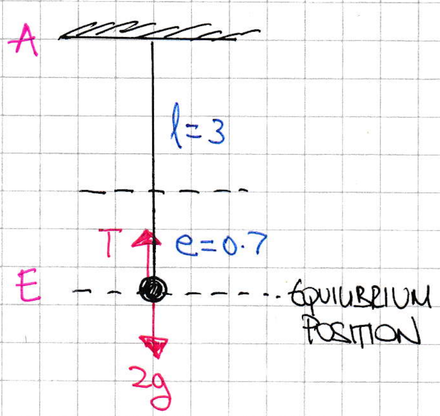
IYGB - FM2 PAPER O - QUESTION 1

a) LOOKING AT THE DIAGRAM



$\Longrightarrow T = 2g$

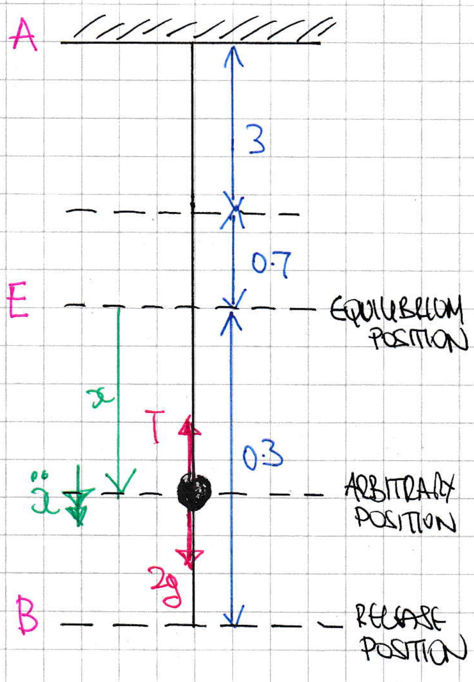
$\Longrightarrow \frac{\lambda}{l}x = 2g$

$\Longrightarrow \frac{\lambda \times 0.7}{3} = 2g$

$\Longrightarrow \lambda = \frac{60}{7}g$

$\Longrightarrow \lambda = 84\text{ N}$

b) LOOKING AT A DIAGRAM WITH THE PARTICLE AT AN ARBITRARY POSITION DURING THE MOTION



! NOT TO SCALE !

$\Longrightarrow m\ddot{x} = 2g - T$

$\Longrightarrow 2\ddot{x} = 2g - \frac{\lambda}{l}(x+e)$

$\Longrightarrow 2\ddot{x} = 19.6 - \frac{84}{3}(x+0.7)$

$\Longrightarrow 2\ddot{x} = 19.6 - 28(x+0.7)$

$\Longrightarrow 2\ddot{x} = \cancel{19.6} - 28x - \cancel{19.6}$

$\Longrightarrow \ddot{x} = -14x$

$\therefore$ S.H.M. ABOUT "E" WITH $\omega^2 = 14$ AND AMPLITUDE $a = 0.3$

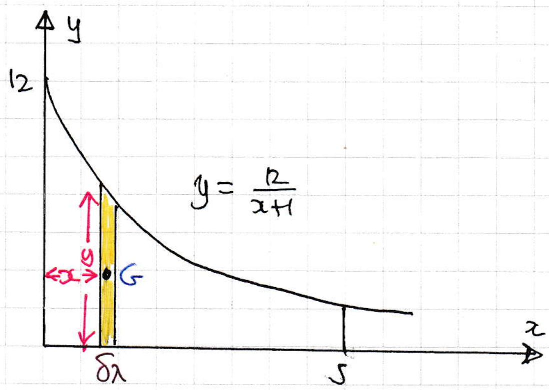
$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{14}} \simeq 1.68\text{s}$

c) USING THE STANDARD FORMULA $V_{MAX} = |\omega a|$

$V_{MAX} = \sqrt{14} \times 0.3 \simeq 1.12\text{ ms}^{-1}$

IYGB – FM2 PAPER O – QUESTION 2

LET ρ BE THE MASS PER UNIT AREA



AREA UNDER THE CURVE IS

$$A = \int_0^5 \frac{12}{x+1}\,dx = \Big[12\ln(x+1)\Big]_0^5 = 12\ln 6 - \cancel{12\ln 1} = 12\ln 6$$

