# C4, NGB, PAPER O

**1. a)**

$$\frac{30}{(x+3)(9-2x)} \equiv \frac{A}{x+3} + \frac{B}{9-2x}$$

$$\boxed{30 \equiv A(9-2x) + B(x+3)}$$

If $x=-3 \Rightarrow 30 = 15A \Rightarrow A = 2$

If $x = \frac{9}{2} \Rightarrow 30 = B\left(\frac{15}{2}\right) \Rightarrow B = 4$

**b)**

$$\int_1^4 \frac{30}{(x+3)(9-2x)}\,dx = \int_1^4 \frac{2}{x+3} + \frac{4}{9-2x}\,dx = \Big[2\ln|x+3| - 2\ln|9-2x|\Big]_1^4$$

$$= (2\ln 7 - \cancel{2\ln 1}) - (2\ln 4 - 2\ln 7) = \ln 49 - \ln 16 + \ln 49$$

$$= \ln\frac{2401}{16}$$

OR

$$\boxed{\begin{aligned} 2\ln 7 + 2\ln 4 + 2\ln 7 &= 4\ln 7 + 2\ln 4 \\ &= 4\ln 7 + 4\ln 2 \\ &= 4\ln\frac{7}{2} \end{aligned}}$$

**2. a)**

$$f(x) = \frac{20}{\sqrt{4+2x}} = 20(4+2x)^{-\frac{1}{2}} = 20\times 4^{-\frac{1}{2}}\left(1+\tfrac{1}{2}x\right)^{-\frac{1}{2}}$$

$$f(x) = 10\left(1+\tfrac{1}{2}x\right)^{-\frac{1}{2}}$$

$$f(x) = 10\left[1 + \frac{-\frac{1}{2}}{1}\left(\tfrac{1}{2}x\right) + \frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{1\times 2}\left(\tfrac{1}{2}x\right)^2 + \frac{-\frac{1}{2}\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\times 2\times 3}\left(\tfrac{1}{2}x\right)^3 + O(x^4)\right]$$

$$f(x) = 10\left[1 - \tfrac{1}{4}x + \tfrac{3}{32}x^2 - \tfrac{5}{128}x^3 + O(x^4)\right]$$

$$f(x) = 10 - \tfrac{5}{2}x + \tfrac{15}{16}x^2 - \tfrac{25}{64}x^3 + O(x^4)$$

**b)** LET $x = \frac{1}{12}$:

$$\frac{20}{\sqrt{4+2\times\frac{1}{12}}} = 10 - \frac{5}{2}\times\frac{1}{12} + \frac{15}{16}\left(\frac{1}{12}\right)^2 - \frac{25}{64}\left(\frac{1}{12}\right)^3 + \dots$$

$$\frac{20}{\sqrt{\frac{25}{6}}} = 10 - \frac{5}{24} + \frac{5}{768} - \frac{25}{110592} + \dots$$

$$4\sqrt{6} = 9.7979510\dots$$

$$\sqrt{6} = 2.44948\dots \qquad \therefore \sqrt{6} \simeq 2.45$$

3. a) I) $x^2 + 4xy + 2y^2 = 7$

Diff with respect to $x$

$$\Rightarrow 2x + 4y + 4x\frac{dy}{dx} + 4y\frac{dy}{dx} = 0$$

$$\Rightarrow (4x + 4y)\frac{dy}{dx} = -2x - 4y$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2x+4y}{4x+4y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x+2y}{2x+2y}$$

AS REQUIRED

II) $\left.\frac{dy}{dx}\right|_{(1,1)} = -\frac{1+2}{2+2} = -\frac{3}{4} \Rightarrow$

$$y - y_0 = m(x - x_0)$$
$$y - 1 = -\frac{3}{4}(x-1)$$
$$4y - 4 = -3x + 3$$
$$4y + 3x = 7$$

AS REQUIRED

b) PARALLEL $\Rightarrow$ SAME GRADIENT I.E $\frac{dy}{dx} = -\frac{3}{4}$

$$-\frac{x+2y}{2x+2y} = -\frac{3}{4}$$

$$\Rightarrow \frac{x+2y}{2x+2y} = \frac{3}{4}$$

$$\Rightarrow 4x + 8y = 6x + 6y$$

$$\Rightarrow 2y = 2x$$

$$\Rightarrow \boxed{y = x}$$

SOLVING SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE

$x^2 + 4xy + 2y^2 = 7$ & $y = x$

$$x^2 + 4x^2 + 2x^2 = 7$$
$$7x^2 = 7$$
$$x^2 = 1$$

POINT P

$x = 1$ or $-1$, $y = 1$ or $-1$

$\therefore Q(-1,-1)$

4. a) $\boxed{\frac{dV}{dt} = 30 \text{ (CONSTANT)}}$

$$\boxed{\begin{aligned} V &= 36h^2 \\ \frac{dV}{dh} &= 72h \\ \frac{dh}{dV} &= \frac{1}{72h} \end{aligned}}$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{1}{72h} \times 30$$

$$\frac{dh}{dt} = \frac{5}{12h}$$

$$\left.\frac{dh}{dt}\right|_{h=3} = \frac{5}{12 \times 3} = \frac{5}{36} \simeq 0.139 \text{ cm s}^{-1}$$

b)

