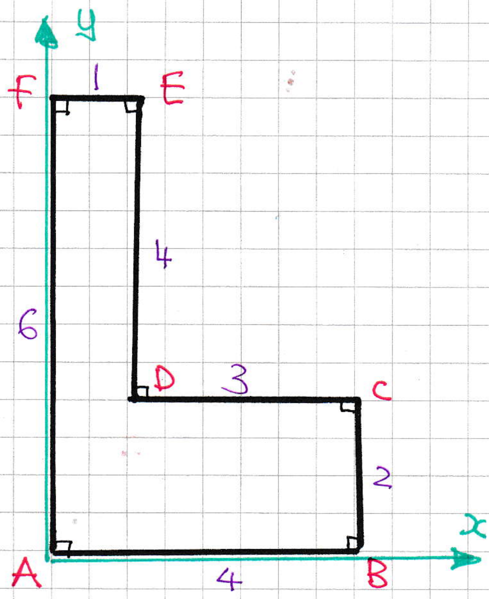
IYGB - FM2 PAPER R - QUESTION 1

a) LOOKING AT THE DIAGRAM BELOW

ROD	MASS RATIO	x	y
AB	4	2	0
BC	2	4	1
CD	3	2.5	2
DE	4	1	4
EF	1	$\frac{1}{2}$	6
FA	6	0	3
TOTAL	20	$\bar{x}$	$\bar{y}$



HENCE WE OBTAIN

$$\begin{cases} 20\bar{x} = (4\times2)+(2\times4)+(3\times2.5)+(4\times1)+(1\times\frac{1}{2})+(6\times0) \\ 20\bar{y} = (4\times0)+(2\times1)+(3\times2)+(4\times4)+(1\times6)+(6\times3) \end{cases}$$

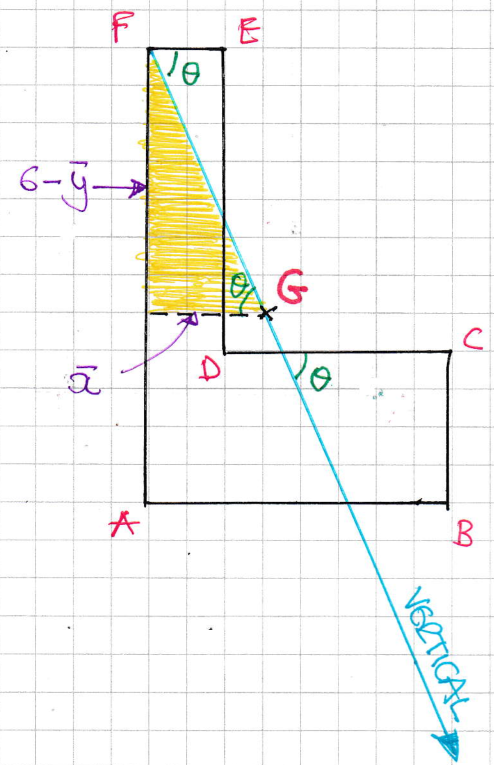
$$\begin{cases} 20\bar{x} = 28 \\ 20\bar{y} = 48 \end{cases}$$

$$\begin{cases} \bar{x} = 1.4 \\ \bar{y} = 2.4 \end{cases}$$

THE CENTRE OF MASS IS 1.4 m FROM AF & 2.4 m FROM AB

IYGB - FM2 PAPER R - QUESTION 1

b) REDRAWING THE FRAMEWORK FOR THE SUSPENSION



$$\tan\theta = \frac{6-\bar{y}}{\bar{x}}$$

$$\tan\theta = \frac{6-2.4}{1.4}$$

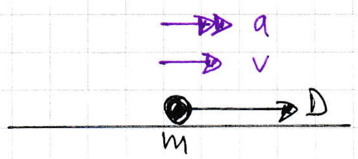
$$\tan\theta = \frac{3.6}{1.4}$$

$$\tan\theta = \frac{36}{14}$$

$$\tan\theta = \frac{18}{7}$$

IYGB - FM2 PAPER R - QUESTION 2

LOOKING AT THE DIAGRAM



$m = 1250$

$P = 31500$

POWER = TRACTIVE FORCE × SPEED

$31500 = D \times v$

$D = \dfrac{31500}{v}$

EQUATION OF MOTION

$m\ddot{x} = D$

FORMING AND SOLVING A DIFFERENTIAL EQUATION

$\Rightarrow \quad 1250\ddot{x} = \dfrac{31500}{v}$

$\Rightarrow \quad 1250\, v\dfrac{dv}{dx} = \dfrac{31500}{v^2}$

$\Rightarrow \quad v^2\, dv = \dfrac{31500}{1250}\, dx$

$\Rightarrow \quad \displaystyle\int_{v=3}^{v=6} v^2\, dv = \int_{x=x_1}^{x_2} \frac{126}{5}\, dx$

