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1. $\boxed{y = e^{-x}\sin x}$

- $\frac{dy}{dx} = -e^{-x}\sin x + e^{-x}\cos x = e^{-x}(\cos x - \sin x)$

SOLVE FOR ZERO

$\cos x - \sin x = 0 \quad (e^{-x} \neq 0)$

$\sin x = \cos x$

$\tan x = 1$

$\boxed{x = \frac{\pi}{4}}$

$\uparrow$

x_1

- $e^{-x}\sin x = e^{-x}$
  $\sin x = 1$) $e^{-x} \neq 0$ SO WE MAY DIVIDE IT

$\boxed{x = \frac{\pi}{2}} \leftarrow x_2$

$\therefore \quad x_2 - x_1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$ // AS REQUIRED

2. a) $f(x) = 2\cos x + \sin x = R\sin(x+\alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$= (R\cos\alpha)\sin x + (R\sin\alpha)\cos x$

$R\cos\alpha = 1$
$R\sin\alpha = 2$ $\Longrightarrow$ SQUARE & ADD $\Longrightarrow R = \sqrt{2^2+1^2} = \sqrt{5}$

DIVIDE EQNS $\tan\alpha = 2 \Longrightarrow \alpha = 1.107^c$

$f(x) = \sqrt{5}\sin(x + 1.107^c)$ //

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b) $g(f(x)) = g(2\cos x + \sin x) = \dfrac{5}{(2\cos x+\sin x)^2+5}$

$$= \frac{5}{\left[\sqrt{5}\sin(x+1.107^c)\right]^2+5} = \frac{5}{5\sin^2(x+1.107)+5}$$

$$= \frac{1}{\sin^2(x+1.107)+1}$$

$$-1 \leqslant \sin(x+1.107^c) \leqslant 1$$

$$0 \leqslant \sin^2(x+1.107) \leqslant 1$$

$$1 \leqslant 1+\sin^2(x+1.107) \leqslant 2$$

$$\frac{1}{2} \leqslant \frac{1}{1+\sin^2(x+1.107)} \leqslant 1$$

$$\frac{1}{2} \leqslant g(f(x)) \leqslant 1$$

3. a) $M = Ae^{-kt}$

$t=0,\ M=10 \Rightarrow 10 = Ae^0$

$\boxed{A=10}$

$t=5,\ M=5 \Rightarrow 5 = 10e^{-k\times 5}$

$\Rightarrow \frac{1}{2} = e^{-5k}$

$\Rightarrow e^{5k} = 2$

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$\Rightarrow 5k = \ln 2$

$\Rightarrow k = \frac{1}{5}\ln 2$

b) $\boxed{M = 10e^{-\left(\frac{1}{5}\ln 2\right)t}}$

$$\frac{dM}{dt} = 10 \times \left(-\frac{1}{5}\ln 2\right) e^{-\left(\frac{1}{5}\ln 2\right)t}$$

$$\frac{dM}{dt} = -2\ln 2 e^{-\frac{1}{5}t\ln 2}$$

Thus

$\Rightarrow -2\ln 2\, e^{-\frac{1}{5}t\ln 2} = \ln\left(\frac{\sqrt{2}}{2}\right)$

$\Rightarrow -2\ln 2\, e^{-\frac{1}{5}t\ln 2} = \ln\sqrt{2} - \ln 2$

$\Rightarrow -2\ln 2\, e^{-\frac{1}{5}t\ln 2} = \ln 2^{\frac{1}{2}} - \ln 2$

$\Rightarrow -2\ln 2\, e^{-\frac{1}{5}t\ln 2} = \frac{1}{2}\ln 2 - \ln 2$) DIVIDE BY $\ln 2$

