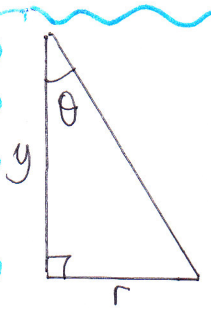
1. a)



$\tan\theta = \frac{1}{\sqrt{3}}$

$\frac{r}{y} = \frac{1}{\sqrt{3}}$

$r = \frac{y}{\sqrt{3}}$

- $\frac{dV}{dt} = 3.2$ CONSTANT

$\therefore V = 3.2t$

- SHAPE IS CONICAL

$\Rightarrow$ "$\frac{1}{3}\pi r^2 h$" $= 3.2t$

$\Rightarrow \frac{1}{3}\pi\left(\frac{y}{\sqrt{3}}\right)^2 y = \frac{16}{5}t$

$\Rightarrow \frac{1}{9}\pi y^3 = \frac{16}{5}t$

$\Rightarrow \quad y^3 = \frac{144t}{5\pi}$

AS REQUIRED

b) I) $y^3 = \frac{144}{5\pi}t$

$\Rightarrow \frac{d}{dt}\left(y^3\right) = \frac{d}{dt}\left(\frac{144}{5\pi}t\right)$

$\Rightarrow 3y^2\frac{dy}{dt} = \frac{144}{5\pi}$

$\Rightarrow y^2\frac{dy}{dt} = \frac{48}{5\pi}$

$\Rightarrow y^6\left(\frac{dy}{dt}\right)^3 = \left(\frac{48}{5\pi}\right)^3$

$\Rightarrow \left(\frac{144t}{5\pi}\right)^2\left(\frac{dy}{dt}\right)^3 = \left(\frac{48}{5\pi}\right)^3$

$\Rightarrow \left(\frac{dy}{dt}\right)^3 = \left(\frac{48}{5\pi}\right)^3 \times \left(\frac{5\pi}{144t}\right)^2$

$\Rightarrow \left(\frac{dy}{dt}\right)^3 = \frac{48^3}{5\pi \times (144t)^2}$

$\Rightarrow \left(\frac{dy}{dt}\right)^3 = \frac{16}{15\pi t^2}$

$\Rightarrow \left[\frac{dy}{dt}\Big|_{t=60}\right]^3 = \frac{16}{15\pi \times 60^2}$

$\Rightarrow \left[\frac{dy}{dt}\Big|_{t=60}\right]^3 = \frac{1}{3375\pi}$

$\Rightarrow \frac{dy}{dt}\Big|_{t=60} = \sqrt[3]{\frac{1}{3375\pi}}$

$\Rightarrow \frac{dy}{dt}\Big|_{t=60} = \frac{1}{15\pi^{1/3}}$

$\Rightarrow \frac{dy}{dt}\Big|_{t=60} = \frac{1}{15}\pi^{-\frac{1}{3}}$

AS REQUIRED

II) NOW $\boxed{\text{"}A = \pi r\left(r^2+h^2\right)^{\frac{1}{2}}\text{"}}$

$$\Rightarrow A = \pi\left(\frac{y}{\sqrt{3}}\right)\left[\left(\frac{y}{\sqrt{3}}\right)^2+y^2\right]^{\frac{1}{2}}$$

$$\Rightarrow A = \pi\left(\frac{y}{\sqrt{3}}\right)\left(\frac{4}{3}y^2\right)^{\frac{1}{2}}$$

$$\Rightarrow A = \pi\left(\frac{y}{\sqrt{3}}\right)\left(\frac{2y}{\sqrt{3}}\right)$$

$$\Rightarrow \boxed{A = \frac{2}{3}\pi y^2}$$

THUS $\frac{dA}{dt} = \frac{dA}{dy} \times \frac{dy}{dt}$

$$\Rightarrow \frac{dA}{dt} = \left(\frac{4}{3}\pi y\right) \times \left(\frac{dy}{dt}\right)$$

$$\Rightarrow \left(\frac{dA}{dt}\right)^3 = \frac{64}{27}\pi^3 y^3 \times \left(\frac{dy}{dt}\right)^3$$

