IYGB – FM2 PAPER Q – QUESTION 1

START BY FINDING THE AREA

$$\text{AREA} = \int_0^1 6x^2\,dx = \Big[2x^3\Big]_0^1 = 2$$

USING "STANDARD" FORMULAS

$$\bar{x} = \frac{\int_a^b xy\,dx}{\int_a^b y\,dx}$$

$$\bar{x} = \frac{\int_0^1 x(6x^2)\,dx}{2}$$

$$\bar{x} = \frac{1}{2}\int_0^1 6x^3\,dx$$

$$\bar{x} = \frac{1}{2}\Big[\tfrac{3}{2}x^4\Big]_0^1$$

$$\bar{x} = \frac{3}{4}$$

$$\bar{y} = \frac{\int \frac{1}{2}y^2\,dx}{\int_a^b y\,dx}$$

$$\bar{y} = \frac{\int_0^1 \frac{1}{2}(6x^2)^2\,dx}{2}$$

$$\bar{y} = \frac{1}{2}\int_0^1 18x^4\,dx$$

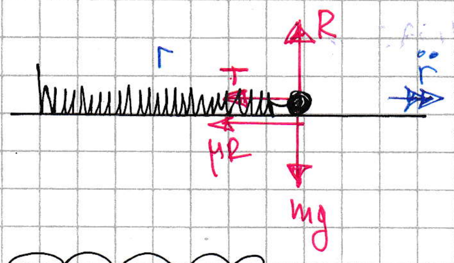
$$\bar{y} = \frac{1}{2}\Big[\tfrac{18}{5}x^5\Big]_0^1$$

$$\bar{y} = \frac{9}{5}$$

$\therefore \left(\frac{3}{4}, \frac{9}{5}\right)$

IYGB - FM2 PAPER Q - QUESTION 2

DRAWING A DIAGRAM WITH PARTICLE SLIPPING OUTWARDS (ω MAX)



$m = 0.5$ kg
$r = 0.75$
$l = 0.6$
$\lambda = 15$
$\mu = 0.45$

- $R = mg$

- $m\ddot{r} = -T - \mu R$

$$m(-\omega^2 r) = -\frac{\lambda}{l}x - \mu(mg)$$

$$m\omega^2 r = \frac{\lambda}{l}x + \mu mg$$

$$0.5\omega^2(0.75) = \frac{15}{0.6}(0.75-0.6) + 0.45\times0.5\times9.8$$

$$\frac{3}{8}\omega^2 = \frac{15}{4} + 2.205$$

$$\frac{3}{8}\omega^2 = 5.955$$

$$\omega^2 = 15.88$$

$$\omega_{MAX} = 3.98 \text{ rad s}^{-1}$$

TO FIND ω_{MIN} EVERYTHING IS THE SAME, EXCEPT FRICTION IS ACTING OUTWARDS

$$\frac{3}{8}\omega^2 = \frac{15}{4} - 2.205$$

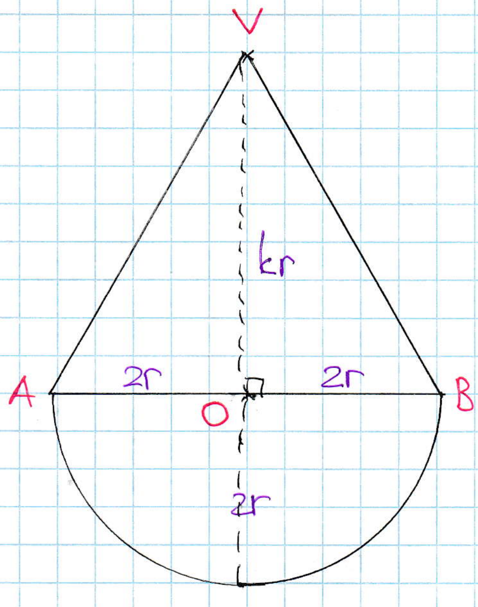
$$\frac{3}{8}\omega^2 = 1.545$$

$$\omega^2 = 4.12$$

$$\omega_{MIN} \simeq 2.03 \text{ rad s}^{-1}$$

IYGB - FM2 PAPER Q - QUESTION 3

a) LOOKING AT THE DIAGRAM



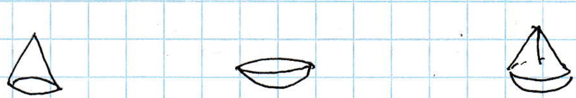
- VOLUME OF THE CONE

$$= \frac{1}{3}\pi(2r)^2(kr)$$
$$= \frac{4}{3}\pi k r^3$$
$$= \left(\frac{4}{3}\pi r^3\right)k$$