- 12.5 MINUTES = 750 SECONDS
- "CONSTANT RATE OF 30 $cm^3$ EVERY SECOND"

$\Longrightarrow$ VOLUME $= 750 \times 30 = 22500 \text{ cm}^3$

- $V = 36h^2$

$$22500 = 36h^2$$

$$625 = h^2$$

$$h = 25$$

HENCE $\left.\frac{dh}{dt}\right|_{t = 12.5 \text{ minutes}} = \left.\frac{dh}{dt}\right|_{h=25} = \frac{5}{12 \times 25} = \frac{1}{60} \simeq 0.0167 \text{ cm s}^{-1}$

5. a) $\underline{a} = (3,0,3)$
$\underline{b} = (4,-1,5)$

$\overrightarrow{AB} = \underline{b} - \underline{a} = (4,-1,5) - (3,0,3) = (1,-1,2)$

$\therefore \underline{r} = (3,0,3) + \lambda(1,-1,2)$

$(x,y,z) = (\lambda+3, -\lambda, 2\lambda+3)$

b) DOTTING THEIR DIRECTION VECTORS.

$(1,-1,2)\cdot(1,3,1) = 1 - 3 + 2 = 0$

INDEED PERPENDICULAR

c) $\underline{r}_1 = (\lambda+3, -\lambda, 2\lambda+3)$
$\underline{r}_2 = (\mu+5, 3\mu+10, \mu+4)$

- EQUATE $\underline{i}$ & $\underline{j}$

$\underline{i}: \quad \lambda+3 = \mu+5$
$\underline{j}: \quad -\lambda = 3\mu+10$
ADD

$3 = 4\mu + 15$
$-12 = 4\mu$
$\boxed{\mu = -3}$

$-\lambda = 3\mu+10$
$-\lambda = -9+10$
$-\lambda = 1$
$\boxed{\lambda = -1}$

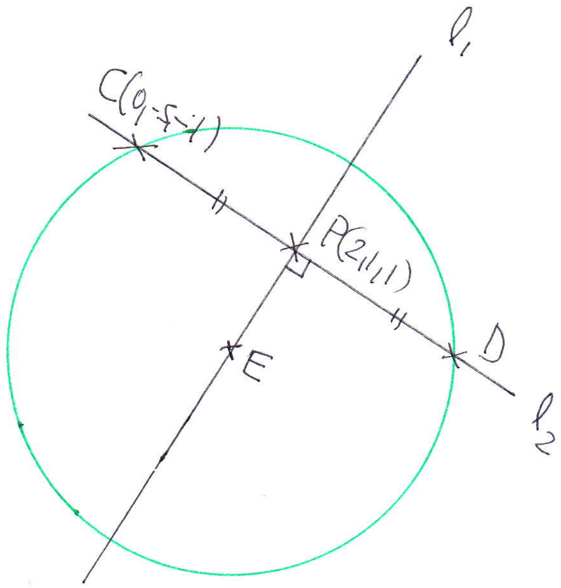
- CHECK $\underline{k}$

$2\lambda+3 = 2(-1)+3 = -2+3 = 1$
$\mu+4 = -3+4 = 1$

AS ALL 3 COMPONENTS AGREE FOR $\lambda=-1$, $\mu=-3$ THE LINES INTERSECT

- USING $\lambda=-1$ GIVES $P(2,1,1)$

d)