$$\Rightarrow \left(\frac{dA}{dt}\right)^3 = \frac{64}{27}\cancel{\pi^3} \times \frac{144t}{5\cancel{\pi}} \times \frac{16}{15\cancel{\pi}t^2}$$

$$\Rightarrow \left(\frac{dA}{dt}\right)^3 = \frac{64\pi \times 144 \times 16}{27 \times 5 \times 15t}$$

$$\Rightarrow \left[\left.\frac{dA}{dt}\right|_{t=60}\right]^3 = \frac{64\pi \times 144 \times 16}{27 \times 5 \times 15 \times 60}$$

$$\Rightarrow \left[\left.\frac{dA}{dt}\right|_{t=60}\right]^3 = \frac{4096\pi}{3375}$$

$$\Rightarrow \left.\frac{dA}{dt}\right|_{t=60} = \sqrt[3]{\frac{4096\pi}{3375}}$$

$$\Rightarrow \left.\frac{dA}{dt}\right|_{t=60} = \frac{16}{15}\pi^{\frac{1}{3}}$$

AS REQUIRED

2. $$\int_0^{\frac{\pi}{4}} \frac{\sqrt{3}}{2+\sin 2x}\,dx = \ldots \text{ BY SUBSTITUTION}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{3}}{2+\sin 2x}\left(\frac{\sqrt{3}\sec^2\theta}{2\sec^2 x}\,d\theta\right)$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{3\sec^2\theta}{4\sec^2 x+2\sin 2x\sec^2 x}\,d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{3\sec^2\theta}{4\sec^2 x+4\sin x\cos x\times\frac{1}{\cos^2 x}}\,d\theta$$

$$= \frac{3}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{\sec^2 x+\frac{\sin x}{\cos x}}\,d\theta$$

$$= \frac{3}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{\sec^2 x+\tan x}\,d\theta$$

$$= \frac{3}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{1+\tan^2 x+\tan x}\,d\theta$$

$$= \frac{3}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{1+\frac{1}{4}(-1+\sqrt{3}\tan\theta)^2+\frac{1}{2}(-1+\sqrt{3}\tan\theta)}\,d\theta$$

$$= \frac{3}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{1+\frac{1}{4}(1-2\sqrt{3}\tan\theta+3\tan^2\theta)-\frac{1}{2}+\frac{\sqrt{3}}{2}\tan\theta}\,d\theta$$

$$= \frac{3}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{1+\frac{1}{4}\cancel{-\frac{\sqrt{3}}{2}\tan\theta}+\frac{3}{4}\tan^2\theta-\frac{1}{2}\cancel{+\frac{\sqrt{3}}{2}\tan\theta}}\,d\theta$$

$$= \frac{3}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{\frac{3}{4}+\frac{3}{4}\tan^2\theta}\,d\theta$$

$\tan x = \frac{1}{2}(-1+\sqrt{3}\tan\theta)$

DIFF W.R.T θ

$\sec^2 x\frac{dx}{d\theta} = \frac{1}{2}\sqrt{3}\sec^2\theta$

$dx = \frac{\sqrt{3}\sec^2\theta}{2\sec^2 x}\,d\theta$

$x=0, \quad 0 = \frac{1}{2}(-1+\sqrt{3}\tan\theta)$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

$x=\frac{\pi}{4}, \quad 1 = \frac{1}{2}(-1+\sqrt{3}\tan\theta)$

$2 = -1+\sqrt{3}\tan\theta$

$3 = \sqrt{3}\tan\theta$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$$= \frac{3}{4}\int_{\pi/6}^{\pi/3} \frac{\cancel{1+\tan^2\theta}}{\frac{3}{4}(\cancel{1+\tan^2\theta})}\,d\theta = \frac{3}{4}\int_{\pi/6}^{\pi/3} \frac{1}{3/4}\,d\theta = \int_{\pi/6}^{\pi/3} 1\,d\theta$$

$$= \Big[\theta\Big]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

3. a) A, B, C ARE COLLINEAR

$A(a,-3,6)$ $B(2,b,2)$ $C(3,3,0)$

$\overrightarrow{AB} = \underline{b} - \underline{a} = (2,b,2) - (a,-3,6) = (2-a,\ b+3,\ -4)$