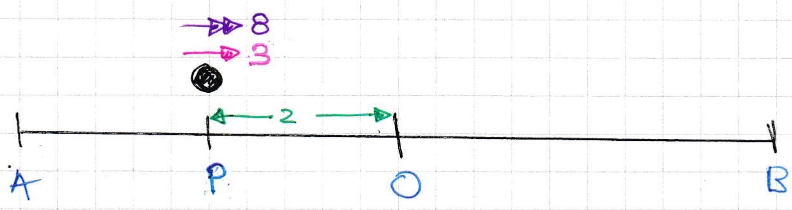
$\Rightarrow \quad \left[\frac{1}{3}v^3\right]_3^6 = \frac{126}{5}(x_2 - x_1)$

$\Rightarrow \quad 72 - 9 = \dfrac{126}{5}d \qquad$ (WHERE $d = x_2 - x_1$)

$\Rightarrow \quad d = 2.5$

IYGB - FM2 PAPER R - QUESTION 3

PUTTING THE INFORMATION INTO A DIAGRAM



$V = 3$
$|\ddot{x}| = 8$
$x = 2$

- $V^2 = \omega^2(a^2 - x^2)$

$9 = \omega^2(a^2 - 2^2)$

$9 = \omega^2(a^2 - 4)$

$9 = 4(a^2 - 4)$

$\frac{9}{4} = a^2 - 4$

$6.25 = a^2$

$\underline{\underline{a = 2.5}}$

- $|\ddot{x}| = \omega^2 x$

$8 = \omega^2 \times 2$

$\omega^2 = 4$

$\underline{\underline{\omega = 2}}$

& PERIOD $T = \frac{2\pi}{\omega} = \pi$

NOW SETTING A DISPLACEMENT EQUATION AS A FUNCTION OF TIME

LET $t=0$ AT THE ORIGIN

$\Rightarrow x = a\cos\omega t$

$\Rightarrow x = 2.5\cos 2t$

$\Rightarrow 2 = 2.5\cos 2t$ TO FIND THE TIME FROM "P" TO "O" OR FROM "O" TO "P"

$\Rightarrow \cos 2t = \frac{4}{5}$

$\Rightarrow 2t = \arccos\left(\frac{4}{5}\right)$ (FIRST TIME)

$\Rightarrow t = \frac{1}{2}\arccos\left(\frac{4}{5}\right)$

THE REQUIRED TIME IS GIVEN BY

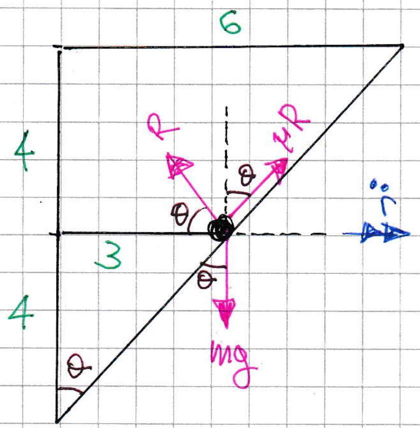
$$\text{"PO"} + \underbrace{\text{"OB"} + \text{"BO"}}_{} + \text{"OP"}$$

$$\frac{1}{2}\arccos\frac{4}{5} + \text{HALF PERIOD} + \frac{1}{2}\arccos\frac{4}{5} = \frac{\pi}{2} + \arccos\frac{4}{5}$$

$$\simeq 2.21\text{ s}$$

IYGB - FM2 PAPER R - QUESTION 4

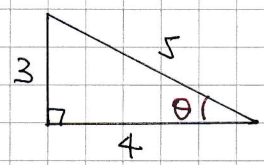
STARTING WITH A DETAILED DIAGRAM



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$



$r = 3$ BY INSPECTION

$(\uparrow)$ $R\sin\theta + \mu R\cos\theta = mg$ (EQUILIBRIUM)

$(\rightarrow)$ $m\ddot{r} = -R\cos\theta + \mu R\sin\theta$ ("F = ma")

