IYGB GCE

Core Mathematics C4

Advanced

Practice Paper F

Difficulty Rating: 3.2600/1.4599

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

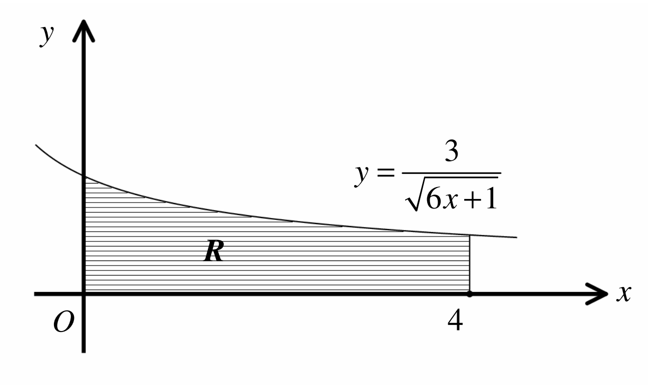
Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



The graph below shows the curve with equation

$$y = \frac{3}{\sqrt{6x+1}}, \ x \neq -\frac{1}{6}.$$

The region R, shown in the figure shaded, is bounded by the curve, the coordinate axes and the straight line with equation $x = 4$.

The shaded region R is rotated by 2π radians about the x axis to form a solid of revolution.

Show that the volume of the solid generated is

$$3\pi \ln 5.$$ **(5)**

Question 2

The curve C has equation

$$yx(2x-y)+1=0.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{y^2-4xy}{2x^2-2xy}. \qquad \textbf{(5)}$$

The point $P(k,2)$ lies on C.

b) Find the value of k. **(3)**

c) Show that P is a stationary point of C. **(2)**

d) Hence, state an equation of the tangent to C at P **(1)**

Question 3

$$f(x)\equiv\frac{18-19x}{(1-x)(2-3x)},\ x\in\mathbb{R},\ |x|<\frac{2}{3}.$$

a) Express $f(x)$ in partial fractions. **(3)**

b) Hence, or otherwise, show that if x is numerically small

$$f(x)\approx 9+13x+19x^2+28x^3. \qquad \textbf{(7)}$$

Question 4

A curve C is defined by the parametric equations

$$x=\frac{t}{1+t^2}, \quad y=\frac{2t^2}{1+t^2}, \quad t\in\mathbb{R}.$$

a) Find a simplified expression for $\frac{dy}{dx}$ in terms of t. **(5)**

The straight line with equation $y=6x-2$ intersects C at the points P and Q.

b) Find the coordinates of P and the coordinates of Q. **(7)**

Question 5

Use the substitution $u=x^2$, followed by integration by parts to find

$$\int x^3 e^{x^2}\,dx.$$ **(6)**

Question 6

Relative to a fixed origin O, the points A and B have respective position vectors

$$3\mathbf{i}-\mathbf{j}+2\mathbf{k} \quad\text{and}\quad -\mathbf{i}+\mathbf{j}+9\mathbf{k}.$$

a) Show that $\overrightarrow{OA}$ and $\overrightarrow{AB}$ are perpendicular. **(3)**

b) Find a vector equation of the straight line l, that passes through A and B. **(2)**

The point C lies on l, so that the areas of the triangles OAB and OBC are equal.

c) Determine the position vector of C. **(2)**

Question 7

The volume of water, V cm^3, in a container is given by the formula

$$V = \sqrt{3x^2 + 2x^3} \ ,$$

where x is the depth of the water in cm.

a) Find the value of $\frac{dV}{dx}$ when $x = 11$. **(4)**

It is further given that the volume of the water in the container is increasing at the constant rate of $14.4 \text{ cm}^3\text{s}^{-1}$

b) Determine the rate at which the depth of the water in the container is increasing when the depth has reached 11 cm. **(2)**

Question 8

Food is placed in a preheated oven maintained at a constant temperature of 200 °C.

Let θ °C be temperature of the food t minutes after it was placed in the oven. It is assumed that θ satisfies the differential equation

$$\frac{d\theta}{dt} = k(200 - \theta) \ ,$$

where k is a positive constant.

a) Solve the differential equation to show that

$$\theta = 200 + A\mathrm{e}^{-kt} \ ,$$

where A is a constant. **(6)**

When a food item was placed in this oven it had a temperature of 20 °C and ten minutes later its temperature had risen to 120 °C.

b) Show further that $k \approx 0.0811$. **(3)**

c) Find the value of t when the food item reaches a temperature of 160 °C. **(3)**

Question 9

By using the substitution $u = \tan x$ and the trigonometric identity $1 + \tan^2 x = \sec^2 x$, show clearly that

$$\int_0^{\frac{\pi}{3}} \sec^4 x \ dx = 2\sqrt{3} . \qquad (\mathbf{6})$$