$\overrightarrow{BC} = \underline{c} - \underline{b} = (3,3,0) - (2,b,2) = (1,\ 3-b,\ -2)$

$$\Rightarrow \frac{2-a}{1} = \frac{b+3}{3-b} = \frac{-4}{-2}$$

$$\Rightarrow 2-a = \frac{b+3}{3-b} = 2$$

$\therefore\ 2-a=2$ & $\frac{b+3}{3-b} = 2$

$\boxed{a=0}$

$b+3 = 6-2b$

$3b = 3$

$\boxed{b=1}$

THUS DIRECTION VECTOR IS $(1,2,-2)$ & $A(0,-3,6)$
$B(2,1,2)$

$\therefore \underline{r} = (0,-3,6) + \lambda(1,2,-2)$

$(x,y,z) = (\lambda,\ 2\lambda-3,\ 6-2\lambda)$

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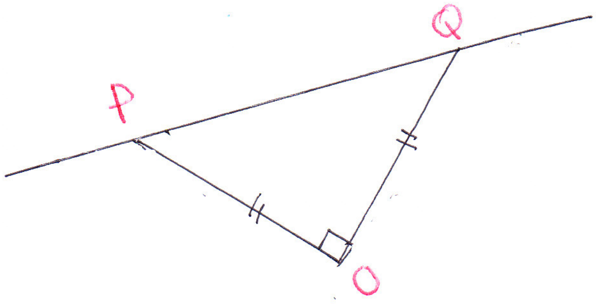
b)



LET $\underline{p} = (p, 2p-3, 6-2p)$
$\underline{q} = (q, 2q-3, 6-2q)$
FOR SOME VALUES
$\lambda = p$ & $\lambda = q$, $p \neq q$

- $\vec{OP} \cdot \vec{OQ} = 0$

$(p, 2p-3, 6-2p) \cdot (q, 2q-3, 6-2q) = 0$

$pq + (2p-3)(2q-3) + (6-2p)(6-2q) = 0$

$pq + 4pq - 6p - 6q + 9 + 36 - 12p - 12q + 4pq = 0$

$9pq - 18p - 18q + 45 = 0$

$\boxed{pq - 2p - 2q + 5 = 0}$

- $|\vec{OP}| = |p, 2p-3, 6-2p|$

$|\vec{OP}| = \sqrt{p^2 + (2p-3)^2 + (6-2p)^2}$

$|\vec{OP}| = \sqrt{p^2 + 4p^2 - 12p + 9 + 36 - 24p + 4p^2}$

$|\vec{OP}| = \sqrt{9p^2 - 36p + 45}$

$\boxed{|\vec{OP}| = 3\sqrt{p^2 - 4p + 5}}$

& IN ANALOGY

$\boxed{|\vec{OQ}| = 3\sqrt{q^2 - 4q + 5}}$

NOW COLLECTING RESULTS & FACTS

$\Rightarrow |\vec{OP}| = |\vec{OQ}|$

$\Rightarrow \cancel{3}\sqrt{p^2 - 4p + 5} = \cancel{3}\sqrt{q^2 - 4q + 5}$

$\Rightarrow p^2 - 4p \cancel{+5} = q^2 - 4q \cancel{+5}$

$\Rightarrow p^2 - q^2 = 4p - 4q$

$\Rightarrow \cancel{(p-q)}(p+q) = 4\cancel{(p-q)}$ $\quad \boxed{p \neq q}$

$\Rightarrow \boxed{p + q = 4}$

&

$\boxed{pq - 2p - 2q + 5 = 0}$

$\Rightarrow pq - 2(p+q) + 5 = 0$

$\Rightarrow pq - 8 + 5 = 0$

$\Rightarrow \boxed{pq = 3}$

THUS $\left.\begin{array}{l} p + q = 4 \\ pq = 3 \end{array}\right\} \Rightarrow$