MANIPULATE THE EQUATION OF MOTION

$$m\left(-\frac{v^2}{r}\right) = -R\cos\theta + \mu R\sin\theta$$

$$-mv^2 = -Rr\cos\theta + \mu Rr\sin\theta$$

DIVIDE THE EQUATIONS SIDE BY SIDE

$$\Rightarrow \frac{mg}{-mv^2} = \frac{R\sin\theta + \mu R\cos\theta}{-Rr\cos\theta + \mu Rr\sin\theta}$$

$$\Rightarrow -\frac{g}{v^2} = \frac{\sin\theta + \mu\cos\theta}{-r\cos\theta + \mu r\sin\theta}$$

$$\Rightarrow -\frac{9.8}{4.2^2} = \frac{0.6 + \mu \times 0.8}{-3\times 0.8 + \mu\times 3\times 0.6}$$

$$\Rightarrow -\frac{5}{9} = \frac{0.6 + 0.8\mu}{1.8\mu - 2.4}$$

$$\Rightarrow -\frac{5}{9} = \frac{3 + 4\mu}{9\mu - 12}$$

$$\Rightarrow -45\mu + 60 = 27 + 36\mu$$

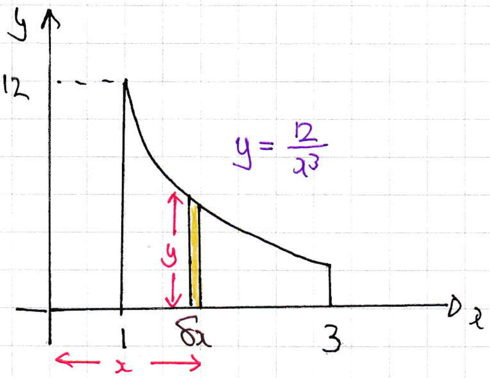
$$\Rightarrow 33 = 81\mu$$

$$\Rightarrow 27\mu = 11$$

$$\Rightarrow \mu = \frac{11}{27}$$

IYGB - FM2 PAPER R - QUESTION 5

LET ρ BE MASS PER UNIT AREA (AREA $\times \rho$ = MASS)



$$\text{AREA} = \int_1^3 \frac{12}{x^3}\,dx$$
$$= \left[-\frac{6}{x^2}\right]_1^3$$
$$= 6\left[\frac{1}{x^2}\right]_3^1$$
$$= 6 - \frac{2}{3}$$
$$= \frac{16}{3}$$

MASS OF THE INFINITESIMAL STRIP OF HEIGHT y AND THICKNESS δx

$$\delta m = \rho(y\,\delta x) = \rho y\,\delta x$$

THE MOMENT OF THE INFINITESIMAL STRIP ABOUT THE x & y AXIS

$$(\rho y\,\delta x)x = \rho x y\,\delta x \quad \& \quad (\rho y\,\delta x)(\tfrac{1}{2}y) = \tfrac{1}{2}\rho y^2\,\delta x$$

SUMMING UP AND TAKING LIMITS

- $M\bar{x} = \int_1^3 \rho x y\,dx$

$$\Rightarrow \frac{16}{3}\rho\bar{x} = \rho\int_1^3 x\left(\frac{12}{x^3}\right)dx$$
$$\Rightarrow \frac{16}{3}\bar{x} = \int_1^3 \frac{12}{x^2}\,dx$$
$$\Rightarrow \frac{16}{3}\bar{x} = \left[-\frac{12}{x}\right]_1^3$$

- $M\bar{y} = \int_1^3 \frac{1}{2}\rho y^2\,dx$

$$\Rightarrow \frac{16}{3}\rho\bar{y} = \frac{1}{2}\rho\int_1^3 \frac{144}{x^6}\,dx$$
$$\Rightarrow \frac{16}{3}\bar{y} = 72\int_1^3 \frac{1}{x^6}\,dx$$
$$\Rightarrow \frac{16}{3}\bar{y} = 72\left[-\frac{1}{5x^5}\right]_1^3$$

IYGB - FM2 PAPER R - QUESTION 5

$$\Rightarrow \frac{16}{3}\bar{x} = 12\left[-\frac{1}{x}\right]_3^1$$

$$\Rightarrow \frac{16}{3}\bar{x} = 12\left(1-\frac{1}{3}\right)$$

$$\Rightarrow \frac{16}{3}\bar{x} = 8$$

$$\Rightarrow \bar{x} = \frac{3}{2}$$

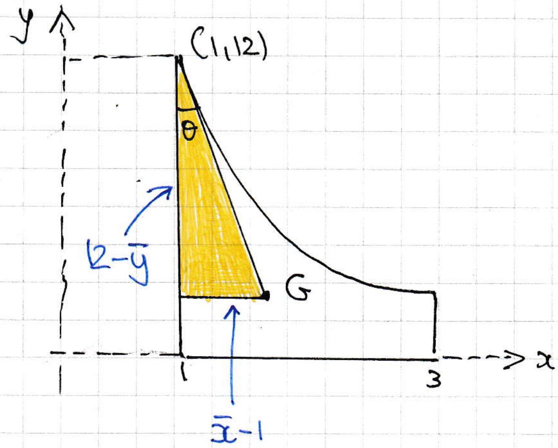
$$\Rightarrow \frac{16}{3}\bar{y} = \frac{72}{5}\left[-\frac{1}{x^5}\right]_3^1$$