BY INSPECTION $P(2,1,1)$ MUST BE THE MIDPOINT OF CD

$0 \underbrace{\quad}_{+2} 2 \underbrace{\quad}_{+2} 4$

$-5 \underbrace{\quad}_{+6} 1 \underbrace{\quad}_{+6} 7$

$-1 \underbrace{\quad}_{+2} 1 \underbrace{\quad}_{+2} 3$

$\therefore D(4,7,3)$

6. a)

$$\frac{dx}{dt} = 2x\sin 2t$$

$$\Rightarrow \frac{1}{x}\,dx = 2\sin 2t\,dt$$

PUT CONDITION INTO THE LIMITS (OR WORK OUT THE "C" AT THE END)

$$\Rightarrow \int_{x=6}^{x} \frac{1}{x}\,dx = \int_{t=0}^{t} 2\sin 2t\,dt$$

$$\Rightarrow \Big[\ln|x|\Big]_6^x = \Big[-\cos 2t\Big]_0^t$$

$$\Rightarrow \ln|x| - \ln 6 = -\cos 2t - (-\cos 0)$$

$$\Rightarrow \ln\left|\frac{x}{6}\right| = 1 - \cos 2t$$

$$\Rightarrow \frac{x}{6} = e^{1-\cos 2t}$$

$$\Rightarrow x = 6e^{1-\cos 2t}$$

AS REQUIRED

b) $x_{MAX}$ WILL OCCUR WHEN THE EXPONENT IS LARGEST

IE $\cos 2t = -1$

$x_{MAX} = 6e^{1-(-1)}$

$x_{MAX} = 6e^2$

$x_{MAX} = 44.3$ cm

C4, IYGB, PAPER O

7. a) $y=0 \implies 0 = 3\sin 2t$

$\sin 2t = 0$

$\begin{cases} 2t = 0 \pm 2n\pi \\ 2t = \pi \pm 2n\pi \end{cases}$ $n = 0,1,2,$

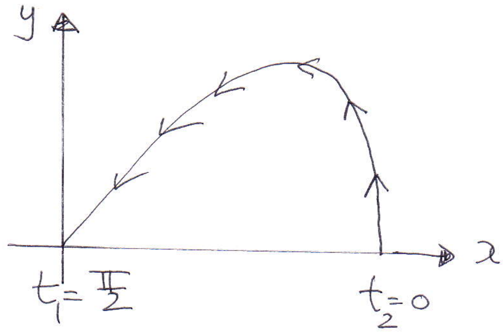
$\begin{cases} t = 0 \pm n\pi \\ t = \frac{\pi}{2} \pm n\pi \end{cases}$

CROSS CHECK WITH $x$ EQUATION

AT O $t = \frac{\pi}{2}$

AT P $t = 0$

b)




$x = 5\cos t$
$y = 3\sin 2t$
$0 \le t \le \frac{\pi}{2}$

$\implies V = \pi \int_{x_1}^{x_2} [y(x)]^2 \, dx = \pi \int_{t_1}^{t_2} [y(t)]^2 \frac{dx}{dt} \, dt$

$\implies V = \pi \int_{\frac{\pi}{2}}^{0} (3\sin 2t)^2 (-5\sin t) \, dt$

$\implies V = \pi \int_{\frac{\pi}{2}}^{0} -45\sin^2 2t \sin t \, dt$

$\implies V = \pi \int_{0}^{\frac{\pi}{2}} 45 (2\sin t\cos t)^2 \sin t \, dt$

$\implies V = \pi \int_{0}^{\frac{\pi}{2}} 45 (4\sin^2 t\cos^2 t) \sin t \, dt$

$\implies V = \pi \int_{0}^{\frac{\pi}{2}} 180\sin^3 t\cos^2 t \, dt$ AS REQUIRED

c)
$$V = \pi \int_0^{\frac{\pi}{2}} 180 \sin^3 t \cos^2 t \, dt$$

$$\Rightarrow V = \pi \int_1^0 180 \sin^3 t \times u^2 \left(\frac{du}{-\sin t}\right)$$

$$\Rightarrow V = \pi \int_1^0 -180 u^2 \sin^2 t \, du$$

$$\Rightarrow V = \pi \int_0^1 180 u^2 \sin^2 t \, du$$

$$\Rightarrow V = \pi \int_0^1 180 u^2 (1 - \cos^2 t) \, du$$

$$\Rightarrow V = \pi \int_0^1 180 u^2 (1 - u^2) \, du$$

$$\Rightarrow V = 180\pi \int_0^1 u^2 - u^4 \, du$$

$$\Rightarrow V = 180\pi \left[ \tfrac{1}{3}u^3 - \tfrac{1}{5}u^5 \right]_0^1$$

$$\Rightarrow V = 180\pi \left[ \left(\tfrac{1}{3} - \tfrac{1}{5}\right) - 0 \right]$$

$$\Rightarrow V = 180\pi \times \frac{2}{15}$$

$$\Rightarrow V = 24\pi$$

$u = \cos t$

$\frac{du}{dt} = -\sin t$

$dt = \frac{du}{-\sin t}$

$t = 0 \quad u = 1$

$t = \frac{\pi}{2} \quad u = 0$