- VOLUME OF HEMISPHERE

$$= \frac{1}{2} \times \frac{4}{3}\pi(2r)^3$$
$$= \frac{16}{3}\pi r^3$$
$$= 4\left(\frac{4}{3}\pi r^3\right)$$

ORGANIZE RESULTS (MOMENTS) IN A TABLE



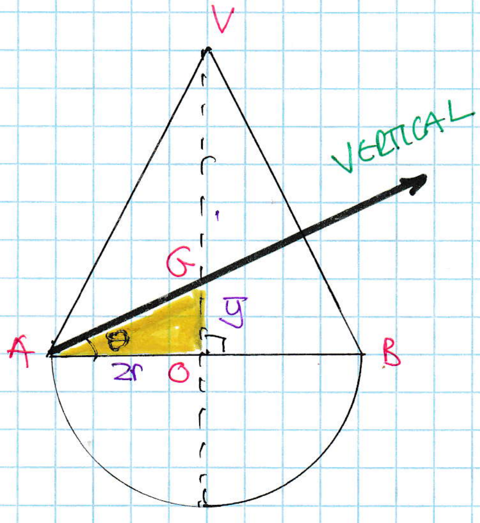
MASS RATIO	k	4	$k+4$
DISTANCE FROM O	$+\frac{1}{4}(kr)$	$-\frac{3}{8}(2r)$	$\bar{y}$

$$\Rightarrow (k+4)\bar{y} = \frac{1}{4}k^2r - 3r$$
$$\Rightarrow (k+4)\bar{y} = \frac{1}{4}r(k^2-12)$$
$$\Rightarrow \bar{y} = \frac{(k^2-12)r}{4(k+4)}$$