THE MASS OF AN INFINITESIMAL STRIP OF HEIGHT y AND THICKNESS δx IS

$$\delta m = \rho y\,\delta x$$

THE "MOMENT" OF THE INFINITESIMAL ABOUT THE x & THE y AXIS ARE

$$(\rho y\delta x)x \quad \& \quad (\rho y \delta x) \times \tfrac{1}{2}y$$

SUMMING UP AND TAKING LIMITS

- $M\bar{x} = \int_0^5 \rho y x\,dx$

$$(12\ln 6)\rho\bar{x} = \rho\int_0^5 \frac{12x}{x+1}\,dx$$

$$\bar{x} = \frac{12}{12\ln 6}\int_0^5 \frac{x+1-1}{x+1}\,dx$$

$$\bar{x} = \frac{1}{\ln 6}\int_0^5 1 - \frac{1}{x+1}\,dx$$

- $M\bar{y} = \int_0^5 \frac{1}{2}\rho y^2\,dx$

$$(12\ln 6)\rho\bar{y} = \frac{1}{2}\rho\int_0^5 \frac{144}{(x+1)^2}$$

$$\bar{y} = \frac{1}{24\ln 6}\left[-\frac{144}{x+1}\right]_0^5$$

$$\bar{y} = \frac{144}{24\ln 6}\left[\frac{1}{x+1}\right]_5^0$$

IYGB

$\Rightarrow \bar{x} = \frac{1}{\ln 6}\left[x - \ln(x+1)\right]_0^5$

$\Rightarrow \bar{x} = \frac{1}{\ln 6}\left[(5 - \ln 6) - (\cancel{\ln 1})\right]$

$\Rightarrow \bar{x} = \frac{5}{\ln 6} - 1$

$\Rightarrow \bar{y} = \frac{6}{\ln 6}\left[1 - \frac{1}{6}\right]$

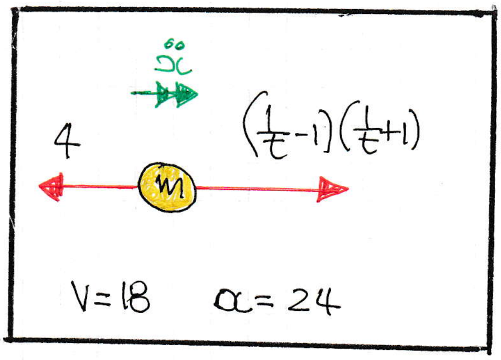
$\Rightarrow \bar{y} = \frac{6}{\ln 6} \times \frac{5}{6}$

$\Rightarrow \bar{y} = \frac{5}{\ln 6}$

$\therefore G\left(\frac{5}{\ln 6} - 1, \frac{5}{\ln 6}\right)$ //

IYGB – FM2 PAPER O – QUESTION 3

- STARTING WITH THE EQUATION OF MOTION



$$\Rightarrow m\ddot{x} = \left(\frac{1}{t}-1\right)\left(\frac{1}{t}+1\right)-4$$

$$\Rightarrow \frac{1}{6}\ddot{x} = \frac{1}{t^2}-1-4$$

$$\Rightarrow \frac{1}{6}\ddot{x} = \frac{1}{t^2}-5$$

$$\Rightarrow \ddot{x} = \frac{6}{t^2}-30$$

$$\Rightarrow \frac{dv}{dt} = \frac{6}{t^2}-30$$

- NOW WHEN $V=18$, $a=24$ $\Rightarrow$

$$24 = \frac{6}{t^2}-30$$

$$54 = \frac{6}{t^2}$$

$$t^2 = \frac{1}{9}$$

$$t = +\frac{1}{3}$$

- SEPARATING VARIABLES, SUBJECT TO THE CONDITION $t=\frac{1}{3}$, $V=18$

$$\Rightarrow 1\,dv = \left(\frac{6}{t^2}-30\right)dt$$

$$\Rightarrow \int_{V=18}^{10} 1\,dv = \int_{t=\frac{1}{3}}^{t} \frac{6}{t^2}-30\,dt$$

$$\Rightarrow \Big[v\Big]_{18}^{10} = \left[-\frac{6}{t}-30t\right]_{\frac{1}{3}}^{t}$$

