IYGB GCE

Core Mathematics C4

Advanced

Practice Paper Q

Difficulty Rating: 3.6067/1.6713

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$f(x)=4x\mathrm{e}^{2x}.$$

a) Use integration by parts to find $\int f(x)\,dx$. **(3)**

b) Find an exact vale for $\int_0^{\ln 2} f(x)\,dx$. **(2)**

Question 2

$$f(x)=(1+ax)^n,\ a\in\mathbb{R},\ n\in\mathbb{R}.$$

It is given that the series expansion of $f(x)$ is

$$1+2x+\frac{1}{2}x^2+bx^3+O\left(x^4\right).$$

a) Show that $a=\frac{3}{2}$ and find the value of n. **(6)**

b) Find the value of b. **(1)**

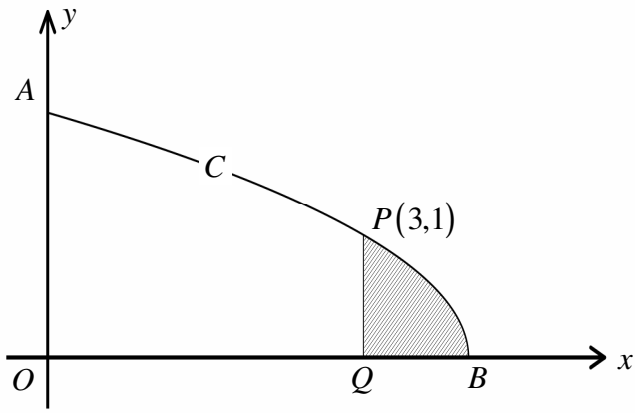
c) State the range of values of x for which the above expansion is valid. **(1)**

Question 3

The side length, x cm, of a cube is increasing at the constant rate of $1.5\ \text{cm}\,\text{s}^{-1}$.

Find the rate at which the volume of the cube is increasing when its side is 6 cm.

Question 4



The figure above shows the curve C, with parametric equations

$$x = 4\sin^2 t \ , \ y = 2\cos t \ , \ 0 \le t \le \frac{\pi}{2}.$$

The curve meets the coordinate axes at the points A and B.

The point $P(3,1)$ lies on C.

The point Q lies on the x axis so that PQ is parallel to the y axis.

a) Show that the area of the shaded region bounded by C, the line PQ and the x axis is given by the integral

$$16\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 t \sin t \ dt . \qquad (\mathbf{6})$$

b) Evaluate the above integral to find the area of the shaded region. $(\mathbf{3})$

Question 5

A curve C is defined implicitly by

$$2y^2 - xy + 4x + x^2 = 7 , \quad x, y \in \mathbb{R} .$$

a) Find an expression for $\frac{dy}{dx}$, in terms of x and y. **(5)**

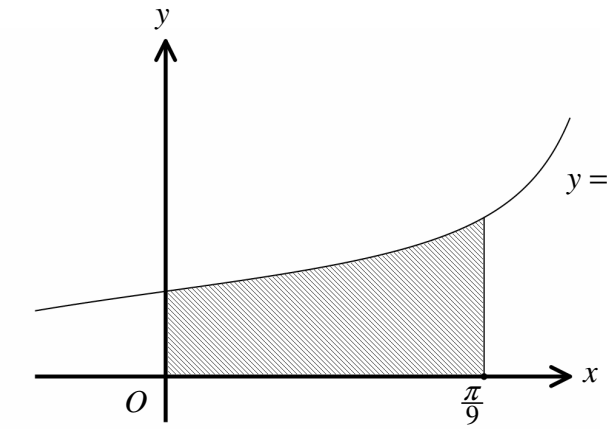
b) Show that $(-1, 2)$ is one of the stationary points of C and determine the exact coordinates of the other stationary point. **(6)**

Question 6

Show that

a) $(2+\tan 3x)^2 = 3+4\tan 3x+\sec^2 3x$ **(2)**

b) $\int \tan x \, dx = \ln|\sec x| + C$ **(4)**



The figure above shows part of the graph of the curve with equation

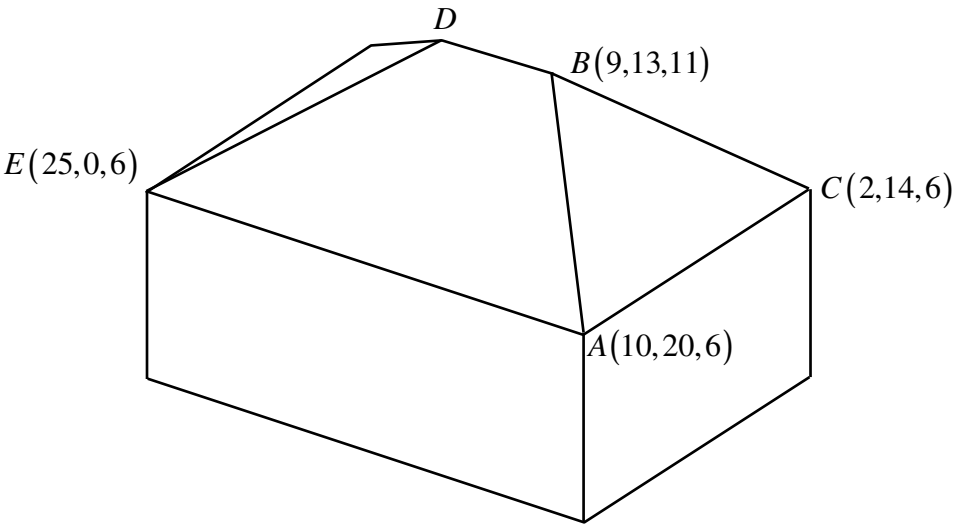
$$y = 2+\tan 3x \, .$$

The shaded region bounded by the curve the coordinate axes and the line $x = \frac{\pi}{9}$ is rotated by 2π radians about the x axis to form a solid of revolution.

c) Show that the volume of the solid is

$$\frac{\pi}{3}\left(\pi + 4\ln 2 + \sqrt{3}\right). \qquad \textbf{(5)}$$

Question 7



The figure above shows a solid, modelling a house with a standard slanted roof, where all the distances are measured in metres. With respect to a fixed origin, the coordinates of some of the vertices of the solid are marked in the diagram.

a) Find a vector equation of AE. **(3)**

b) Show that AE is perpendicular to AC. **(2)**

c) Find the cosine of the angle ABC. **(4)**

The straight line BD is parallel to AE. The length of BD is 10 metres.

d) Determine the coordinates of D. **(4)**

Question 8

During a chemical reaction a compound is formed, whose mass m grams in time t minutes satisfies the differential equation

$$\frac{dm}{dt} = k(m-6)(m-3),$$

where k is a positive constant.

a) Solve the differential equation to show that

$$\frac{m-6}{m-3} = A\mathrm{e}^{3kt},$$

where A is a non zero constant. **(7)**

When the chemical reaction started there was no compound present, and when $t = \ln 16$ the mass of the compound has risen to 2 grams.

b) Show further that

$$m = \frac{6-6\mathrm{e}^{-\frac{1}{4}t}}{2-\mathrm{e}^{-\frac{1}{4}t}}.$$ **(6)**

c) Show that in practice, 3 grams of the compound can never be produced. **(1)**