$\Rightarrow p^2 + pq = 4p$

$\Rightarrow p^2 + 3 = 4p$

$\Rightarrow p^2 - 4p + 3 = 0$

$\Rightarrow (p-1)(p-3) = 0$

$\Rightarrow p = \left\langle \begin{array}{l} 1 \\ 3 \end{array}\right. \quad q = \left\langle \begin{array}{l} 3 \\ 1 \end{array}\right.$

(SYMMETRICAL SOLUTION)

$\therefore P(1, -1, 4)$ & $Q(3, 3, 0)$

(IN EITHER ORDER)

NOTE THAT ONE POINT IS IN FACT POINT A

4. $f(x) = \dfrac{3x^4 + x^3 + 2x^2 + x - 2}{(x+1)x^5}$

$$= \frac{-2 + x + 2x^2 + x^3 + 3x^4}{1+x} \times \frac{1}{x^5}$$

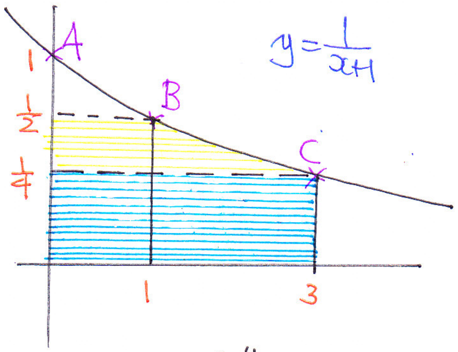
"NOW DIVIDE IN "ASCENDING" ORDER"

$$
\begin{array}{r|l}
 & -2 + 3x - x^2 + 2x^3 + x^4 \\
\hline
1+x & -2 + x + 2x^2 + x^3 + 3x^4 \\
 & +2 + 2x \\
\hline
 & 3x + 2x^2 + x^3 + 3x^4 \\
 & 3x - 3x^2 \\
\hline
 & -x^2 + x^3 + 3x^4 \\
 & -x^2 + x^3 \\
\hline
 & +2x^3 + 3x^4 \\
 & -2x^3 - 2x^4 \\
\hline
 & x^4 \\
 & -x^4 - x^5 \\
\hline
 & -x^5
\end{array}
$$

$$\ldots = \left[-2 + 3x - x^2 + 2x^3 + x^4 - \frac{x^5}{1+x}\right] \times \frac{1}{x^5}$$

$$= -\frac{2}{x^5} + \frac{3}{x^4} - \frac{1}{x^3} + \frac{2}{x^2} + \frac{1}{x} - \frac{1}{1+x}$$

5.

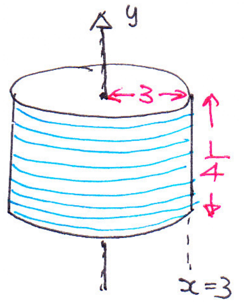


$A(0,1)$

$B(1,\frac{1}{2})$

$C(3,\frac{1}{4})$

a)

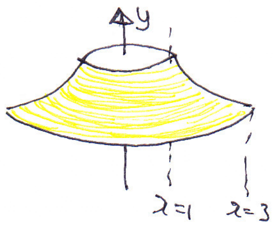


$$V = \pi r^2 h = \pi \times 3^2 \times \frac{1}{4} = \frac{9}{4}\pi$$

b)

$$y = \frac{1}{x+1}$$
$$x+1 = \frac{1}{y}$$
$$x = \frac{1}{y} - 1$$
$$x = \frac{1-y}{y}$$

VOLUME OF THE "YELLOW" SOLID



$$= \pi \int_{y_1}^{y_2} x^2 \, dy$$

$$= \pi \int_{\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{y} - 1\right)^2 dy$$

$$= \pi \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{y^2} - \frac{2}{y} + 1 \, dy$$

$$= \pi \left[-\frac{1}{y} - 2\ln|y| + y \right]_{\frac{1}{4}}^{\frac{1}{2}}$$

TIDYING UP THE EXPRESSION

$$= \pi \left[\left(-2 - 2\ln\frac{1}{2} + \frac{1}{2}\right) - \left(-4 - 2\ln\frac{1}{4} + \frac{1}{4}\right) \right]$$