$$\Rightarrow 10-18 = \left[\frac{6}{t}+30t\right]_{t}^{\frac{1}{3}}$$

IYGB - FM2 PAPER O - QUESTION 3

$\Rightarrow -8 = (18 + 10t) - \left(\frac{6}{t} + 30t\right)$

$\Rightarrow -8 = 18 + 10 - \frac{6}{t} - 30t$

$\Rightarrow 0 = 36 - \frac{6}{t} - 30t$ $\quad \div 6$

$\Rightarrow 0 = 6 - \frac{1}{t} - 5t$

$\Rightarrow 0 = 6t - 1 - 5t^2$

$\Rightarrow 5t^2 - 6t + 1 = 0$

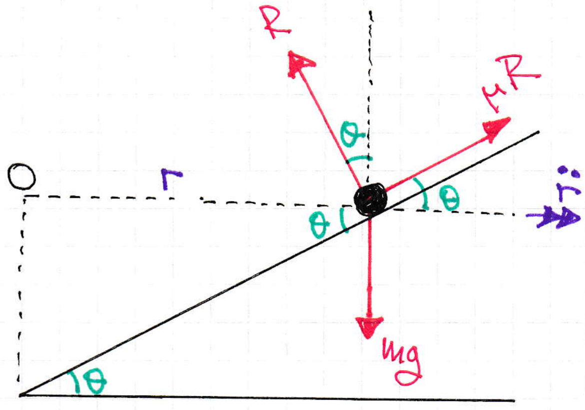
$\Rightarrow (5t - 1)(t - 1)$

$\Rightarrow t = \begin{cases} 1 \\ \frac{1}{5} \end{cases}$ //

IYGB – FM2 PAPER N – QUESTION 4

- VERTICALLY WE HAVE EQUILIBRIUM

$$\boxed{R\cos\theta + \mu R\sin\theta = mg}$$



- IN THE RADIAL DIRECTION, AWAY FROM O

$$\boxed{m\ddot{r} = \mu R\cos\theta - R\sin\theta}$$

- DIVIDING THE TWO EQUATIONS WE OBTAIN

$$\Rightarrow \frac{m\ddot{r}}{mg} = \frac{\mu R\cos\theta - R\sin\theta}{R\cos\theta + \mu R\sin\theta}$$

$$\Rightarrow \frac{\ddot{r}}{g} = \frac{\mu\cos\theta - \sin\theta}{\cos\theta + \mu\sin\theta}$$

$$\Rightarrow \frac{-\frac{v^2}{r}}{g} = \frac{\frac{\mu\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \mu\frac{\sin\theta}{\cos\theta}}$$

$$\Rightarrow -\frac{v^2}{rg} = \frac{\mu - \tan\theta}{1 + \mu\tan\theta}$$

$$\Rightarrow \frac{v^2}{rg} = \frac{\tan\theta - \mu}{\mu\tan\theta + 1}$$

$$\Rightarrow v^2 = \frac{rg(\tan\theta - \mu)}{1 + \mu\tan\theta}$$

IYGB - FM2 PAPER O - QUESTION 5

$$\frac{d^2x}{dt^2} = -k^2 x$$

PERIOD $= \frac{2\pi}{k} = 2$ $\therefore k = \pi$

AMPLITUDE $= 0.6$

USING THE STANDARD EQUATION FOR RELEASE AT THE ENDPOINT

$\Rightarrow x = a\cos kt$

$\Rightarrow x = 0.6\cos \pi t$

$\Rightarrow 0.3 = 0.6\cos \pi t$

$\Rightarrow 0.5 = \cos \pi t$

$\arccos(0.5) = \frac{\pi}{3}$

$\Rightarrow \begin{cases} \pi t = \frac{\pi}{3} \pm 2n\pi \\ \pi t = \frac{5\pi}{3} \pm 2n\pi \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\Rightarrow \begin{cases} t = \frac{1}{3} \pm 2n \\ t = \frac{5}{3} \pm 2n \end{cases}$

$\therefore t = \frac{1}{3}, 2\frac{1}{3}, \frac{5}{3}, 2+\frac{5}{3}$

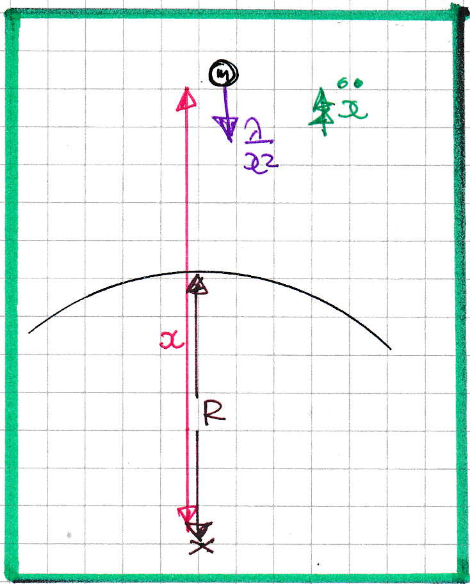
$\therefore t = \frac{1}{3}, \frac{5}{3}, \frac{7}{3}, \frac{11}{3}$

