IYGB GCE

Core Mathematics C3

Advanced

Practice Paper M

Difficulty Rating: 3.3467/1.5075

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$1-\frac{1}{x-2}+\frac{3}{x^2-x-2}, \quad x \neq 2, \ x \neq 1 .$$

Write the above algebraic expression as a single simplified fraction **(5)**

Question 2

A cubic equation has the following equation.

$$x^3+1=4x, \quad x \in \mathbb{R} .$$

a) Show that the above equation has a root α, which lies between 0 and 1. **(3)**

b) Show further that the above equation can be written as

$$x=\frac{1}{4-x^2} . \qquad \textbf{(1)}$$

An iterative formula, based on the rearrangement of part **(b)**, is to be used to find α.

c) Starting with $x_1=0.1$, find to 4 decimal places, the value of x_2, x_3 and x_4. **(2)**

d) By considering the graph of a suitable function in an appropriate interval, show that $\alpha=0.25410$, correct to 5 decimal places. **(3)**

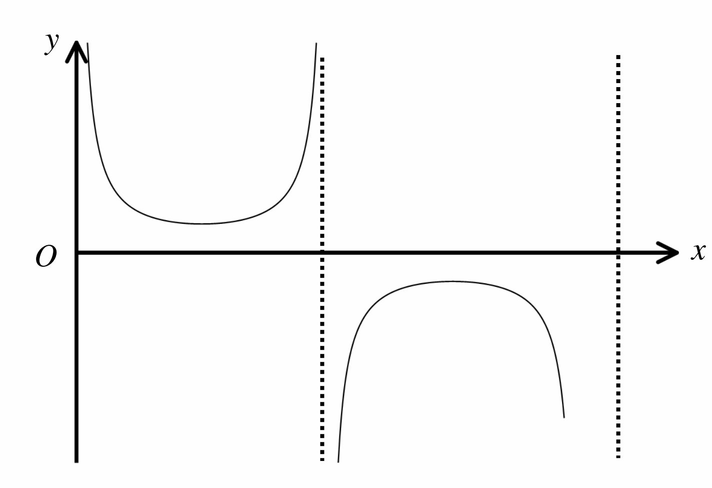
Question 3

A curve C has equation

$$y=\ln\left(\frac{x}{4}\right), \quad x>0 .$$

Find an equation of the normal to C at the point where $x=4$ **(5)**

Question 4



The figure above shows the graph of

$$y = \operatorname{cosec} x \text{ , for } 0 \le x < 360° .$$

a) Sketch the graph of

$$y = |\operatorname{cosec} x| \text{ , for } 0 \le x < 360° .$$

The sketch must include the equations of any asymptotes and the coordinates of any stationary points. **(3)**

b) Solve the equation

$$\operatorname{cosec} x = 2 \text{ , for } 0 \le x < 360° . \qquad \textbf{(3)}$$

c) Hence solve the equation

$$|\operatorname{cosec} x| = 2 \text{ for } 0 \le x < 360° . \qquad \textbf{(2)}$$

Question 5

$$\sin\theta = \frac{8}{17} \quad \text{and} \quad \cos\varphi = \frac{5}{13}.$$

If θ is obtuse and φ is acute, show that

$$\cos(\theta+\varphi) = -\frac{171}{221}. \qquad (5)$$

Question 6

A curve has equation

$$x = \sqrt{2y+1}, \;\; y \geq -\frac{1}{2}.$$

a) Find $\frac{dy}{dx}$ in **terms of** $\boldsymbol{x}$, by **first** finding $\frac{dx}{dy}$. **(5)**

b) By making y the subject of the above equation and differentiating the resulting equation, verify the result of part **(a)**. **(3)**

Question 7

A hot metal rod is cooled down by dipping it into a large pool of water which is maintained at constant temperature.

The temperature of the metal rod, T °C, is given by

$$T = 20 + 480e^{-0.1t}, \ t \geq 0$$

where t is time in minutes since the rod was dipped in the water.

a) State the temperature …

i. … of the rod before it enters the water. **(1)**

ii. … of the water. **(1)**

b) Determine the value of t when the rod reaches a temperature of 260 °C. **(5)**

c) Find the value of t when the rod is cooling at the rate of 0.533 °C per minute. **(5)**

d) Show clearly that

$$\frac{dT}{dt} = -\frac{1}{10}(T - 20).$$ **(3)**

Question 8

It is given that

$$\sin P + \sin Q \equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right).$$

a) Prove the validity of the above trigonometric identity, by using the compound angle identities for $\sin(A+B)$ and $\sin(A-B)$. **(4)**

b) Hence, or otherwise, solve the trigonometric equation

$$\sin 4\theta + \sin 2\theta = \cos\theta, \ \ 0 \leq \theta < \pi,$$

giving the answers in terms of π. **(7)**

Question 9

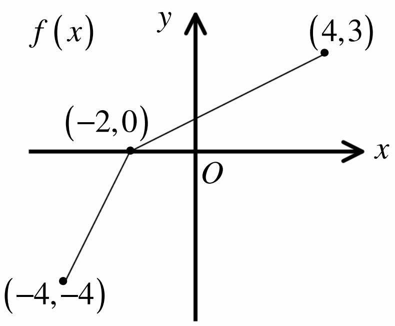


figure 1

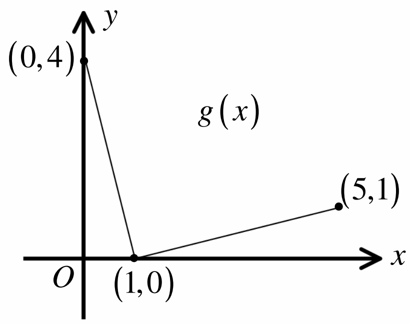


figure 2

Figure 1 and figure 2 above, show the graphs of two piecewise continuous functions $f(x)$ and $g(x)$, respectively.

Each graph consists of two straight line segments joining the points with the coordinates shown in each figure.

a) Sketch on the same set of axes the graphs of $f(x)$ and its inverse $f^{-1}(x)$, stating the domain and range of $f^{-1}(x)$. **(4)**

b) Evaluate …

i. … $fg\left(\frac{1}{2}\right)$. **(2)**

ii. … $fgf^{-1}(1)$. **(3)**