IYGB GCE

Core Mathematics C4

Advanced

Practice Paper W

Difficulty Rating: 4.0600/2.0619

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

A curve C has implicit equation

$$ax^2 + xy - 2y^2 + b = 0,$$

where a and b are constants.

The normal to the curve at the point $P(1,4)$ has equation

$$2y + 3x = 11.$$

Determine the value of a and the value of b. **(8)**

Question 2

Relative to a fixed origin O, the points A and B have respective position vectors

$$2\mathbf{i} + 10\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

The angle AOB is θ.

a) Show that $\sin\theta = \dfrac{\sqrt{6}}{3}$. **(4)**

b) Calculate the exact area of the triangle AOB. **(2)**

c) Show further that the shortest distance of …

i. … A from the straight line OB is $6\sqrt{2}$, **(3)**

ii. … the straight line AB from O is $2\sqrt{2}$. **(3)**

Question 3

The surface area S of a sphere is increasing at the constant rate of $16 \text{ cm}^2 \text{ s}^{-1}$.

Find the rate at which the volume V of the sphere is increasing, when the sphere's surface area is $625\pi \text{ cm}^2$. **(7)**

$\left[\text{surface area of a sphere of radius } r \text{ is given by } 4\pi r^2\right]$

$\left[\text{volume of a sphere of radius } r \text{ is given by } \tfrac{4}{3}\pi r^3\right]$

Question 4

By using the substitution $u = \mathrm{e}^x + 1$, or otherwise, find

$$\int \frac{\mathrm{e}^{2x} - 2\mathrm{e}^x}{\mathrm{e}^x + 1} \, dx .$$ **(7)**

Question 5

A population P, in millions, at a given time t years, is growing at a rate equal to the product of the population squared and the difference of the population from one million.

Initially the population is one quarter of a million.

a) Form and solve a differential equation to show that **(11)**

$$t = \ln\left|\frac{3P}{1-P}\right| - \frac{1}{P} + 4 .$$

b) State the limiting value for this population. **(1)**

Question 6

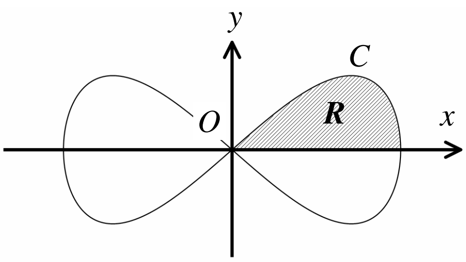


figure 1

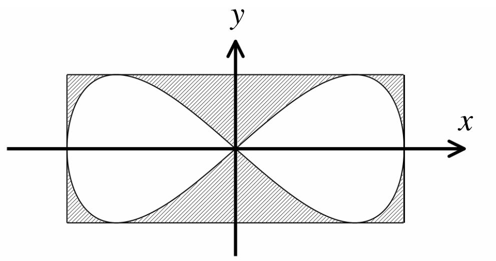


figure 2

The figure 1 above, shows the curve C with parametric equations

$$x = 6\cos t, \quad y = 12\sin 2t, \quad 0 \le t \le 2\pi .$$

The curve is symmetrical in the x axis and in the y axis.

The region R, shown shaded on the figure 1, is bounded by the part of C in the first quadrant and the coordinate axes.

a) Show that the area of R is given by

$$\int_0^{\frac{\pi}{2}} 144\cos t\sin^2 t \; dt . \qquad \textbf{(6)}$$

b) Hence find the area enclosed by C in all four quadrants. **(3)**

The area enclosed by the entire curve is to be cut out of a piece of rectangular card, as shown in the figure 2. This is modelled by a rectangle whose sides are tangents to the curve, parallel to the coordinate axes.

The area of the card left over after the curve was cut out is shown shaded in figure 2.

c) Show that the area of the card left over is exactly 96 square units. **(3)**

Question 7

$$f(x) \equiv \frac{1}{\sqrt{1-ax}} - \sqrt{1+bx},$$

where a and b are constants so that $a > b > 0$.

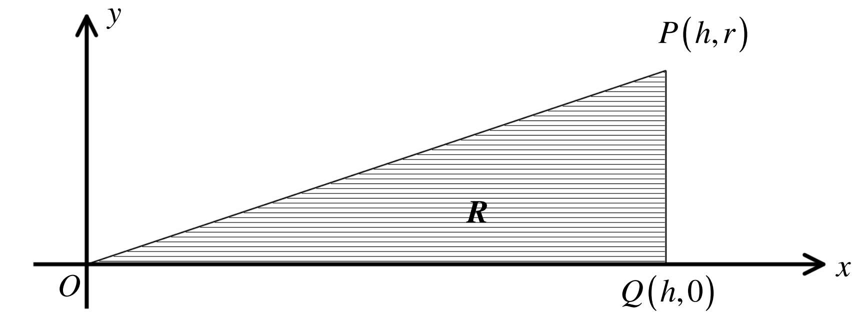
The function f is defined in a suitable domain of x, and furthermore the values of x are small enough so that $f(x)$ has a binomial series expansion.

Given that

$$f(x) \approx 2x + 26x^2,$$

determine the value of a and the value of b. **(10)**

Question 8



The figure above shows the straight line segment OP, joining the origin to the point $P(h,r)$, where h and r are positive coordinates.

The point $Q(h,0)$ lies on the x axis.

The shaded region R is bounded by the straight line segments OP, PQ and OQ.

The region R is rotated by 2π radians in the x axis to form a solid cone of height h and radius r.

Show by integration that the volume of the cone V is given by

$$V = \frac{1}{3}\pi r^2 h.$$ **(7)**