$\Rightarrow -2e^{-\frac{1}{5}t\ln 2} = \frac{1}{2} - 1$

$\Rightarrow -2e^{-\frac{1}{5}t\ln 2} = -\frac{1}{2}$

$\Rightarrow e^{-\frac{1}{5}t\ln 2} = \frac{1}{4}$

$\Rightarrow e^{\frac{1}{5}t\ln 2} = 4$

$\Rightarrow \frac{1}{5}t\ln 2 = \ln 4$

$\Rightarrow \frac{1}{5}t\cancel{\ln 2} = 2\cancel{\ln 2}$

$\Rightarrow t = 10$

4. a) $\frac{d}{dx}(\tan 2x) = \frac{d}{dx}\left(\frac{\sin 2x}{\cos 2x}\right)$ = BY QUOTIENT RULE

$$= \frac{(\cos 2x)(2\cos 2x) - (\sin 2x)(-2\sin 2x)}{(\cos 2x)^2}$$

$$= \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x} = \frac{2(\cos^2 2x + \sin^2 2x)}{\cos^2 2x}$$

$$= \frac{2}{\cos^2 2x} = 2\sec^2 2x$$ AS REQUIRED

b) $y = 6x\tan 2x$

- $\frac{dy}{dx} = 6\times\tan 2x + 6x \times 2\sec^2 2x$

$\frac{dy}{dx} = 6\tan 2x + 12x\sec^2 2x$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{8}} = 6\times 1 + 12\times\frac{\pi}{8}\times 2 = \boxed{6+3\pi}$

- WHEN $x = \frac{\pi}{8}$ $\quad y = 6\times\frac{\pi}{8}\times 1 = \frac{3\pi}{4}$ $\quad \therefore \boxed{\left(\frac{\pi}{8}, \frac{3\pi}{4}\right)}$

- $y - y_0 = m(x - x_0)$

$y - \frac{3\pi}{4} = (6+3\pi)\left(x - \frac{\pi}{8}\right)$

when $x = 0$

$y - \frac{3\pi}{4} = (6+3\pi)\left(-\frac{\pi}{8}\right)$

$y = \cancel{\frac{3\pi}{4}} - \cancel{\frac{3\pi}{4}} - \frac{3}{8}\pi^2$ $\quad \therefore \left(0, -\frac{3}{8}\pi^2\right)$

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5. a) $f(x) = \sqrt{1-(2x-1)^2} \quad 0 \le x \le a$

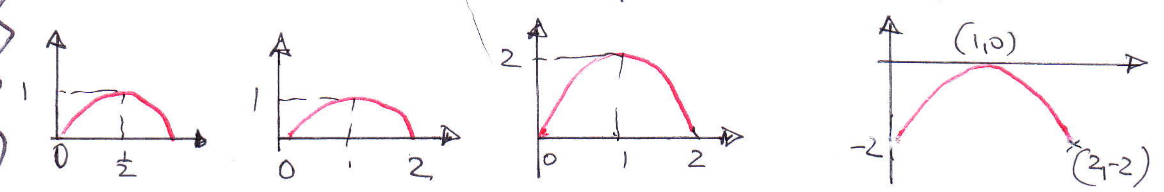
- $y=0$

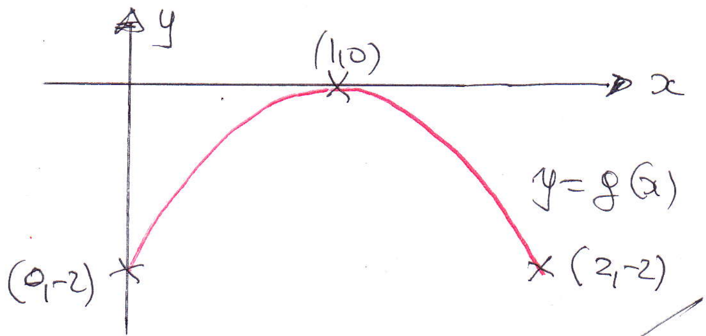
$$0 = \sqrt{1-(2x-1)^2}$$
$$0 = 1-(2x-1)^2$$
$$(2x-1)^2 = 1$$
$$2x-1 = \begin{cases} 1 \\ -1 \end{cases}$$
$$2x = \begin{cases} 2 \\ 0 \end{cases}$$
$$x = \begin{cases} 1 \\ 0 \end{cases} \qquad \therefore a = 1$$