IYGB - FM2 PAPER O - QUESTION 6

STARTING WITH A DIAGRAM &

CONSIDER THE PARTICLE ON THE SURFACE

OF THE EARTH, TO START WITH



WHEN $x=R$ $\quad \frac{\lambda}{R^2} = mg$

$$\lambda = mgR^2$$

NEXT CONSIDER THE ARBITRARY CASE

$$\Rightarrow m\ddot{x} = -\frac{\lambda}{x^2}$$

$$\Rightarrow m\ddot{x} = -\frac{mgR}{x^2}$$

$$\Rightarrow \ddot{x} = -\frac{gR}{x^2}$$

$$\Rightarrow v\frac{dv}{dx} = -\frac{gR}{x^2}$$

INTEGRATE SUBJECT TO THE $x = R+\frac{1}{2}R$, $v=\sqrt{gR}$

$$\Rightarrow \int_{v=\sqrt{gR}}^{v=0} v \, dv = \int_{x=\frac{3}{2}R}^{x=d} -\frac{gR^2}{x^2} \, dx$$

$$\Rightarrow \left[\frac{1}{2}v^2\right]_{\sqrt{gR}}^{0} = \left[\frac{gR^2}{x}\right]_{x=3/2R}^{x=d}$$

$$\Rightarrow 0-\frac{1}{2}\cancel{g}R = \cancel{g}R^2\left[\frac{1}{d} - \frac{1}{\frac{3}{2}R}\right]$$

$$\Rightarrow -\frac{1}{2R} = \frac{1}{d} - \frac{2}{3R}$$

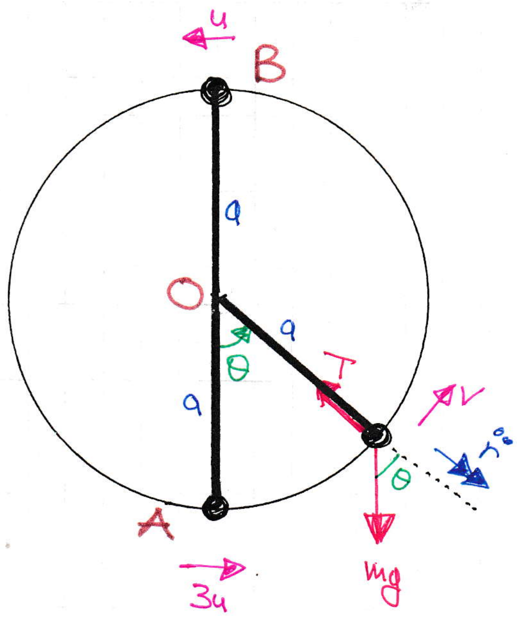
$$\Rightarrow \frac{2}{3R} - \frac{1}{2R} = \frac{1}{d}$$

$$\Rightarrow \frac{1}{6R} = \frac{1}{d}$$

$$\therefore d = 6R$$

IYGB – FM2 PAPER O – QUESTION 7

- LET THE MASS OF THE PARTICLE BE m AND THE LENGTH OF THE ROD BE a



- BY ENERGIES, TAKING THE LEVEL OF A AS THE ZERO POTENTIAL LEVEL WE HAVE

$$\Rightarrow KE_A + \cancel{PE_A} = KE_B + PE_B$$

$$\Rightarrow \tfrac{1}{2}\cancel{m}(3u)^2 + 0 = \tfrac{1}{2}\cancel{m}u^2 + \cancel{m}g(2a)$$

$$\Rightarrow \tfrac{9}{2}u^2 = \tfrac{1}{2}u^2 + 2ag$$

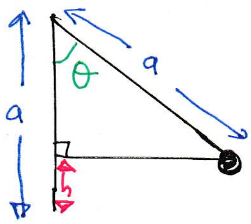
$$\Rightarrow 4u^2 = 2ag$$

$$\Rightarrow \boxed{u^2 = \tfrac{1}{2}ag}$$

- BY ENERGIES AGAIN BETWEEN A & SOME ARBITRARY POSITION "θ"

