C3, IYGB, PAPER S

1. $$\left[e-\left(\frac{e^{x+\frac{1}{2}}}{e^{-2x}}\right)^2\right]\times\frac{1}{e^{3x}+\ln e}=\left[e-\frac{e^{2x+1}}{e^{-4x}}\right]\times\frac{1}{e^{3x}+1}$$

$$=\left[e-e^{6x+1}\right]\times\frac{1}{e^{3x}+1}=e\left[1-e^{6x}\right]\left[\frac{1}{e^{3x}+1}\right]$$

DIFFERENCE OF SQUARES

$$=e\left[\frac{1-(e^{3x})^2}{1+e^{3x}}\right]=e\left[\frac{(1-e^{3x})\cancel{(1+e^{3x})}}{\cancel{1+e^{3x}}}\right]$$

$$=e(1-e^{3x})=e-e^{3x+1}$$

AS REQUIRED

2. $$f(x)=\frac{(3-2\cos^2x)(1+6\sin^2x)^{\frac{1}{2}}}{(1+\tan x)^2}$$

- TAKING LOGS

$$\ln[f(x)]=\ln\left[\frac{(3-2\cos^2x)(1+6\sin^2x)^{\frac{1}{2}}}{(1+\tan x)^2}\right]$$

$$\ln[f(x)]=\ln(3-2\cos^2x)+\tfrac{1}{2}\ln(1+6\sin^2x)-2\ln(1+\tan x)$$

- DIFFERENTIATE WITH RESPECT TO x

$$\frac{1}{f(x)}f'(x)=\frac{1}{3-2\cos^2x}(4\cos x\sin x)+\frac{1}{2}\left(\frac{1}{1+6\sin^2x}\right)(12\sin x\cos x)-2\left(\frac{1}{1+\tan x}\right)\sec^2x$$

$$\frac{1}{f(x)}f'(x)=\frac{2\sin 2x}{3-2\cos^2x}+\frac{3\sin 2x}{1+6\sin^2x}-\frac{2\sec^2x}{1+\tan x}$$

$$f'(x)=f(x)\left[\frac{2\sin 2x}{3-2\cos^2x}+\frac{3\sin 2x}{1+6\sin^2x}-\frac{2(1+\tan^2x)}{1+\tan x}\right]$$

- FIRSTLY $f\left(\frac{\pi}{4}\right)=\dfrac{\left[3-2\times\frac{1}{2}\right]\left[1+6\times\frac{1}{2}\right]^{\frac{1}{2}}}{(1+1)^2}=\dfrac{2\times2}{4}=1$

THUS $f'\left(\frac{\pi}{4}\right)=1\times\left[\dfrac{2\times1}{3-2\times\frac{1}{2}}+\dfrac{3\times1}{1+6\times\frac{1}{2}}-\dfrac{2(1+1)}{1+1}\right]=1+\frac{3}{4}-2=-\frac{1}{4}$

3.

$$\sin x\cos x + \frac{1}{2} = \cos^2\left(\frac{x}{2}-\frac{\pi}{6}\right)$$
$$2\sin x\cos x + 1 = 2\cos^2\left(\frac{x}{2}-\frac{\pi}{6}\right)$$
$$\sin 2x = 2\cos^2\left(\frac{x}{2}-\frac{\pi}{6}\right) - 1$$

← DOUBLE ANGLE FOR COSINE $\cos 2A = 2\cos^2 A - 1$

$$\sin 2x = \cos\left[2\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]$$
$$\sin 2x = \cos\left[x-\frac{\pi}{3}\right]$$
$$\sin 2x = \sin\left[\frac{\pi}{2}-\left(x-\frac{\pi}{3}\right)\right]$$

← $\boxed{\cos\theta \equiv \sin\left(\frac{\pi}{2}-\theta\right)}$

$$\sin 2x = \sin\left(\frac{5\pi}{6}-x\right)$$

$$\begin{cases} 2x = \left(\frac{5\pi}{6}-x\right) \pm 2n\pi \\ 2x = \pi - \left(\frac{5\pi}{6}-x\right) \pm 2n\pi \end{cases}$$