b) BY SYMMETRY $f(\tfrac{1}{2}) = 1$

$$\therefore 0 \le f(x) \le 1$$

c) $f(x)$ $\quad f(\tfrac{1}{2}x)$ $\quad 2f(\tfrac{1}{2}x)$ $\quad 2f(x)-2$





d) DOMAIN $0 \le x \le 2$

RANGE : $-2 \le g(x) \le 0$

6. a) $LHS = \dfrac{2\cot\theta}{1+\cot^2\theta} = \dfrac{2\cot\theta}{\operatorname{cosec}^2\theta} = \dfrac{\frac{2\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$

$= \dfrac{2\cos\theta\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}} = 2\cos\theta\sin\theta = \sin 2\theta = RHS$

b) $4\cot^2\theta + 1 = 2\sin 2\theta\,(1+\cot^2\theta)$

FROM PART (a)

$\Rightarrow 4\cot^2\theta + 1 = 2\left(\dfrac{2\cot\theta}{\cancel{1+\cot^2\theta}}\right)\cancel{(1+\cot^2\theta)}$

$\Rightarrow 4\cot^2\theta + 1 = 4\cot\theta$

$\Rightarrow 4\cot^2\theta - 4\cot\theta + 1 = 0$

$\Rightarrow (2\cot\theta - 1)^2 = 0$

$\Rightarrow \cot\theta = \frac{1}{2}$

$\Rightarrow \tan\theta = 2$

$\arctan 2 \simeq 63.4°$

$\Rightarrow \theta \simeq 63.4° \pm 180n \quad n = 0, 1, 2, 3, \ldots$

$\theta_1 \simeq 63.4°$

$\theta_2 \simeq 243.4°$

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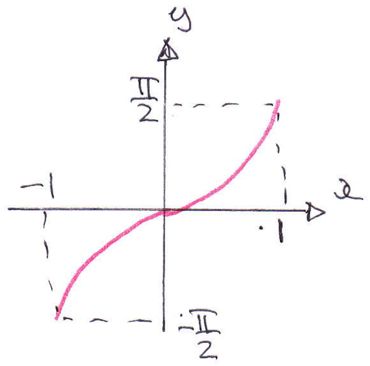
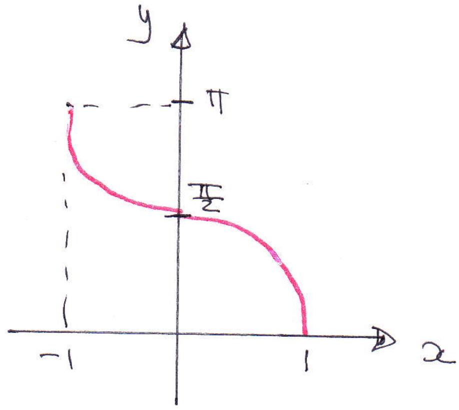
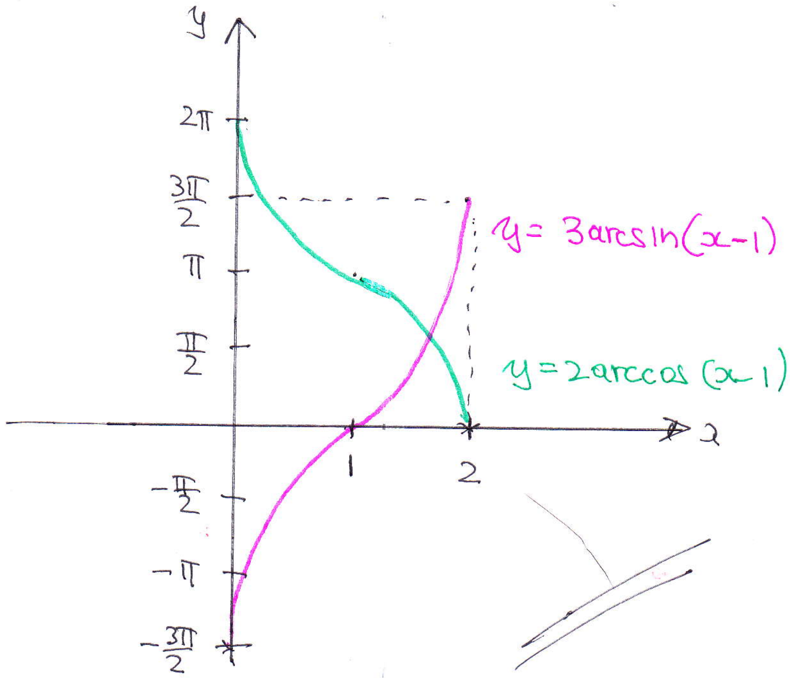
7. a)



