## Created by T. Madas

## IYGB GCE

Core Mathematics C4 Advanced Practice Paper A<br>Difficulty Rating: 2.7333/1.2245<br>\section*{Time: 1 hour 30 minutes}<br>Candidates may use any calculator allowed by the<br>Regulations of the Joint Council for Qualifications.

## Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used. Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2). There are 10 questions in this question paper.
The total mark for this paper is 75 .
Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy. The examiner may refuse to mark any parts of questions if deemed not to be legible.

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## Question 1

The binomial expression $(1+x)^{-2}$ is to be expanded as an infinite convergent series, in ascending powers of $x$.
a) Determine the expansion of $(1+x)^{-2}$ up and including the term in $x^{3}$.
b) Use part (a) to find the expansion of $(1+2 x)^{-2}$, up and including the term in $x^{3}$, stating the range of values of $x$ for which this expansion is valid.

## Question 2

A curve is given implicitly by the equation

$$
y^{2}+3 x y+x^{2}=20 .
$$

Find an equation for the tangent to the curve at the point $P(2,2)$.

## Question 3

Find a general solution of the differential equation

$$
3 y^{2} \frac{d y}{d x}+2 x=1
$$

giving the answer in the form $y^{3}=f(x)$.
$\qquad$

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## Question 4

$$
\frac{5 x+13}{(2 x+1)(x+4)} \equiv \frac{A}{2 x+1}+\frac{B}{x+4} .
$$

a) Determine the value of each of the constants $A$ and $B$.
b) Evaluate

$$
\int_{0}^{4} \frac{5 x+13}{(2 x+1)(x+4)} d x
$$

giving the answer as a single simplified natural logarithm.
(6)

## Question 5

The surface area, $S \mathrm{~cm}^{2}$, of a sphere is increasing at the constant rate of $512 \mathrm{~cm}^{2} \mathrm{~s}^{-1}$.

The surface area of a sphere is given by

$$
S=4 \pi r^{2},
$$

where $r \mathrm{~cm}$ is its radius.

Find the rate at which the radius $r$ of the sphere is increasing, when the sphere's radius has reached 8 cm .

## Question 6

By using the substitution $u=1+4 \ln x$, or otherwise, find

$$
\begin{equation*}
\int \frac{4}{x(1+4 \ln x)^{2}} d x \tag{6}
\end{equation*}
$$

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## Question 7



The diagram above shows the graph of the curve with equation

$$
y=\frac{6}{x+3}, x \neq-3 .
$$

The region $R$, shown shaded in the figure above, is bounded by the curve, the coordinate axes and the straight lines with equations $x=-1$ and $x=3$.

The region $R$ is rotated by $360^{\circ}$ about the $x$ axis to form a solid of revolution.

Show that the volume of the solid generated is $12 \pi$.

## Question 8

The points $A(2,10,7)$ and $B(0,15,12)$ are given.
a) Determine the vector equation of the straight line $l_{1}$ that passes through the points $A$ and $B$.

The vector equation of the straight line $l_{2}$ is

$$
\mathbf{r}_{2}=4 \mathbf{i}+\mathbf{j}-6 \mathbf{k}+\mu(2 \mathbf{i}-\mathbf{j}+3 \mathbf{k})
$$

where $\mu$ is a scalar parameter.
b) Show that $l_{1}$ and $l_{2}$ intersect at some point $P$ and find its coordinates.
c) Calculate the acute angle between $l_{1}$ and $l_{2}$.

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## Question 9

| $x$ | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5 \pi}{12}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0 | 0.1309 | 0.4534 | 0.7854 |  | 0.6545 |

The table above shows tabulated values for the equation

$$
y=x \sin 2 x, 0 \leq x \leq \frac{5 \pi}{12}
$$

a) Complete the missing value in the table.
b) Use the trapezium rule with all the values from the table to find an approximate value for

$$
\begin{equation*}
\int_{0}^{\frac{5 \pi}{12}} x \sin 2 x d x \tag{3}
\end{equation*}
$$

c) Use integration by parts to find an exact value for
(6)

$$
\int_{0}^{\frac{5 \pi}{12}} x \sin 2 x d x
$$

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## Question 10



The figure above shows a cycloid $C$, whose parametric equations are

$$
x=\theta-\sin \theta, \quad y=1-\cos \theta, \text { for } 0 \leq \theta \leq 2 \pi .
$$

The finite region $R$ is bounded by $C$ and the $x$ axis.
a) Show, with full justification, that the area of $R$ is given by

$$
\begin{equation*}
\int_{0}^{2 \pi}(1-\cos \theta)^{2} d \theta \tag{3}
\end{equation*}
$$

b) Hence find the area of $R$.