$n = 0, 1, 2, 3, \ldots$

$$\begin{cases} 3x = \frac{5\pi}{6} \pm 2n\pi \\ x = \frac{\pi}{6} \pm 2n\pi \end{cases}$$

$$\begin{cases} x = \frac{5\pi}{18} \pm \frac{2}{3}n\pi \\ x = \frac{\pi}{6} \pm 2n\pi \end{cases}$$

$\therefore x = \frac{5\pi}{18}, \frac{17}{18}\pi, \frac{\pi}{6}$

4.

$$y = |x^2-16| + 2x$$

- IF $x^2-16 \geq 0$, $x^2 \geq 16$, $x \leq -4$ OR $x \geq 4$

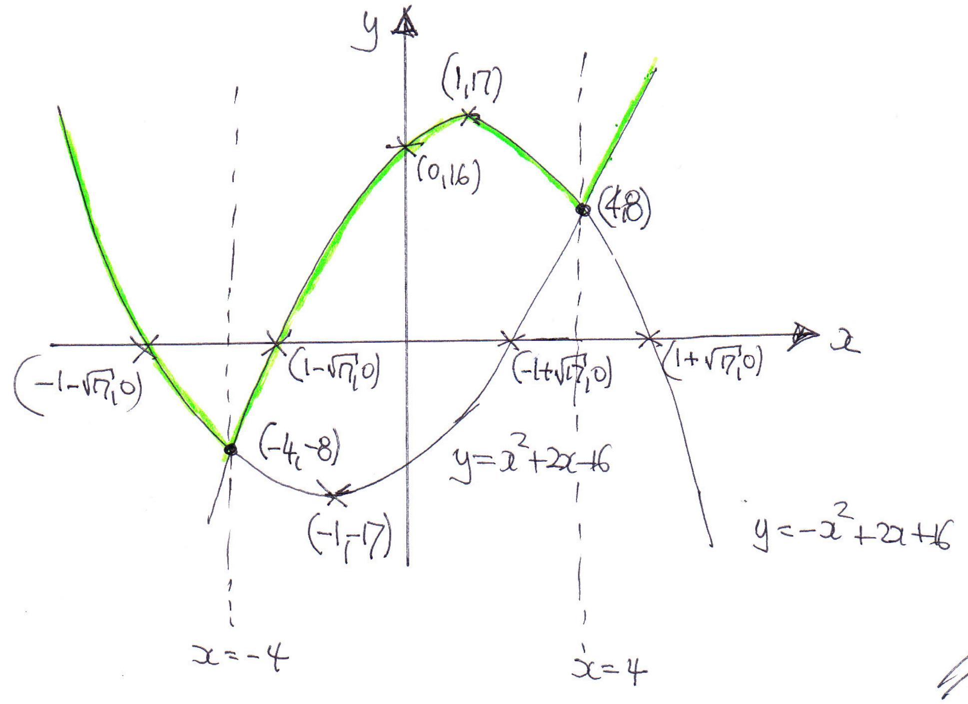
$$y = (x^2-16)+2x$$
$$y = x^2+2x-16$$
$$\boxed{y = (x+1)^2-17}$$

$\left(\begin{array}{c} y=0 \\ x = -1 \pm \sqrt{17} \end{array}\right)$

- IF $x^2-16 \leq 0$, $x^2 \leq 16$, $-4 \leq x \leq 4$

$$y = -(x^2-16)+2x$$
$$y = -x^2+2x+16$$
$$-y = x^2-2x-16$$
$$-y = (x-1)^2-17$$
$$\boxed{y = 17-(x-1)^2}$$

$\left(\begin{array}{c} y=0 \\ x = 1 \pm \sqrt{17} \end{array}\right)$



5. a) $f(x) = \begin{cases} x^2 & 0 \le x \le 1 \\ 2-x & 1 < x \le 2 \end{cases}$ $\quad g(x) = \frac{1}{x} \quad x \in \mathbb{R},\ x \neq 0$

- $f(g(x)) = f\left(\frac{1}{x}\right) = \begin{cases} \frac{1}{x^2} & D: 0 < \frac{1}{x} \le 1 \Rightarrow x \ge 1 \\ 2 - \frac{1}{x} & D: 1 < \frac{1}{x} \le 2 \Rightarrow \frac{1}{2} \le x < 1 \end{cases}$