$y = \arcsin x$



$y = \arccos x$



b)

$$\left.\begin{aligned} y &= 3\arcsin(x-1) \\ y &= 2\arccos(x-1) \end{aligned}\right\} \Rightarrow 3\arcsin(x-1) = 2\arccos(x-1)$$

$$\Rightarrow \arcsin(x-1) = \frac{2}{3}\arccos(x-1)$$

$$\Rightarrow x-1 = \sin\left[\frac{2}{3}\arccos(x-1)\right]$$

$$\Rightarrow x = 1 + \sin\left[\frac{2}{3}\arccos(x-1)\right]$$

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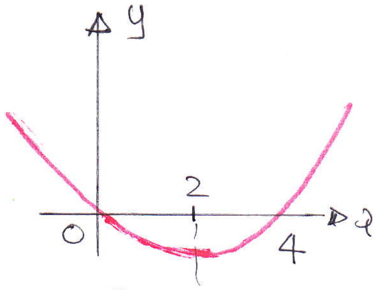
$$x_{n+1} = 1 + \sin\left[\tfrac{2}{3}\arccos(x_n - 1)\right]$$

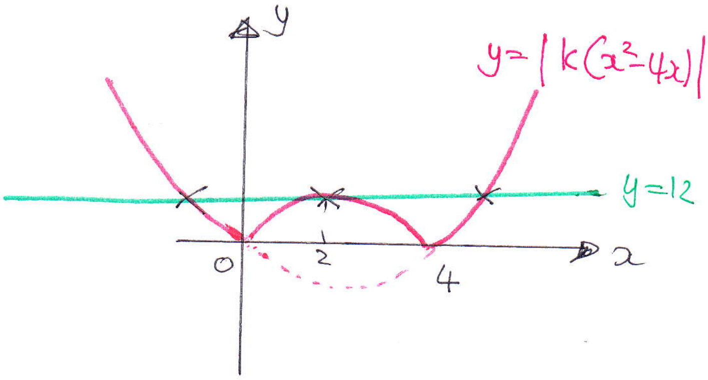
$x_1 = 1.6$
$x_2 = 1.57957$
$x_3 = 1.59323$
$x_4 = 1.58414$
$x_5 = 1.59021$
$x_6 = 1.58617$
$x_7 = 1.58886$

$\therefore\ x \simeq 1.59$ 2 d.p.

8. a)



$y = k(x^2 - 4x)$
$y = kx(x-4)$



- when $x = 2$

$|f(x)| = |k(2^2 - 4\times 2)|$
$= |k(-4)|$
$= 4k$

$\therefore\ 4k = 12$
$k = 3$

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b) $\boxed{y = |3(x^2 - 4x)|}$

$$|3x^2 - 12x| = 12$$

$3x^2 - 12x = 12$

$3x^2 - 12x - 12 = 0$

$x^2 - 4x - 4 = 0$

$(x-2)^2 - 8 = 0$

$(x-2)^2 = 8$

$x - 2 = \pm 2\sqrt{2}$

$x = 2 \pm \sqrt{2}$

OR

$3x^2 - 12x = -12$

$3x^2 - 12x + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

$$\therefore x = \begin{cases} 2 \\ 2 + \sqrt{2} \\ 2 - \sqrt{2} \end{cases}$$