$$= \pi \left[\left(-\frac{3}{2} + 2\ln 2\right) - \left(-\frac{15}{4} + 2\ln 4\right) \right]$$

$$= \pi \left[\frac{15}{4} - \frac{3}{2} + 2\ln 2 - 4\ln 2 \right]$$

$$= \pi \left[\frac{9}{4} - 2\ln 2 \right]$$

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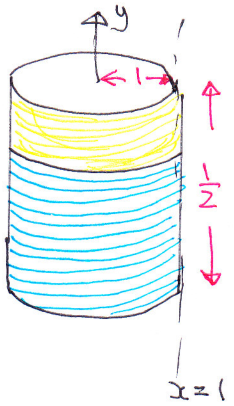
● 

$V = \pi r^2 h = \pi \times 1^2 \times \frac{1}{2} = \frac{1}{2}\pi$

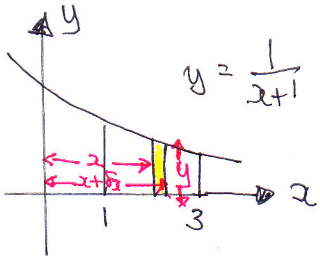
● REQUIRED VOLUME IS $\frac{9}{4}\pi + \pi\left[\frac{9}{4} - 2\ln 2\right] - \frac{1}{2}\pi$

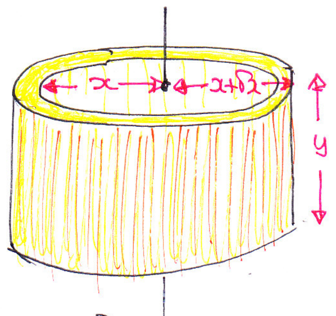
$= \frac{9}{4}\pi + \frac{9}{4}\pi - \frac{1}{2}\pi - \pi\ln 4$

$= 4\pi - \ln 4$

$= \pi(4 - \ln 4)$

ALTERNATIVE FROM "FIRST PRINCIPLES"





$\delta V = \left[\pi(x+\delta x)^2 - \pi x^2\right] y$

$\delta V = \pi y\left[\cancel{x^2} + 2x\delta x + \delta x^2 - \cancel{x^2}\right]$

$\delta V = \pi y\left[2x\delta x + \delta x^2\right]$

$\boxed{\delta V \approx \pi y(2x\delta x)}$

HENCE

$V = \int_{x=1}^{x=3} 2\pi y x \, dx$

$V = \int_1^3 2\pi x\left(\frac{1}{x+1}\right) dx$

$V = 2\pi \int_1^3 \frac{x}{x+1} \, dx$

$V = 2\pi \int_1^3 \frac{(x+1)-1}{(x+1)} \, dx$

$V = 2\pi \int_1^3 1 - \frac{1}{x+1} \, dx$

P.T.O

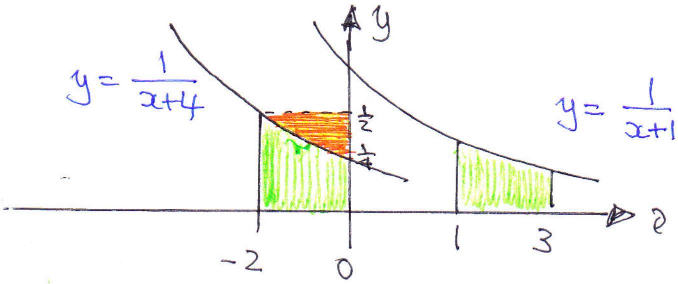
$$V = 2\pi\left[x - \ln|x+1|\right]_1^3 = 2\pi\left[(3-\ln 4)-(1-\ln 2)\right]$$

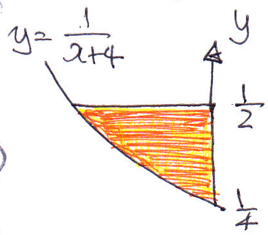
$$= 2\pi\left[3-\ln 4 - 1 + \ln 2\right] = 2\pi\left[2 - 2\ln 2 + \ln 2\right]$$

$$= 2\pi\left[2-\ln 2\right] = \pi\left[4 - 2\ln 2\right] = \pi\left[4 - \ln 4\right]$$