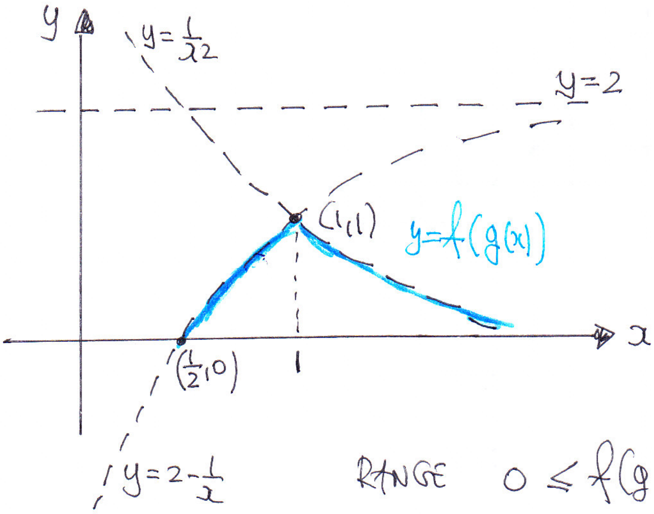
$\therefore f(g(x)) = \begin{cases} 2 - \frac{1}{x} & \frac{1}{2} \le x < 1 \\ \frac{1}{x^2} & x \ge 1 \end{cases}$

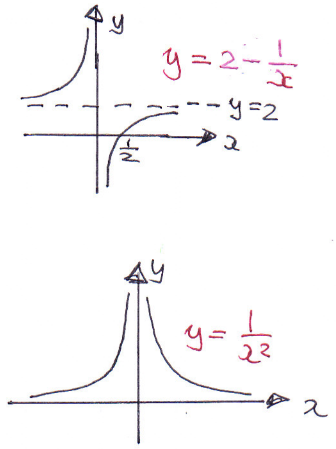
- $g(f(x)) = g(x^2 \text{ OR } 2-x) = \begin{cases} \frac{1}{x^2} & D: 0 < x \le 1 \\ \frac{1}{2-x} & D: 1 < x < 2 \end{cases}$

ASYMPTOTE

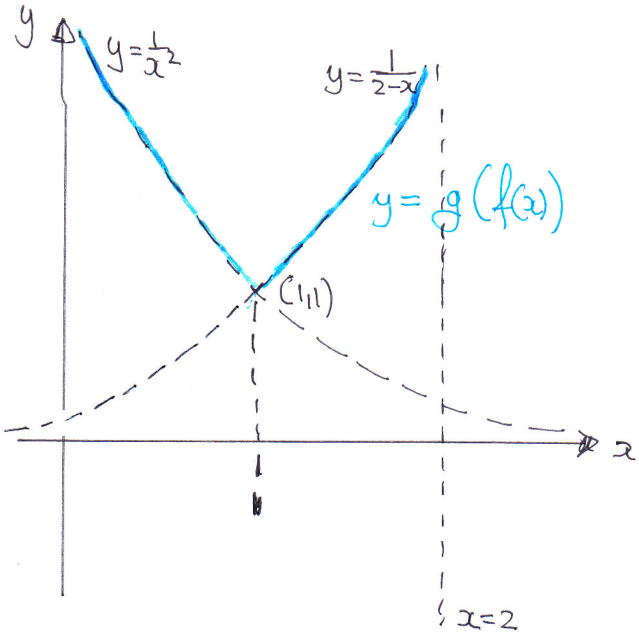
$\therefore g(f(x)) = \begin{cases} \frac{1}{x^2} & 0 < x \le 1 \\ \frac{1}{2-x} & 1 \le x < 2 \end{cases}$

