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IYGB GCE

Core Mathematics C4

Advanced

Practice Paper U

Difficulty Rating: 3.9533/1.9544

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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Question 1

The binomial expression $(1+12x)^{\frac{3}{4}}$ is to be expanded as an infinite convergent series, in ascending powers of x.

a) Find the first 4 terms in the expansion of $(1+12x)^{\frac{3}{4}}$. **(3)**

b) State the range of values of x for which the expansion is valid. **(1)**

c) By substituting a suitable value for x in the expansion show that

$$\left(\frac{53}{50}\right)^{\frac{3}{4}} \approx 1.04467 .$$ **(3)**

Question 2

By using the substitution $u = 2x^{\frac{5}{2}} + 1$, or otherwise, find an exact simplified value for

$$\int_0^1 \frac{10x^4}{2x^{\frac{5}{2}}+1}\, dx .$$ **(7)**

Question 3

The number of foxes N, in thousands, living within an urban area t years after a given instant, can be modelled by the differential equation

$$\frac{dN}{dt} = 2N - N^2, \quad t \geq 0 .$$

Initially it is thought 1000 foxes lived within this urban area.

a) Find a solution of the differential equation, in the form $N = f(t)$. **(11)**

b) Find the long term prospects of this population of foxes, as predicted by this model, clearly showing your reasoning. **(2)**

Question 4

The point $P(4,2)$ lies on the curve with equation

$$2^x y + 2^y x = 6xy .$$

Show that the gradient of the curve at P is

$$\frac{1 - a\ln 2}{b\ln 2 - 1},$$

where a and b are positive integers to be found. **(8)**

Question 5

A curve is defined by the parametric equations

$$x = t^2 + t , \quad y = 2t - 1 , \quad t \in \mathbb{R} .$$

a) Show that an equation of the tangent to the curve at the point P where $t = p$ can be written as

$$y(2p+1) = 2x + 2p^2 - 2p - 1 . \qquad \textbf{(5)}$$

The tangents to curve at the points $(2,1)$ and $(0,-3)$ meet at the point Q.

b) Find the coordinates of Q. **(5)**

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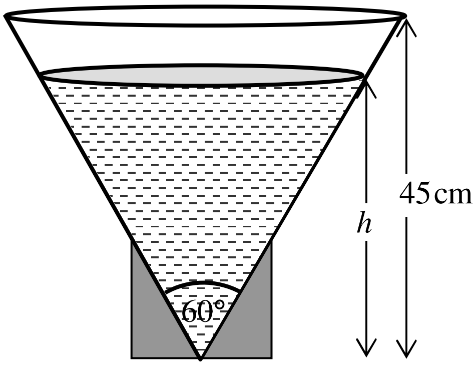
Question 6



A container, in the shape of a hollow inverted cone, is filled up the water.

The height of the container is 45 cm and the angle between the sides of the cone, when viewed as a cross section, is $60°$.

a) Show that the volume, V cm^3, of the water in the container is given by

$$V = \frac{1}{9}\pi h^3 ,$$

where h cm is the height of the water in the container. **(4)**

The container is filled up with water to the rim and then the water is allowed to leak from a small hole at the bottom of the cone, at the **constant** rate of $80 \text{ cm}^3\text{s}^{-1}$.

b) Determine the rate at which the height of the water is decreasing …

i. … when the height of the water is 20 cm. **(4)**

ii. … five minutes after the leaking started. **(6)**

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Question 7

Relative to a fixed origin O, the straight lines L and M have vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 4 \\ 10 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 0 \\ 14 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix},$$

where λ and μ are scalar parameters.

a) Show that L and M represent the same straight line and find a linear relationship between λ and μ, giving the answer in the form $\lambda = f(\mu)$. **(5)**

The points A, B and C lie on L, where $\lambda = 3$, $\lambda = 5$ and $\lambda = 8$ respectively.

b) State the ratio $AB : BC$. **(2)**

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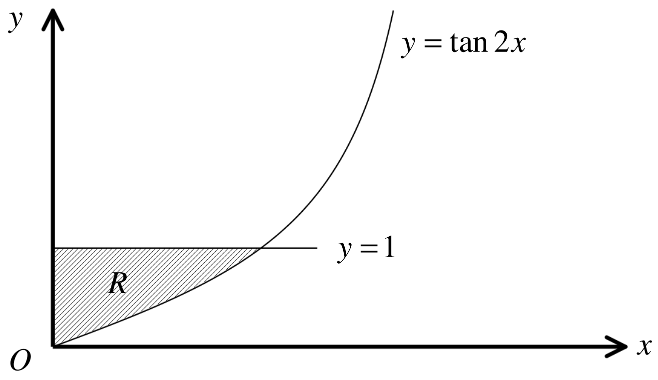
Question 8



The figure above shows the graph of the curve with equation

$$y = \tan 2x, \ 0 \le x \le \frac{\pi}{4}.$$

The finite region R is bounded by the curve, the y axis and the horizontal line with equation $y = 1$.

The region R is rotated by 2π radians about the straight line with equation $y = 1$ forming a solid of revolution.

Determine an exact volume for this solid. **(9)**

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