IYGB GCE

Core Mathematics C4

Advanced

Practice Paper X

Difficulty Rating: 4.1066/2.1127

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

A curve C has implicit equation

$$y\mathrm{e}^{y} = x^{x}, \ x > 0$$

By taking logarithms on both sides of this equation, show that

$$\frac{dy}{dx} = \frac{y(1+\ln x)}{1+y}.$$ **(7)**

Question 2

The function $f(x)$ is defined in terms of the non zero constant n, by

$$f(x) = (3+2x)^{n}, \ -\frac{3}{2} < x < \frac{3}{2}.$$

a) Given that n is not a positive integer, find in terms of n the ratio of the coefficient of x^3 to the coefficient of x^2 in binomial expansion of $f(x)$. **(5)**

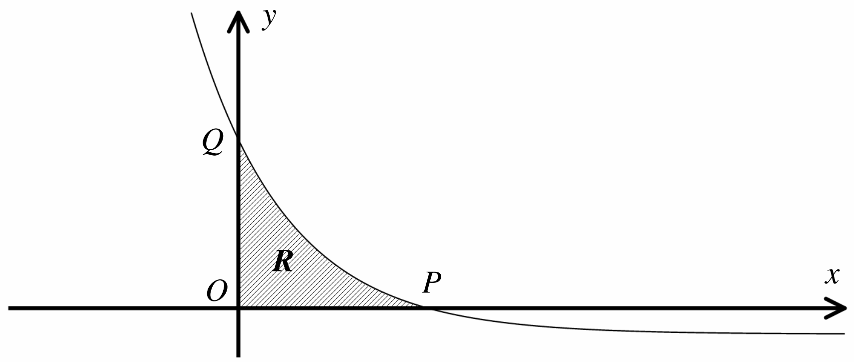
It is now given that $n = \frac{7}{2}$.

b) Evaluate the ratio found in part **(a)**. **(1)**

The coefficient of x^r in the binomial expansion of $f(x)$ is negative.

c) Find the smallest value of r. **(2)**

Question 3



The figure above shows the graph of the curve with parametric equations

$$x = 2 - \frac{1}{4}t, \quad y = 2^t - 2, \quad t \in \mathbb{R}.$$

The curve meets the x axis at the point P and the y axis at the point Q.

a) Find the coordinates of P and Q. **(4)**

The finite region R is bounded by the curve and the coordinate axes, and is shown shaded in the figure above.

b) Show that the area R is given by the integral **(4)**

$$\int_1^8 2^{t-2} - \frac{1}{2} \; dt.$$

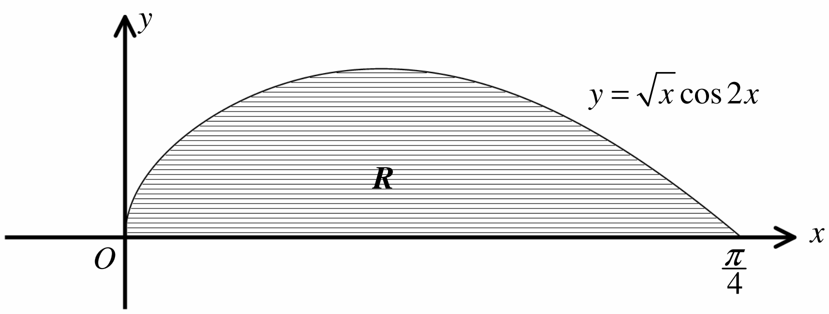
c) Hence find an exact value for R. **(3)**

Question 4

$$f(x) = \frac{1}{8}(4x + \sin 4x), \ x \in \mathbb{R}, \ 0 \le x \le \frac{\pi}{4}.$$

a) Show that

$$f'(x) = \cos^2 2x. \qquad \textbf{(4)}$$



The figure above shows part of the graph of a curve C with equation

$$y = \sqrt{x} \cos 2x, \ x > 0.$$

The curve meets the x axis at the origin and at the point where $x = \frac{\pi}{4}$.

The shaded region R is bounded by the curve and the x axis. The region R is rotated by 2π radians about the x axis, forming a solid of revolution S.

b) Show further that the volume of S is

$$\frac{\pi}{64}(\pi^2 - 4). \qquad \textbf{(8)}$$

Question 5

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - \mathbf{k})$$

$$\mathbf{r}_2 = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$$

where λ and μ are scalar parameters.

The point A is the intersection of l_1 and l_2.

The point $B(b,1,-1)$, where b is a scalar constant, lies on l_1.

The point $D(4,d,3)$, where d is a scalar constant, lies on l_2.

a) Find the value of b and the value of d. **(3)**

b) Calculate the cosine of θ, where θ is the acute angle formed by l_1 and l_2. **(3)**

The point C is such so that $ABCD$ is a parallelogram.

c) Determine the coordinates of C. **(3)**

d) Show that the area of the parallelogram $ABCD$ is $48\sqrt{2}$ square units. **(4)**

Question 6

The surface area of a sphere is decreasing at the rate of $6 \text{ cm}^2 \text{ s}^{-1}$ at the instant when its radius is 12 cm.

Find the rate at which the volume of the sphere is decreasing at that instant. **(7)**

[surface area of a sphere of radius r is given by $4\pi r^2$]

[volume of a sphere of radius r is given by $\frac{4}{3}\pi r^3$]

Question 7

There are $20,000$ chickens in a farm and some of them have been infected by a virus. Let x be the number of infected chickens **in thousands**, and t the time in hours since the infection was first discovered.

The rate at which chickens are infected is proportional to the product of the number of chickens infected and the number of chickens not yet infected.

a) Form a differential equation in terms of x, t and a suitable proportionality constant k. **(2)**

When the disease was first discovered 4000 chickens were infected, and chickens were infected at the rate of 32 chickens per hour.

b) Solve the differential equation to show that

$$t = 100\ln\left[\frac{4x}{20-x}\right].$$ **(10)**

c) Rearrange the answer in part **(b)** to show further that

$$x = \frac{20}{1+4e^{-0.01t}}.$$ **(3)**

d) If a vet cannot attend the farm for 24 hours, since the infection was first discovered, find how many extra chickens will be infected by the time the vet arrives. **(2)**