AS REQUIRED

b) CAN BE DONE DIRECTLY BY FIRST PRINCIPLES AS SHOWN ABOVE OR BY TRANSLATING THE CURVE 3 UNITS TO THE "LEFT"





$$y = \frac{1}{x+4}$$

$$x+4 = \frac{1}{y}$$

$$x = \frac{1}{y} - 4$$

$$V = \pi\int_{y_1}^{y_2} x^2\,dy = \pi\int_{\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{y}-4\right)^2 dy$$

$$= \pi\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{y^2} - \frac{8}{y} + 16\,dy = \pi\left[-\frac{1}{y} - 8\ln|y| + 16y\right]_{\frac{1}{4}}^{\frac{1}{2}}$$

$$= \pi\left[\left(-2 - 8\ln\tfrac{1}{2} + 8\right) - \left(\cancel{-4} - 8\ln\tfrac{1}{4} \cancel{+4}\right)\right]$$

$$= \pi\left[6 + 8\ln 2 - 8\ln 4\right]$$

$$= \pi\left[6 + 8\ln 2 - 16\ln 2\right]$$

$$= \pi\left[6 - 8\ln 2\right]$$

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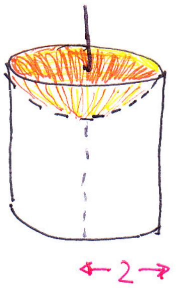
FINALLY



$V = \pi r^2 h = \pi \times 2^2 \times \frac{1}{2} = 2\pi$

$\therefore$ REQUIRED VOLUME $= 2\pi - \pi(6 - 8\ln 2)$

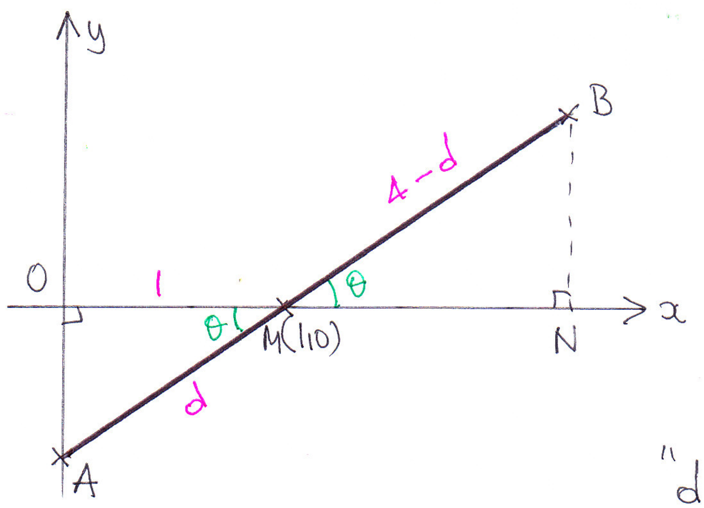
$= 2\pi - 6\pi + 8\pi \ln 2$

$= -4\pi + 8\pi \ln 2$

$= 4\pi[-1 + 2\ln 2]$

$= 4\pi[-1 + \ln 4]$

6. a)



- LOOKING AT $\triangle AOM$

$\frac{|OM|}{|AM|} = \cos\theta$

$\frac{1}{|AM|} = \cos\theta$

"d" $|AM| = \frac{1}{\cos\theta}$

THUS LOOKING AT $\hat{BMN}$ & THE CO-ORDINATES OF B

- $x = |OM| + |MN| = 1 + (4-d)\cos\theta = 1 + \left(4 - \frac{1}{\cos\theta}\right)\cos\theta$

$= 1 + 4\cos\theta - 1 = 4\cos\theta$

- $y = |BN| = (4-d)\sin\theta = \left(4 - \frac{1}{\cos\theta}\right)\sin\theta = 4\sin\theta - \frac{\sin\theta}{\cos\theta}$

$= 4\sin\theta - \tan\theta$

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b) $x = 4\cos\theta \qquad y = 4\sin\theta - \tan\theta$

