C4, IYGB, PAPER F,

1.
$$\boxed{V = \pi \int_{x_1}^{x_2} [y(x)]^2 \, dx}$$

$$V = \pi \int_0^4 \left[\frac{3}{\sqrt{6x+1}}\right]^2 dx = \pi \int_0^4 \frac{9}{6x+1} \, dx = \pi\left[\frac{9}{6}\ln|6x+1|\right]_0^4$$

$$= \frac{3\pi}{2}\left[\ln|6x+1|\right]_0^4 = \frac{3\pi}{2}\left(\ln 25 - \cancel{\ln 1}\right) = \frac{3\pi}{2}\ln 25$$

$$= \frac{3\pi}{2} \times 2\ln 5 = 3\pi \ln 5$$

AS REQUIRED

2. a)
$$yx(2x-y) + 1 = 0$$
$$2x^2y - xy^2 + 1 = 0$$
$$\frac{d}{dx}(2x^2y) - \frac{d}{dx}(xy^2) + \frac{d}{dx}(1) = \frac{d}{dx}(0)$$
$$4xy + 2x^2\frac{dy}{dx} - \left(y^2 + x \times 2y\frac{dy}{dx}\right) = 0$$
$$4xy + 2x^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$$
$$(2x^2 - 2xy)\frac{dy}{dx} = y^2 - 4xy$$
$$\frac{dy}{dx} = \frac{y^2 - 4xy}{2x^2 - 2xy}$$

AS REQUIRED

b)
$$y = 2$$
$$2x^2y - xy^2 + 1 = 0$$
$$4x^2 - 4x + 1 = 0$$
$$(2x-1)^2 = 0$$
$$x = \frac{1}{2}$$
$$\therefore k = \frac{1}{2}$$

c) $P(\tfrac{1}{2}, 2)$

$$\left.\frac{dy}{dx}\right|_{(\frac{1}{2},2)} = \frac{2^2 - 4 \times \frac{1}{2} \times 2}{2 \times (\frac{1}{2})^2 - 2 \times \frac{1}{2} \times 2}$$
$$= \frac{4 - 4}{\frac{1}{2} - 2}$$
$$= 0$$

INDEED STATIONARY

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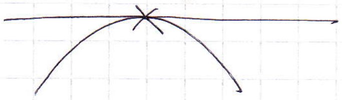
d) $P(\frac{1}{2}, 2)$



$y = 2$

3. a) $$\frac{18-19x}{(1-x)(2-3x)} \equiv \frac{A}{1-x} + \frac{B}{2-3x}$$

$$A(2-3x) + B(1-x) \equiv 18-19x$$

- IF $x=1 \Rightarrow -A=-1 \Rightarrow A=1$
- IF $x=0 \Rightarrow 2A+B=18 \Rightarrow B=16$

$$\therefore f(x) = \frac{1}{1-x} + \frac{16}{2-3x}$$

b) $$\frac{1}{1-x} = (1-x)^{-1} = 1 + \frac{-1}{1}(-x) + \frac{-1(-2)}{1\times 2}(-x)^2 + \frac{(-1)(-2)(-3)}{1\times 2\times 3}(-x)^3 + O(x^4)$$

$$= 1 + x + x^2 + x^3 + O(x^4)$$

$$\frac{16}{2-3x} = 16(2-3x)^{-1} = 16\times 2^{-1}\left(1-\frac{3}{2}x\right)^{-1} = 8\left(1-\frac{3}{2}x\right)^{-1}$$

EITHER EXPAND AGAIN OR USE ABOVE LINE

$$= 8\left[1 + \left(\frac{3}{2}x\right) + \left(\frac{3}{2}x\right)^2 + \left(\frac{3}{2}x\right)^3 + O(x^4)\right]$$

$$= 8\left[1 + \frac{3}{2}x + \frac{9}{4}x^2 + \frac{27}{8}x^3 + O(x^4)\right]$$

$$= 8 + 12x + 18x^2 + 27x^3 + O(x^4)$$

$$\therefore f(x) = \begin{array}{l} 1 + x + x^2 + x^3 + O(x^4) \\ 8 + 12x + 18x^2 + 27x^3 + O(x^4) \\ \hline \end{array}$$

$$f(x) \approx 9 + 13x + 19x^2 + 28x^3$$

AS REQUIRED

4. a) $x = \frac{t}{1+t^2}$ $\quad \frac{dx}{dt} = \frac{(1+t^2)\times 1 - t(2t)}{(1+t^2)^2} = \frac{t^2+1-2t^2}{(1+t^2)^2} = \frac{1-t^2}{(1+t^2)^2}$

