IYGB GCE

Core Mathematics C4

Advanced

Practice Paper G

Difficulty Rating: 3.2867/1.4742

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$f(x) \equiv \frac{x-5}{x^2+5x+4}.$$

a) Express $f(x)$ in partial fractions. **(4)**

b) Find the value of

$$\int_0^2 f(x)\ dx\ ,$$

giving the answer as a single simplified logarithm. **(3)**

Question 2 (+)**

$$f(x) = \frac{2x}{(1+2x)^3},\ x \neq -\frac{1}{2}.$$

a) Find the first 4 terms in the series expansion of $f(x)$. **(5)**

b) State the range of values of x for which the expansion of $f(x)$ is valid. **(1)**

Question 3

$$\frac{dy}{dx}\cos^2 4x = y,\quad y > 0.$$

Show that the solution of the above differential equation subject to the boundary condition $y = e^3$ at $x = \frac{\pi}{16}$ is given by

$$y = e^{\frac{1}{4}(11+\tan 4x)}.$$ **(8)**

Question 4

By using the substitution $u^2 = 16 - 7x^2$, or otherwise, show that

$$\int_0^1 \frac{x}{\sqrt{16-7x^2}}\,dx = \frac{1}{7}. \qquad (6)$$

Question 5

Relative to a fixed origin O, the point A has position vector $7\mathbf{i}+4\mathbf{j}$ and the point B has position vector $-3\mathbf{j}+7\mathbf{k}$. The straight line L_1 passes through the points A and B.

a) Find a vector equation for L_1. **(3)**

The straight line L_2 has a vector equation

$$\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} - 4\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}),$$

where μ is a scalar parameter.

b) Show that L_1 and L_2 intersect at a point C, and find its position vector. **(6)**

c) Show further that L_1 and L_2 are perpendicular. **(2)**

The point D has position vector $4\mathbf{i}-\mathbf{k}$.

d) Verify that D lies on L_2. **(2)**

The point E is the image of D after reflection about L_1.

e) Find the position vector of E. **(2)**

Question 6

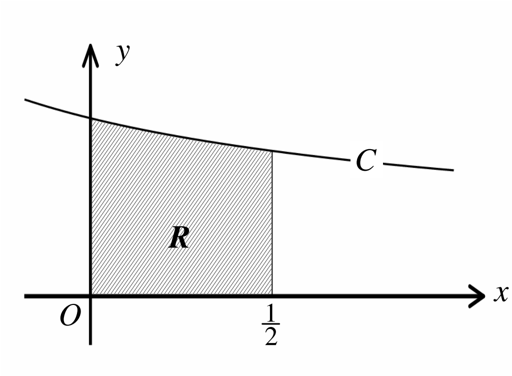
A curve C has equation

$$y = 2^{\sin 2x}, \ x \in \mathbb{R}.$$

a) By taking logarithms on both sides of this equation, or otherwise, find an expression for $\frac{dy}{dx}$ in terms of x. **(6)**

b) Find an equation of the tangent to the curve at the point where $x = \frac{\pi}{4}$. **(3)**

Question 7



The figure above shows part of the curve C, with parametric equations

$$x = \cos 2\theta, \quad y = \sec\theta \ , \ 0 < \theta < \frac{\pi}{2}.$$

The finite region R is bounded by C, the straight line with equation $x = \frac{1}{2}$ and the coordinate axes.

a) Show that the area of R is given by the integral

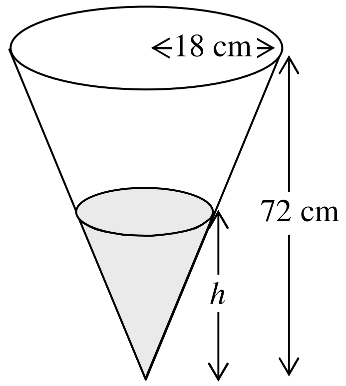
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 4\sin\theta \ d\theta. \qquad (5)$$

b) Evaluate the above integral to find an exact value for R. **(2)**

The region R is rotated by 2π radians in the x axis to form a solid of revolution S.

c) Use parametric integration to find an exact value for the volume of S. **(5)**

Question 8



Flowers at a florists' are stored in vases which are in the shape of hollow inverted right circular cones with height 72 cm and radius 18 cm.

One such vase is initially empty and placed under a tap where the water is flowing into the vase at the constant rate of $6\pi \text{ cm}^3\text{s}^{-1}$.

a) Show that the volume, V cm^3, of the water in the vase is given by

$$V = \frac{1}{48}\pi h^3, \qquad \textbf{(3)}$$

where, h cm, is the height of the water in the vase.

b) Find the rate at which h is rising when $h = 4$ cm. **(4)**

c) Determine the rate at which h is rising 12.5 **minutes** after the vase was placed under the tap. **(5)**

$\left[\text{volume of a cone of radius } r \text{ and height } h \text{ is given by } \frac{1}{3}\pi r^2 h\right]$