$\Rightarrow y = 4\sin\theta - \tan\theta$

$\Rightarrow y = 4\sin\theta - \frac{\sin\theta}{\cos\theta}$

$\Rightarrow y = \sin\theta\left(4 - \frac{1}{\cos\theta}\right)$

$\Rightarrow y = \sin\theta\left(\frac{4\cos\theta - 1}{\cos\theta}\right)$

$\Rightarrow y^2 = \sin^2\theta \frac{(4\cos\theta - 1)^2}{\cos^2\theta}$

$\Rightarrow y^2 = \frac{(1-\cos^2\theta)(4\cos\theta - 1)^2}{\cos^2\theta}$

BUT $\cos\theta = \frac{x}{4}$

$\Rightarrow y^2 = \frac{\left(1 - \frac{x^2}{16}\right)(x-1)^2}{\frac{x^2}{16}}$

$\Rightarrow y^2 = \frac{\frac{1}{16}(16-x^2)(x-1)^2}{\frac{x^2}{16}}$

$\Rightarrow y^2 = \frac{(16-x^2)(x-1)^2}{x^2}$

AS REQUIRED

P.T.O

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7. a) • FIRSTLY THE VOLUME OF THE WATER (REGARDLESS OF SALT)

1 LITRE IN & 1.5 LITRES OUT $\Longrightarrow$ NET OUT OF $\frac{1}{2}$ LITRE

• BY INSPECTION

$$\boxed{V = 25 - 0.5t}$$

IF THE TANK EMPTIES IN 50 MINUTES

• NOW THE SALT

PROPORTION OF SALT IN THE TOTAL VOLUME

PER

$$\frac{dx}{dt} = 0.2 \times 1 - \frac{x}{V} \times 1.5$$

1.5 litres "out"

PROPORTION PER 1 litre "IN"

$$\Longrightarrow \frac{dx}{dt} = \frac{1}{5} - \frac{3x}{2V}$$

$$\Longrightarrow \frac{dx}{dt} = \frac{1}{5} - \frac{3x}{2(25-0.5t)}$$

$$\Longrightarrow \frac{dx}{dt} = \frac{1}{5} - \frac{3x}{50-t}$$

AS REQUIRED

OR FROM FIRST PRINCIPLES

LET x BE THE SALT AT TIME t

THEN AT TIME $t+\delta t$

$$\underbrace{x+\delta x}_{\text{MASS}} = x + 0.2 \times 1 \times \delta t - \frac{x}{V} \times 1.5 \times \delta t$$

MASS + $\frac{\text{MASS}}{\text{VOLUME}} \times \frac{\text{VOLUME}}{\text{TIME}} \times$ TIME $-$ $\frac{\text{MASS}}{\text{VOLUME}} \times \frac{\text{VOLUME}}{\text{TIME}} \times$ TIME

DIMENSIONAL CHECK

$$\Longrightarrow \delta x = 0.2\,\delta t - \frac{3x}{2V}\delta t$$

$$\Longrightarrow \frac{\delta x}{\delta t} = 0.2 - \frac{3x}{2V}$$

TAKE LIMITS

$$\Longrightarrow \frac{dx}{dt} = \frac{1}{5} - \frac{3x}{2V}$$ & FINISH OFF AS BEFORE

b) $x = \frac{1}{10}(50-t) + A(50-t)^3$

$$\frac{dx}{dt} = -\frac{1}{10} - 3A(50-t)^2$$

NOW $A(50-t)^3 = x - \frac{1}{10}(50-t)$

$$A = \frac{x}{(50-t)^3} - \frac{1}{10(50-t)^2}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{1}{10} - 3\underbrace{\left[\frac{x}{(50-t)^3} - \frac{1}{10(50-t)^2}\right]}_{A}(50-t)^2$$

$$\Rightarrow \frac{dx}{dt} = -\frac{1}{10} - \frac{3x}{50-t} + \frac{3}{10}$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{5} - \frac{3x}{50-t}$$ // AS REQUIRED