$y = \frac{2t^2}{1+t^2}$ $\quad \frac{dy}{dt} = \frac{(1+t^2)(4t) - 2t^2(2t)}{(1+t^2)^2} = \frac{4t+4t^3-4t^3}{(1+t^2)^2} = \frac{4t}{(1+t^2)^2}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{4t}{(1+t^2)^2}}{\frac{1-t^2}{(1+t^2)^2}} = \frac{4t}{1-t^2}$$

b) $y = 6x-2$ $\quad x = \frac{t}{1+t^2}$ $\quad y = \frac{2t^2}{1+t^2}$

SOLVING SIMULTANEOUSLY

$\Rightarrow \frac{2t^2}{1+t^2} = 6\left(\frac{t}{1+t^2}\right) - 2$

$\Rightarrow \frac{2t^2}{1+t^2} = \frac{6t}{1+t^2} - 2$

MULTIPLY THROUGH BY $(1+t^2)$

$\Rightarrow 2t^2 = 6t - 2(1+t^2)$

$\Rightarrow 2t^2 = 6t - 2 - 2t^2$

$\Rightarrow 4t^2 - 6t + 2 = 0$

$\Rightarrow 2t^2 - 3t + 1 = 0$

$\Rightarrow (2t-1)(t-1) = 0$

$t = \begin{cases} 1 \\ \frac{1}{2} \end{cases} \quad x = \begin{cases} \frac{1}{2} \\ \frac{2}{5} \end{cases} \quad y = \begin{cases} 1 \\ \frac{2}{5} \end{cases}$

$\therefore \left(\frac{1}{2}, 1\right)$ & $\left(\frac{2}{5}, \frac{2}{5}\right)$

(P.T.O)

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5. $\int x^3 e^{x^2}\,dx = \dots$ substitution

$$= \int x^3 e^u \times \frac{du}{2x} = \int \tfrac{1}{2}x^2 e^u \, du$$

$$= \int \tfrac{1}{2} u e^u \, du = \dots \text{by parts}$$

$$= \tfrac{1}{2}ue^u - \int \tfrac{1}{2}e^u \, du$$

$$= \tfrac{1}{2}ue^u - \tfrac{1}{2}e^u + C$$

$$= \tfrac{1}{2}x^2 e^{x^2} - \tfrac{1}{2}e^{x^2} + C$$

$$\left[\text{OR } \tfrac{1}{2}e^{x^2}(x^2-1) + C \right]$$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\frac{1}{2}u$	$\frac{1}{2}$
e^u	e^u

6. a) $\vec{AB} = \underline{b} - \underline{a} = (-1,1,9) - (3,-1,2) = (-4,2,7)$

$\vec{OA} \cdot \vec{AB} = (3,-1,2)\cdot(-4,2,7) = -12-2+14 = 0$

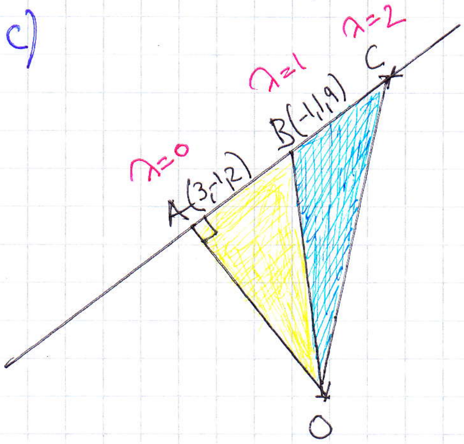
INDEED PERPENDICULAR

b) $\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (3,-1,2) + \lambda(-4,2,7)$

$\underline{r} = (3-4\lambda,\ 2\lambda-1,\ 7\lambda+2)$

c)