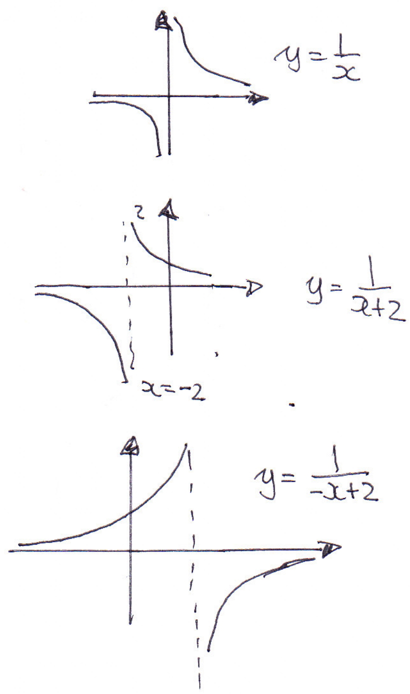
b)





RANGE $0 \leq f(g(x)) \leq 1$





RANGE $g(f(x)) \geq 1$

6\. a) $\dfrac{\sin 2\theta}{1+\sin\theta} = 1 - \sin\phi$

$\theta + \phi = \pi$

$\phi = \pi - \theta$

$\sin\phi = \sin(\pi - \theta)$

$\sin\phi = \sin\theta$

$\Rightarrow \dfrac{\sin 2\theta}{1+\sin\theta} = 1 - \sin\theta$

$\Rightarrow \sin 2\theta = (1-\sin\theta)(1+\sin\theta)$

$\Rightarrow 2\sin\theta\cos\theta = 1 - \sin^2\theta$

$\Rightarrow 2\sin\theta\cos\theta = \cos^2\theta$

$\Rightarrow \dfrac{2\sin\theta\cos\theta}{\cos^2\theta} = 1$ $\quad$ ($\cos\theta \neq 0$ AS $\theta \neq \frac{\pi}{2}$, $0 < \theta < \pi$)

$\Rightarrow 2\tan\theta = 1$

$\Rightarrow \tan\theta = \frac{1}{2}$

b) $\tan(3\theta + 5\phi) = \tan(3\theta + 5(\pi - \theta)) = \tan(-2\theta + \cancel{5\pi})$ PERIOD π

$= \tan(-2\theta) = -\tan 2\theta$ (ODD FUNCTION)

$= -\dfrac{2\tan\theta}{1-\tan^2\theta}$

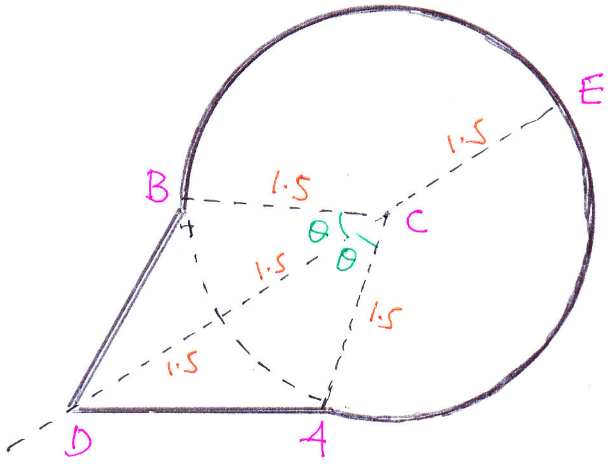
$= -\dfrac{2 \times \frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} = -\dfrac{1}{1-\frac{1}{4}}$

$= -\dfrac{1}{\frac{3}{4}} = -\dfrac{4}{3}$

AS REQUIRED

C3, IYGB, PAPER S

7.



- BY THE COSINE RULE OF $\overset{\triangle}{BCD}$

$$|BD|^2 = |BC|^2 + |CD|^2 - 2|BC||CD|\cos\theta$$
$$|BD|^2 = 1.5^2 + 3^2 - 2\times1.5\times3\cos\theta$$
$$|BD|^2 = \frac{9}{4} + 9 - 9\cos\theta$$
$$|BD|^2 = \frac{45}{4} - 9\cos\theta$$
$$|BD|^2 = \frac{9}{4}(5 - 4\cos\theta)$$
$$|BD| = \frac{3}{2}(5-4\cos\theta)^{\frac{1}{2}}$$