AS REQUIRED

IYGB – FM2 PAPER Q – QUESTION 3

b) LOOKING AT THE DIAGRAM AGAIN



$$\Rightarrow \tan\theta = \frac{\bar{y}}{2r}$$

$$\Rightarrow \frac{3}{10} = \frac{\frac{(k^2-12)r}{4(k+4)}}{2r}$$

$$\Rightarrow \frac{3}{10} = \frac{(k^2-12)\cancel{r}}{8\cancel{r}(k+4)}$$

$$\Rightarrow 24(k+4) = 10(k^2-12)$$

$$\Rightarrow 12(k+4) = 5(k^2-12)$$

$$\Rightarrow 12k+48 = 5k^2-60$$

$$\Rightarrow 0 = 5k^2-12k-108$$

QUADRATIC FORMULA OR FACTORIZATION YIELDS

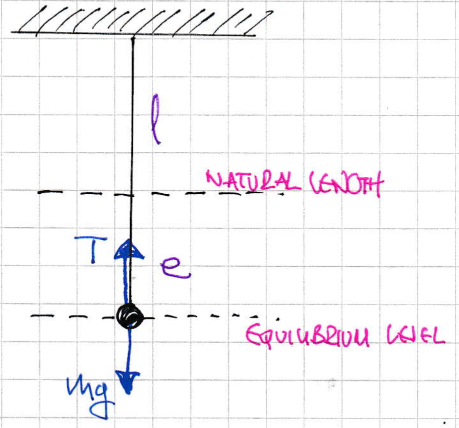
$$\Rightarrow 0 = (5k+18)(k-6)$$

$$\Rightarrow k = \begin{cases} 6 \\ \cancel{-\frac{18}{5}} \end{cases}$$

$$\therefore k = 6$$

IYGB – FM2 PAPER Q – QUESTION 4

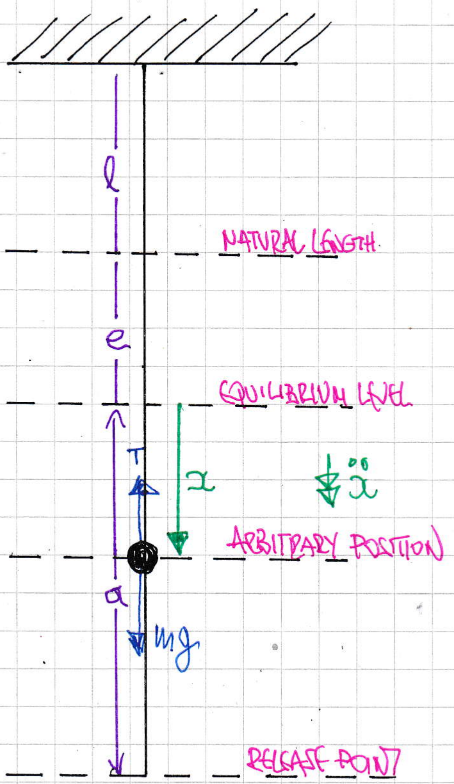
a) DIAGRAM IN EQUILIBRIUM



$m = 0.5$
$l = 0.8$
$\lambda = 90$

$$\Rightarrow T = mg$$
$$\Rightarrow \frac{\lambda}{l}e = mg$$
$$\Rightarrow e = \frac{mgl}{\lambda}$$
$$\Rightarrow e = \frac{0.5 \times 9.8 \times 0.8}{90}$$
$$\Rightarrow e = \frac{49}{1125}$$

DIAGRAM WITH PARTICLE IN AN ARBITRARY POSITION BELOW EQUILIBRIUM LEVEL



$$\Rightarrow m\ddot{x} = mg - T$$
$$\Rightarrow m\ddot{x} = mg - \frac{\lambda}{l}(e + x)$$
$$\Rightarrow m\ddot{x} = mg - \frac{\lambda e}{l} - \frac{\lambda}{l}x$$
$$\Rightarrow m\ddot{x} = mg - \frac{\lambda}{l}\left(\frac{mgl}{\lambda}\right) - \frac{\lambda}{l}x$$
$$\Rightarrow m\ddot{x} = \cancel{mg} - \cancel{mg} - \frac{\lambda}{l}x$$
$$\Rightarrow \ddot{x} = -\frac{\lambda}{ml}x$$
$$\Rightarrow \ddot{x} = -\frac{90}{0.5 \times 0.8}x$$
$$\Rightarrow \ddot{x} = -225x$$

IE. INDEED S.H.M WITH $\omega = 15$, ABOUT THE EQUILIBRIUM POSITION

IYGB – FM2 PAPER Q – QUESTION 4

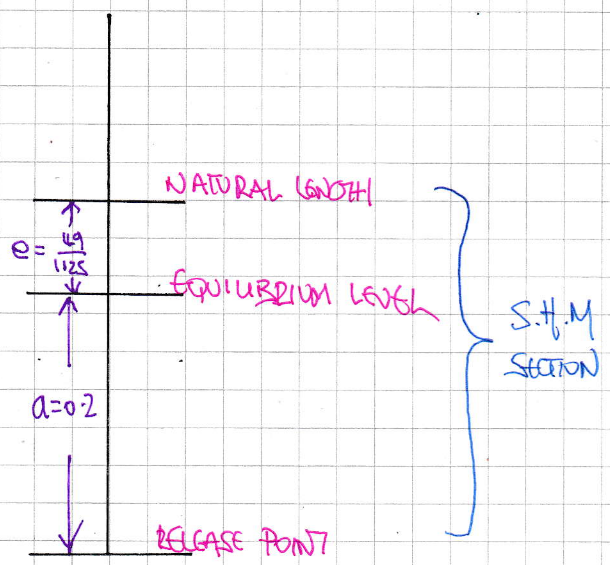
b) IF SPEED THROUGH EQUILIBRIUM POSITION IS $3\ \text{ms}^{-1}$, THEN $V_{MAX} = 3$

$$\Rightarrow |V|_{MAX} = a\omega$$

$$\Rightarrow 3 = a \times 15$$

$$\Rightarrow a = 0.2$$

LOOKING AT THE DIAGRAM



- $v^2 = \omega^2(a^2 - x^2)$

$v^2 = 225\left(0.2^2 - \left(\frac{49}{1125}\right)^2\right)$

$v^2 = 8.573155\ldots$

$v = 2.927995143\ldots$

- NEXT FALLING UNDER GRAVITY

$u = 2.9279\ldots$
$a = -9.8$
$s = ?$
$t =$
$v = 0$

$v^2 = u^2 + 2as$

$0 = 8.573155\ldots + 2(-9.8)s$

$s = 0.437405\ldots$

- REQUIRED TOTAL DISTANCE

$0.2 + \frac{49}{1125} + 0.4374\ldots$

$\approx 0.681\ \text{m}$ (3 s.f.)