- THE TWO TRIANGLES HAS THE SAME HEIGHT, $|OA|$
- IF THEY ARE TO HAVE THE SAME AREA, THEIR BASES MUST BE EQUAL

$|AB| = |BC|$, IE B IS THE MIDPOINT OF A & C

$\therefore$ BY INSPECTION $C(-5,3,16)$

7. a) $V = \sqrt{3x^2+2x^3} = (3x^2+2x^3)^{\frac{1}{2}}$

$\Rightarrow \frac{dV}{dx} = \frac{1}{2}(3x^2+2x^3)^{-\frac{1}{2}}(6x+6x^2)$

$\Rightarrow \left.\frac{dV}{dx}\right|_{x=11} = \frac{1}{2}\left[3\times11^2+2\times11^3\right]^{-\frac{1}{2}} \times \left[6\times11+6\times11^2\right]$

$\Rightarrow \left.\frac{dV}{dx}\right|_{x=11} = \frac{1}{2}\times\frac{1}{55}\times 792 = \frac{36}{5} = 7.2$

b) $\boxed{\frac{dV}{dt} = 14.4}$

$\Rightarrow \frac{dx}{dt} = \frac{dx}{dV}\times\frac{dV}{dt}$

$\Rightarrow \frac{dx}{dt} = \frac{dx}{dV}\times 14.4$

$\Rightarrow \left.\frac{dx}{dt}\right|_{x=11} = \left.\frac{dx}{dV}\right|_{x=11}\times 14.4$

$\Rightarrow \left.\frac{dx}{dt}\right|_{x=11} = \frac{5}{36}\times 14.4$

$\Rightarrow \left.\frac{dx}{dt}\right|_{x=11} = 2$

8. a) $\frac{d\theta}{dt} = k(200-\theta)$

$\Rightarrow d\theta = k(200-\theta)\,dt$

$\Rightarrow \frac{1}{200-\theta}\,d\theta = k\,dt$

$\Rightarrow \int \frac{1}{200-\theta}\,d\theta = \int k\,dt$

$\Rightarrow -\ln|200-\theta| = kt + C$

$\Rightarrow \ln|200-\theta| = -kt + C$

$\Rightarrow 200-\theta = e^{-kt+C}$

$\Rightarrow 200-\theta = Be^{-kt} \quad (B=e^C)$

$\Rightarrow -\theta = -200 + Be^{-kt}$

$\Rightarrow \theta = 200 + Ae^{-kt}$

AS REQUIRED

b) $t=0 \quad \theta=20 \implies 20 = 200 + Ae^{0}$

$20 = 200 + A$

$\boxed{A = -180}$

$\therefore \theta = 200 - 180e^{-kt}$

$t=10 \quad \theta=120 \implies 120 = 200 - 180e^{-10k}$

$\implies 180e^{-10k} = 80$

$\implies e^{-10k} = \frac{4}{9}$

$\implies e^{10k} = \frac{9}{4}$

$\implies 10k = \ln\frac{9}{4}$

$\implies k = \frac{1}{10}\ln(2.25) \simeq 0.0811$ (3 sf)

c) $\theta = 200 - 180e^{-0.811t}$

$\implies 160 = 200 - 180e^{-0.811t}$

$\implies 180e^{-0.811t} = 40$

$\implies e^{-0.811t} = \frac{2}{9}$

$\implies e^{0.811t} = \frac{9}{2}$

$\implies 0.811t = \ln\left(\frac{9}{2}\right)$

$\implies t \simeq 18.547\ldots$

$\implies t \simeq 18.5$ min

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9. $\int_0^{\frac{\pi}{3}} \sec^4 x \, dx = \ldots$

$$\int_0^{\sqrt{3}} \sec^4 x \frac{du}{\sec^2 x} = \int_0^{\sqrt{3}} \sec^2 x \, du$$

$$= \int_0^{\sqrt{3}} 1 + \tan^2 x \, du$$

$$= \int_0^{\sqrt{3}} 1 + u^2 \, du$$

$$= \left[u + \frac{1}{3}u^3 \right]_0^{\sqrt{3}}$$

$$= \left[\sqrt{3} + \frac{1}{3}(\sqrt{3})^3 \right] - \left[0 \right]$$

$$= \sqrt{3} + \sqrt{3}$$

$$= 2\sqrt{3}$$

AS REQUIRED

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$x = 0, \quad u = 0$

$x = \frac{\pi}{3}, \quad u = \sqrt{3}$