$$\Rightarrow \frac{16}{3}\bar{y} = \frac{72}{5}\left[1-\frac{1}{243}\right]$$

$$\Rightarrow \frac{16}{3}\bar{y} = \frac{1936}{135}.$$

$$\Rightarrow \bar{y} = \frac{121}{45}$$

FINALLY WE HAVE TO FIND THE ANGLE, MARKED AS θ



$$\tan\theta = \frac{\bar{x}-1}{12-\bar{y}}$$

$$\tan\theta = \frac{\frac{3}{2}-1}{12-\frac{121}{45}}$$

$$\tan\theta = \frac{135-90}{1080-242}$$

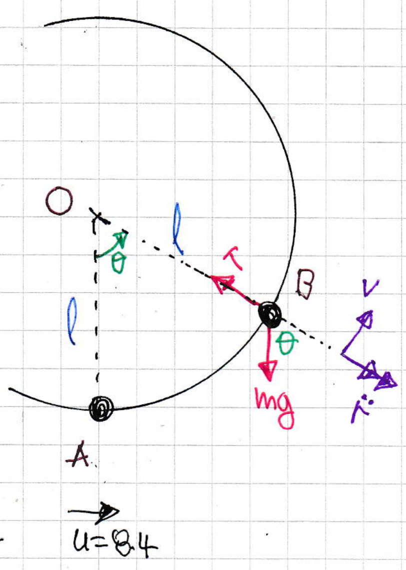
$$\tan\theta = \frac{45}{838}$$

$$\theta \simeq 86.92\ldots$$

$$\therefore \theta \simeq 87^\circ$$

IYGB – FM2 PAPER R – QUESTION 6

a) CONSIDERING ENERGIES TAKING THE LEVEL OF O AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL



$$\Rightarrow KE_A + PE_A = KE_B + PE_B$$

$$\Rightarrow \tfrac{1}{2}mu^2 - mgl = \tfrac{1}{2}mv^2 - mg(l\cos\theta)$$

$$\Rightarrow u^2 - 2gl = v^2 - 2gl\cos\theta$$

$$\Rightarrow v^2 = u^2 - 2gl + 2gl\cos\theta$$

$$\Rightarrow \boxed{v^2 = u^2 + 2gl(\cos\theta - 1)}$$

EQUATION OF MOTION RADIALLY YIELDS

$$\Rightarrow m\ddot{r} = mg\cos\theta - T$$

$$\Rightarrow m\left(-\frac{v^2}{l}\right) = mg\cos\theta - T$$

WHEN $v = 3.5$, $T = 0$

$$\Rightarrow -\frac{\cancel{m}(3.5)^2}{l} = \cancel{m}g\cos\theta$$

$$\Rightarrow -gl\cos\theta = 12.25$$

$$\Rightarrow \boxed{l\cos\theta = -\frac{5}{4}}$$

FINALLY FROM THE ENERGY EQUATION $u = 8.4$ $v = 3.5$

$$\Rightarrow (3.5)^2 = (8.4)^2 + 2gl(\cos\theta - 1)$$

$$\Rightarrow 2gl(\cos\theta - 1) = -58.31$$

$$l(\cos\theta - 1) = -2.975$$

$$l\cos\theta - l = -2.975$$

$$-1.25 - l = -2.975$$

$$\therefore l = \frac{69}{40} = 1.725\text{ m}$$

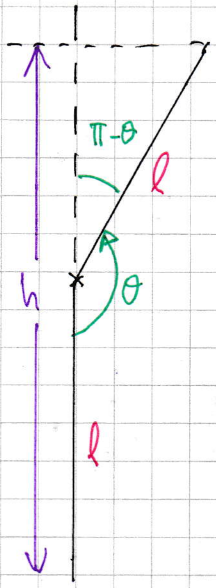
IYGB – FM2 PAPER R – QUESTION 6

b) LOOKING AT THE DIAGRAM

$$l\cos\theta = -\frac{5}{4}$$

$$\frac{69}{40}\cos\theta = -\frac{5}{4}$$

$$\cos\theta = -\frac{50}{69}$$



VERTICAL DISPLACEMENT IS h

$$h = l + l\cos\theta$$

$$h = l(1+\cos\theta)$$

$$h = 1.725\left(1 + \frac{50}{69}\right)$$

$$h = 2.975\text{ m}$$

OR

$$h = l + l\cos(\pi-\theta)$$

$$h = l + l\left[\cos\pi\cos\theta + \cancel{\sin\pi\sin\theta}\right]$$

$$h = l + l(-\cos\theta)$$

$$h = l - l\cos\theta$$

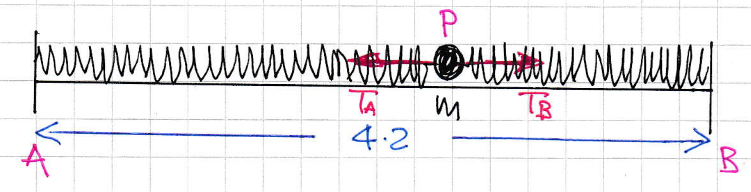
$$h = l(1-\cos\theta)$$

$$h = 1.725\left(1+\frac{50}{69}\right)$$

$$h = 2.975$$

IYGB – FM2 PAPER R – QUESTION 7

a) LOOKING AT A DIAGRAM



$m = 0.25$

$l_A = 1.8$

$l_B = 1.2$

$\lambda_A = 20$

$\lambda_B = 40$

- $T_A = T_B$

$$\frac{\lambda_A}{l_A} x_A = \frac{\lambda_B}{l_B} x_B$$

$$\frac{20}{1.8} x_A = \frac{40}{1.2} x_B$$

$$x_A = 3x_B$$

- $l_A + l_B + x_A + x_B = 4.2$

$$1.8 + 1.2 + x_A + x_B = 4.2$$

$$x_A + x_B = 1.2$$

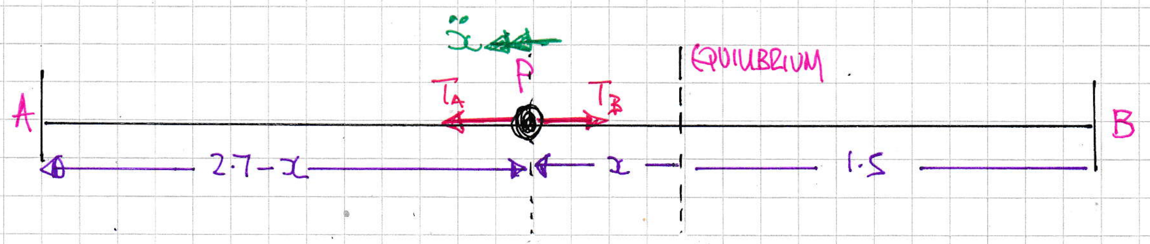
$$3x_B + x_B = 1.2$$

$$4x_B = 1.2$$

$$x_B = 0.3 \quad \& \quad x_A = 0.9$$

$\therefore$ REQUIRED DISTANCE IS $l_A + x_A = 1.8 + 0.9 = 2.7\text{ m}$

b) LOOKING AT A NEW DIAGRAM WITH THE PARTICLE IN AN ARBITRARY POSITION, SAY x TO "THE LEFT" OF THE EQUILIBRIUM POSITION



$$\Rightarrow m\ddot{x} = T_A - T_B$$

$$\Rightarrow \frac{1}{4}\ddot{x} = \frac{\lambda_A}{l_A}(2.7 - x - l_A) - \frac{\lambda_B}{l_B}(1.5 + x - l_B)$$

$$\Rightarrow \frac{1}{4}\ddot{x} = \frac{20}{1.8}(2.7 - x - 1.8) - \frac{40}{1.2}(1.5 + x - 1.2)$$

$$\Rightarrow \frac{1}{4}\ddot{x} = \frac{100}{9}(0.9 - x) - \frac{100}{3}(0.3 + x)$$

IYGB – FM2 PAPER R – QUESTION 7

$$\Rightarrow \frac{1}{4}\ddot{x} = 10 - \frac{100}{9}x - \left(10 + \frac{100}{3}x\right)$$

$$\Rightarrow \frac{1}{4}\ddot{x} = \cancel{10} - \frac{100}{9}x \cancel{-10} - \frac{100}{3}x$$

$$\Rightarrow \frac{1}{4}\ddot{x} = -\frac{400}{9}x$$

$$\Rightarrow \ddot{x} = -\frac{1600}{9}x$$

I.E S.H.M WITH $\omega^2 = \frac{1600}{9}$, IE $\omega = \frac{400}{3}$

$\therefore$ PERIOD $= \frac{2\pi}{\omega} = 2\pi \times \frac{3}{400} = \frac{3\pi}{20} \approx 0.471$