IYGB - FM2 PAPER Q - QUESTION 5

a) STANDARD METHODOLOGY USING CALCULUS

$$\ddot{x} = \frac{60}{(t+3)^2} \qquad t=0,\ v=\dot{x}=0,\ x=0$$

ARBITRARY

$$\Rightarrow \frac{dv}{dt} = \frac{60}{(t+3)^2}$$

$$\Rightarrow 1\,dv = \frac{60}{(t+3)^2}\,dt$$

$$\Rightarrow \int_{v=0}^{v} 1\,dv = \int_{t=0}^{t} 60(t+3)^{-2}\,dt$$

$$\Rightarrow \big[v\big]_0^v = \big[-60(t+3)^{-1}\big]_0^t$$

$$\Rightarrow v - 0 = \left[\frac{60}{t+3}\right]_t^0$$

$$\Rightarrow v = 20 - \frac{60}{t+3}$$

b) CONTINUING BY WRITING $v = dx/dt$

$$\Rightarrow \frac{dx}{dt} = 20 - \frac{60}{t+3}$$

$$\Rightarrow 1\,dx = \left(20 - \frac{60}{t+3}\right)dt$$

$$\Rightarrow \int_{x=0}^{x} 1\,dx = \int_{t=0}^{t=6} 20 - \frac{60}{t+3}\,dt$$

$$\Rightarrow \big[x\big]_0^x = \big[20t - 60\ln(t+3)\big]_0^6$$

$$\Rightarrow x - 0 = (120 - 60\ln 9) - (0 - 60\ln 3)$$

$$\Rightarrow x = 120 - 60\ln 9 + 60\ln 3$$

$$\Rightarrow x = 120 - 60\left[\ln 9 - \ln 3\right]$$

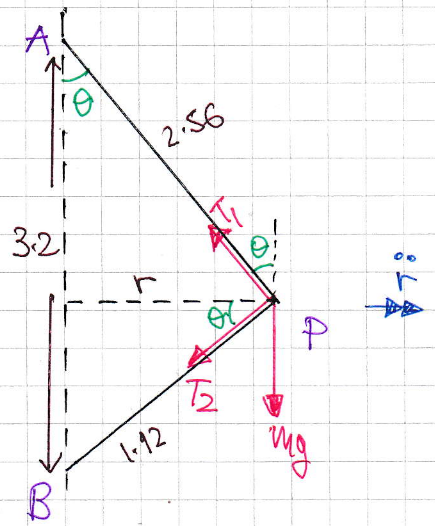
$$\Rightarrow x = 120 - 60\ln 3$$

$$\Rightarrow x = 60(2 - \ln 3)$$

AS REQUIRED

IYGB - FM2 PAPER Q - QUESTION 6

START WITH A DETAILED DIAGRAM



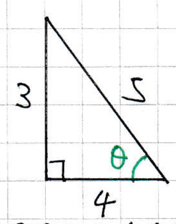
BY INSPECTION THE TRIANGLE IS 3 : 4 : 5
(OR USE THE COSINE RULE TO FIND $\cos\theta$)

$$1.92 : 2.56 : 3.2$$
$$192 : 256 : 320 \quad \times 10$$
$$3 : 4 : 5 \quad \div 64$$

$\therefore \sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$



VERTICALLY (EQUILIBRIUM)

$$T_1\cos\theta = T_2\sin\theta + mg$$
$$\frac{4}{5}T_1 = \frac{3}{5}T_2 + mg \quad \times 5$$
$$4T_1 = 3T_2 + 5mg \quad \times 3$$
$$12T_1 = 9T_2 + 15mg$$