- $\overset{\frown}{BEA} = 1.5 \times (2\pi - 2\theta)$

$$\overset{\frown}{BEA} = 3\pi - 3\theta$$

- HENCE

$$L = \underbrace{3\pi - 3\theta}_{\overset{\frown}{BEA}} + \underbrace{3(5-4\cos\theta)^{\frac{1}{2}}}_{|BD|+|DA|}$$

$$\frac{dL}{d\theta} = -3 + \frac{3}{2}(5-4\cos\theta)^{-\frac{1}{2}}(4\sin\theta)$$

$$\boxed{\frac{dL}{d\theta} = -3 + \frac{6\sin\theta}{\sqrt{5-4\cos\theta}}}$$

- SOLVE FOR ZERO

$$\Rightarrow \frac{6\sin\theta}{\sqrt{5-4\cos\theta}} = 3$$
$$\Rightarrow \frac{2\sin\theta}{\sqrt{5-4\cos\theta}} = 1$$
$$\Rightarrow 2\sin\theta = \sqrt{5-4\cos\theta}$$
$$\Rightarrow 4\sin^2\theta = 5 - 4\cos\theta$$
$$\Rightarrow 4(1-\cos^2\theta) = 5 - 4\cos\theta$$
$$\Rightarrow 4 - 4\cos^2\theta = 5 - 4\cos\theta$$
$$\Rightarrow 0 = 4\cos^2\theta - 4\cos\theta + 1$$
$$\Rightarrow 0 = (2\cos\theta - 1)^2$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3} \quad \text{ONLY}$$

(IT SATISFIES "ORIGINAL" BEFORE SQUARING)

◉ NEXT FIND L IF $\theta = \frac{\pi}{3}$, $\cos\theta = \frac{1}{2}$ $\sin\theta = \frac{\sqrt{3}}{2}$

$$L = 3\pi - 3\theta + 3(5 - 4\cos\theta)^{\frac{1}{2}}$$
$$L = 3\pi - 3 \times \frac{\pi}{3} + 3\left(5 - 4 \times \frac{1}{2}\right)^{\frac{1}{2}}$$
$$L = 3\pi - \pi + 3\sqrt{3}$$
$$L = 2\pi + 3\sqrt{3}$$

◉ NATURE

$$\frac{dL}{d\theta} = -3 + \frac{6\sin\theta}{(5 - 4\cos\theta)^{\frac{1}{2}}}$$

$$\frac{d^2L}{d\theta^2} = \frac{(5-4\cos\theta)^{\frac{1}{2}}(6\cos\theta) - 6\sin\theta(5-4\cos\theta)^{-\frac{1}{2}}(2\sin\theta)}{(5-4\cos\theta)}$$

$$\frac{d^2L}{d\theta^2} = \frac{(5-4\cos\theta)^{-\frac{1}{2}}\left[6\cos\theta(5-4\cos\theta) - 12\sin^2\theta\right]}{5-4\cos\theta}$$

$$\boxed{\frac{d^2L}{d\theta^2} = \frac{30\cos\theta - 24\cos^2\theta - 12\sin^2\theta}{(5-4\cos\theta)^{3/2}} = \frac{30\cos\theta - 12\cos^2\theta - 12}{(5-4\cos\theta)^{\frac{3}{2}}}}$$

$$\left.\frac{d^2L}{d\theta^2}\right|_{\theta=\frac{\pi}{3}} = \frac{30 \times \frac{1}{2} - 12 \times \frac{1}{4} - 12}{\left(5 - 4 \times \frac{1}{2}\right)^{3/2}} = \frac{15 - 3 - 12}{3\sqrt{3}} = 0$$

$$\frac{d^3L}{d\theta^3} = \frac{(5-4\cos\theta)^{3/2}(-30\sin\theta + 24\sin\theta\cos\theta) - (30\cos\theta - 12\cos^2\theta - 12)\,6(5-4\cos\theta)^{\frac{1}{2}}\sin\theta}{(5-4\cos\theta)^3}$$

$$\left.\frac{d^3L}{d\theta^3}\right|_{\theta=\frac{\pi}{3}} = \frac{3\sqrt{3}(-15\sqrt{3} + 6\sqrt{3}) - \cancel{6(15 - 3 - 12)(\sqrt{3})\frac{\sqrt{3}}{2}}}{27}$$

$$= \frac{3\sqrt{3}(-9\sqrt{3})}{27} = -3 \neq 0$$

∴ POINT OF INFLEXION