c) APPLY CONDITION $t=0$ $x=0$

$$0 = \frac{1}{10} \times 50 + A \times 50^3$$

$$0 = 5 + 125000A$$

$$A = -\frac{1}{25000}$$

$$\therefore \frac{dx}{dt} = -\frac{1}{10} + \frac{3}{25000}(50-t)^2$$

SOLVE FOR ZERO

$$\Rightarrow 0 = -\frac{1}{10} + \frac{3}{25000}(50-t)^2$$

$$\Rightarrow \frac{1}{10} = \frac{3}{25000}(50-t)^2$$

$$\Rightarrow (50-t)^2 = \frac{2500}{3}$$

$$\Rightarrow (t-50)^2 = \frac{2500}{3}$$

$$\Rightarrow t-50 = \pm\frac{50}{\sqrt{3}}$$

$$\Rightarrow t = \begin{cases} \cancel{50 + \frac{50}{\sqrt{3}}} \\ 50 - \frac{50}{\sqrt{3}} \end{cases}$$

$\leftarrow$ NOT PHYSICALLY POSSIBLE AS THE TANK EMPTIES IN 50 MINUTES

$$\Rightarrow \boxed{\frac{50}{\sqrt{3}} = 50 - t}$$

HENCE $x = \frac{1}{10}(50-t) - \frac{1}{25000}(50-t)^3$

$$x = \frac{1}{10} \times \frac{50}{\sqrt{3}} - \frac{1}{25000} \times \left(\frac{50}{\sqrt{3}}\right)^3$$

$$x = \frac{5}{\sqrt{3}} - \frac{125000}{25000 \times 3\sqrt{3}}$$

$$x = \frac{5}{\sqrt{3}} - \frac{5}{3\sqrt{3}}$$

$$x = \frac{5}{\sqrt{3}}\left(1 - \frac{1}{3}\right)$$

$$x = \frac{5}{\sqrt{3}} \times \frac{2}{3}$$

$$x = \frac{10}{3\sqrt{3}} \quad \text{OR} \quad \frac{10}{9}\sqrt{3}$$

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8. $$1+\frac{1}{24}+\frac{1\times4}{24\times48}+\frac{1\times4\times7}{24\times48\times72}+\frac{1\times4\times7\times10}{24\times48\times72\times96}+\dots$$

- CREATE FACTORIALS AT THE BOTTOM FIRST

$$=1+\frac{1}{24(1)}+\frac{1\times4}{24^2(1\times2)}+\frac{1\times4\times7}{24^3(1\times2\times3)}+\frac{1\times4\times7\times10}{24^4(1\times2\times3\times4)}+\dots$$

- BY INSPECTION THE NUMERATOR "JUMP IN 3s", SO IN A BINOMIAL IT LOOKS LIKE $-\frac{1}{3}, -\frac{4}{3}, -\frac{7}{3}, -\frac{10}{3}$ ETC
  SO GET "3" TO THE TOP

$$=1+\frac{\frac{1}{3}}{8(1!)}+\frac{\frac{1}{3}\left(\frac{4}{3}\right)}{8^2(2!)}+\frac{\frac{1}{3}\left(\frac{4}{3}\right)\left(\frac{7}{3}\right)}{8^3(3!)}+\frac{\frac{1}{3}\left(\frac{4}{3}\right)\left(\frac{7}{3}\right)\left(\frac{10}{3}\right)}{8^4(4!)}+\dots$$

- SORT OUT THE MINUS SIGNS & REWRITE AS

$$=1+\frac{\left(-\frac{1}{3}\right)}{1!}\left(-\frac{1}{8}\right)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(-\frac{1}{8}\right)^2+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}\left(-\frac{1}{8}\right)^3+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)\left(-\frac{10}{3}\right)}{4!}\left(-\frac{1}{8}\right)^4+\dots$$

- WHICH IS THE BINOMIAL EXPANSION OF $(1+x)^{-\frac{1}{3}}$ WITH $x=-\frac{1}{8}$

$$=\left(1-\frac{1}{8}\right)^{-\frac{1}{3}}=\left(\frac{7}{8}\right)^{-\frac{1}{3}}=\left(\frac{8}{7}\right)^{\frac{1}{3}}=\sqrt[3]{\frac{8}{7}}=\frac{2}{\sqrt[3]{7}}$$

AS REQUIRED