IYGB GCE

Core Mathematics C4

Advanced

Practice Paper N

Difficulty Rating: 3.5067/1.6043

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

By using the substitution $u^2 = 1 - x^2$, or otherwise, show that

$$\int_0^1 5x\left(1-x^2\right)^{\frac{3}{2}}\,dx = 1.$$ **(6)**

Question 2

$$f(x) = 2\sqrt{1+4x} + \frac{4}{1+x}.$$

a) By combining the first 4 terms in the expansions of $(1+x)^{-1}$ and $(1+4x)^{\frac{1}{2}}$ show that

$$f(x) \approx 6 + 4x^3.$$ **(8)**

b) State range of values of x for which the expansion of $f(x)$ is valid. **(1)**

Question 3

A curve C is given parametrically by the equations

$$x = 2t^2 + \frac{1}{t}, \quad y = 2t^2 - \frac{1}{t}, \quad t \in \mathbb{R}, \; t \neq 0.$$

a) Show that at the point on C where $t = \frac{1}{2}$, the gradient is -3. **(5)**

b) By considering $(x+y)$ and $(x-y)$, show that a Cartesian equation of C is

$$(x+y)(x-y)^2 = 16.$$ **(3)**

Question 4

The variables y, x and t are related by the equations

$$y = 10e^{\frac{1}{5}x - 1} \quad \text{and} \quad x = \sqrt{6t+1}, \ \ t \ge 0.$$

Find the value of $\frac{dy}{dt}$, when $t = 4$. **(7)**

Question 5

A curve C has implicit equation

$$y = \frac{2x+1}{xy+3}.$$

a) Find an expression for $\frac{dy}{dx}$, in terms of x and y. **(5)**

b) Show that there is **no** point on C, where the tangent is parallel to the y axis. **(7)**

Question 6

With respect to a fixed origin O, the points A and B have coordinates $(5,-1,-1)$ and $(1,-5,7)$, respectively.

a) Find a vector equation of the straight line l which passes through A and B. **(3)**

The point C has coordinates $(4,-2,1)$.

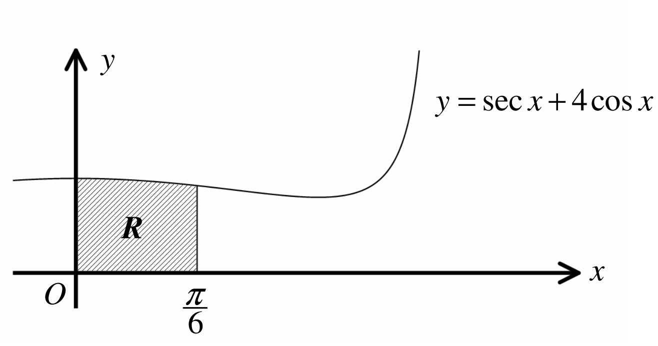
b) Show that C lies on l. **(1)**

c) Show further that $\overrightarrow{OC}$ is perpendicular to l. **(2)**

The point D lie on l so that $\left|\overrightarrow{CD}\right| = 2\left|\overrightarrow{CA}\right|$.

d) Find the **two** possible sets for the coordinates of D. **(4)**

Question 7



The figure above shows part of the curve with equation

$$y = \sec x + 4\cos x .$$

The shaded region, labelled R, bounded by the curve, the coordinate axes and the straight line with equation $x = \frac{\pi}{6}$ is rotated by 2π radians in the x axis to form a solid of revolution.

Show that the solid has a volume of

$$\frac{\pi}{3}\left(8\pi + 7\sqrt{3}\right).$$ **(9)**

Question 8

The population of a herd of zebra, P thousands, in time t years is thought to be governed by the differential equation

$$\frac{dP}{dt} = \frac{1}{20}P(2P-1)\cos t .$$

It is assumed that since P is large it can be modelled as a continuous variable, and its initial value is 8.

a) Solve the differential equation to show that

$$P = \frac{8}{16 - 15e^{\frac{1}{20}\sin t}} .$$ **(11)**

b) Find the maximum and minimum population of the herd. **(3)**