$$\Rightarrow KE_A + \cancel{P.E_A} = KE_\theta + P.E_\theta$$

$$\Rightarrow \tfrac{1}{2}\cancel{m}(3u)^2 + 0 = \tfrac{1}{2}\cancel{m}v^2 + \cancel{m}g\underbrace{(a - a\cos\theta)}_{h}$$



$$\Rightarrow \tfrac{9}{2}u^2 = \tfrac{1}{2}v^2 + ag(1-\cos\theta)$$

$$\Rightarrow 9u^2 = v^2 + 2ag(1-\cos\theta)$$

$$\Rightarrow v^2 = 9u^2 - 2ag(1-\cos\theta)$$

$$\Rightarrow v^2 = 9\left(\tfrac{1}{2}ag\right) - 2ag(1-\cos\theta)$$

$$\Rightarrow v^2 = \tfrac{9}{2}ag - 2ag + 2ag\cos\theta$$

$$\Rightarrow \boxed{v^2 = \tfrac{5}{2}ag + 2ag\cos\theta}$$

IYGB - FM2 PAPER O - QUESTION 7

- FINALLY LOOKING AT THE EQUATION OF MOTION (RADIALLY)

$$\Longrightarrow m\ddot{r} = mg\cos\theta - T$$

$$\Longrightarrow T = mg\cos\theta - m\ddot{r}$$

$$\Longrightarrow T = mg\cos\theta - m\left(-\frac{v^2}{a}\right)$$

$$\Longrightarrow T = mg\cos\theta + \frac{m}{a}v^2$$

$$\Longrightarrow T = mg\cos\theta + \frac{m}{a}\left[\frac{5}{2}ag + 2ag\cos\theta\right]$$

$$\Longrightarrow T = mg\cos\theta + \frac{5}{2}mg + 2mg\cos\theta$$

$$\Longrightarrow T = 3mg\cos\theta + \frac{5}{2}mg$$

$$\Longrightarrow T = \frac{1}{2}mg\left[6\cos\theta + 5\right]$$

- WHEN THE TENSION IS ZERO

$$\Longrightarrow 6\cos\theta + 5 = 0$$

$$\Longrightarrow \cos\theta = -\frac{5}{6}$$

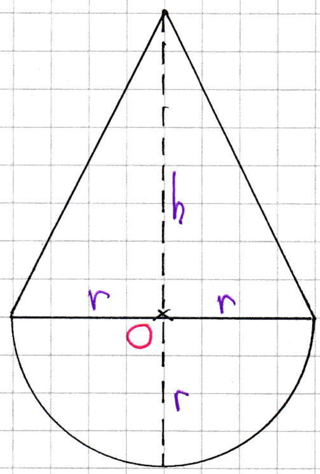
$$\Longrightarrow \theta = \arccos\left(-\frac{\pi}{6}\right)$$

OR

$$\left[\theta = \pi - \arccos\frac{\pi}{6}\right]$$

IYGB - FM2 PAPER O - QUESTION 8

LOOKING AT THE DIAGRAM WE HAVE



VOLUME OF THE CONE $\frac{1}{3}\pi r^2 h$

VOLUME OF THE HEMISPHERE $\frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$

RATIO OF VOLUMES

$$\frac{1}{\cancel{3}}\cancel{\pi} r^2 h \;:\; \frac{2}{\cancel{3}}\cancel{\pi} r^3$$

$$h \;:\; 2r$$

RATIO OF MASSES

$$h \;:\; 4r$$

FORMING A "MOMENTS TABLE"

	CONE	HEMISPHERE	COMPOSITE
MASS RATIO	h	$4r$	$h+4r$
DISTANCE OF CENTRE OF MASS FROM O	$+\frac{1}{4}h$	$-\frac{3}{8}r$	$\frac{19}{180}h$

$$\Rightarrow \frac{1}{4}h^2 - \frac{3}{2}r^2 = \frac{19h}{180}(h+4r)$$

$$\Rightarrow 45h^2 - 270r^2 = 19h^2 + 76hr$$

$$\Rightarrow 26h^2 - 76rh - 270r^2 = 0$$

$$\Rightarrow 13h^2 - 38rh - 135h = 0$$

BY THE QUADRATIC FORMULA OR INSPECTION

$$\Rightarrow (13h + 27r)(h - 5r) = 0$$

$$\Rightarrow h = \begin{cases} \cancel{-\frac{27}{13}r} \\ 5r \end{cases}$$