RADIALLY (ACCELERATION)

$$m\ddot{r} = -T_1\sin\theta - T_2\cos\theta$$
$$m(-\omega^2 r) = -\frac{3}{5}T_1 - \frac{4}{5}T_2 \quad \times 5$$
$$5m\omega^2 r = 3T_1 + 4T_2 \quad \times 4$$
$$20m\omega^2 r = 12T_1 + 16T_2$$

SOLVING BY SUBSTITUTION — WE ONLY NEED T_2 TO SET $\geqslant 0$

$$\Longrightarrow 20m\omega^2 r = 9T_2 + 15mg + 16T_2$$
$$\Longrightarrow 20m\omega^2 r - 15mg = 25T_2$$

NOW T_2 "MUST HAVE TENSION", i.e. $T_2 \geqslant 0$

$$\Longrightarrow 20m\omega^2 r - 15mg \geqslant 0$$
$$\Longrightarrow 4\omega^2 r - 3g \geqslant 0$$

IYGB - FM2 PAPER Q - QUESTION 6

NOW LOOKING AT THE DIAGRAM

$r = 2.56 \sin\theta$

$r = 2.56 \times \frac{3}{5}$

$r = 1.536$

$\Rightarrow \quad 4\omega^2 r \geqslant 3g$

$\Rightarrow \quad 4\omega^2 \times 1.536 \geqslant 3 \times 9.8$

$\Rightarrow \quad \omega^2 \geqslant \frac{1225}{256}$

$\Rightarrow \quad \omega \geqslant \frac{35}{16}$

$\Rightarrow \quad \frac{1}{\omega} \leqslant \frac{16}{35}$

$\Rightarrow \quad \frac{2\pi}{\omega} \leqslant \frac{32\pi}{35}$

$\Rightarrow \quad \text{PERIOD} \leqslant \frac{32\pi}{35}$

AS REQUIRED

IYGB - FM2 PAPER Q - QUESTION 7

- $T = \frac{\pi}{3}$

$\Rightarrow \frac{2\pi}{\omega} = \frac{\pi}{3}$

$\Rightarrow \omega = 6$

- $v^2 = \omega^2(a^2 - x^2)$

$\Rightarrow 2.16^2 = 6^2(a^2 - 0.48^2)$

$\Rightarrow 0.1296 = a^2 - 0.48^2$

$\Rightarrow a^2 = 0.36$

$\Rightarrow a = 0.6$

- NEXT FIND THE VALUE OF x WHEN $v = 2.88$

$\Rightarrow v^2 = \omega^2(a^2 - x^2)$

$\Rightarrow 2.88^2 = 6^2(0.6^2 - x^2)$

$\Rightarrow 0.2304 = 0.6^2 - x^2$

$\Rightarrow x^2 = 0.1296$

$\Rightarrow |x| = 0.36$

- NOW USING A DISPLACEMENT-TIME RELATIONSHIP WITH $t=0$ AT THE END-POINT OF THE OSCILLATION

$x = a\cos\omega t$

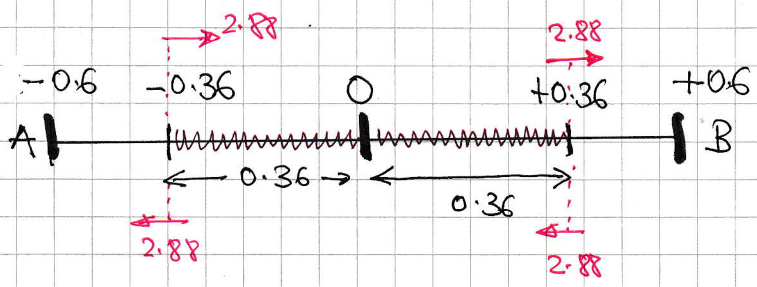
$x = \frac{3}{5}\cos 6t$

$0.36 = 0.6\cos 6t$

$\cos 6t = 0.6$

$6t = \arccos(0.6)$ (FIRST POSITIVE SOLUTION)

$t = \frac{1}{6}\arccos(0.6) \approx 0.154549\ldots$



$\therefore$ REQUIRED TIME $=$ PERIOD $- 4(0.154549\ldots) \approx 0.429$