# IYGB GCE

## Core Mathematics C3

## Advanced

### Practice Paper L

Difficulty Rating: 3.1400/1.3986

### Time: 1 hour 30 minutes

**Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.**

**Information for Candidates**

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper.
The total mark for this paper is 75.

**Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

**Question 1**

$$x^3 + 3x = 5, \ x \in \mathbb{R}.$$

**a)** Show that the above equation has a root $\alpha$ between $1$ and $2$. **(2)**

An attempt is made to find $\alpha$ using the iterative formula

$$x_{n+1} = \frac{5 - x_n^{\ 3}}{3}, \ x_1 = 1.$$

**b)** Find, correct to $2$ decimal places, the value of $x_2$, $x_3$, $x_4$, $x_5$ and $x_6$, discussing further the behaviour of this sequence of approximations to $\alpha$. **(2)**

**c)** Use the iterative formula

$$x_{n+1} = \sqrt[3]{5 - 3x_n}, \ x_1 = 1,$$

to find, to $2$ decimal places, the value of $x_2$, $x_3$, $x_4$, $x_5$ and $x_6$. **(2)**

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**Question 2**

A curve has equation

$$y = \left(x^2 + 3x + 2\right)\cos 2x.$$

Determine an equation of the tangent to the curve at the point where the curve crosses the $y$ axis. **(6)**

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**Question 3**

Find, in exact form where appropriate, the solution of each of the following equations.

**a)** $4 - 3\mathrm{e}^{2x} = 3$ **(3)**

**b)** $\ln(2w+1) = 1 + \ln(w-1)$ **(5)**

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**Question 4**

$$f(x) \equiv 1 + \frac{4x}{2x-5} - \frac{15}{2x^2-7x+5}, \ x \in \mathbb{R}, \ x \neq \frac{5}{2}, \ x \neq k.$$

**a)** Show that

$$f(x) \equiv \frac{3x+2}{x-k}, \ x \in \mathbb{R}, \ x \neq k,$$

stating the value of $k$. **(5)**

**b)** Express $f(x)$ in the form

$$A + \frac{B}{x-k}, \ x \in \mathbb{R}, \ x \neq k,$$

where $A$ and $B$ are integers to be found. **(3)**

**c)** Sketch on separate set of axes the graph of …

**i.** … $y = f(x)$. **(4)**

**ii.** … $y = |f(x)|$. **(1)**

**iii.** … $y = f(|x|)$. **(1)**

Each sketch must include the coordinates of any points where the graph meets the coordinate axes and the equations of any asymptotes.

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**Question 5**

Given that $\cos x° = \sin(x-45)°$, show that

$$\tan x° = 1 + \sqrt{2}.$$ **(5)**

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**Question 6**

Differentiate each of the following expressions with respect to $x$, writing the final answer as a simplified fraction.

**a)** $y = \dfrac{\ln x}{1 + \ln x}$. **(3)**

**b)** $y = \ln\left(\dfrac{1}{x^2 + 9}\right)$. **(5)**

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**Question 7**

$$\operatorname{cosec}\theta + 8\cos\theta = 0, \ 0° \le \theta < 360° .$$

Find the solutions of the above trigonometric equation, giving the answers in degrees correct to one decimal place. **(6)**

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**Question 8**

Solve the equation

$$\frac{2|x| + 1}{3} - \frac{|x| - 1}{2} = 1 . \quad \textbf{(4)}$$

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**Question 9**

A curve has equation

$$y = \pi - \arccos(x+1),\ -2 \le x \le 0.$$

**a)** Describe geometrically the 3 transformations that map the graph of

$$y = \arccos x,\ -1 \le x \le 1,$$

onto the graph of

$$y = \pi - \arccos(x+1),\ -2 \le x \le 0. \qquad (3)$$

**b)** Sketch the graph of

$$y = \pi - \arccos(x+1),\ -2 \le x \le 0.$$

The sketch must include the coordinates of any points where the graph meets the coordinate axes. **(2)**

**c)** Use symmetry arguments to find the area of the finite region bounded by

$$y = \pi - \arccos(x+1),\ -2 \le x \le 0,$$

and the coordinate axes. **(2)**

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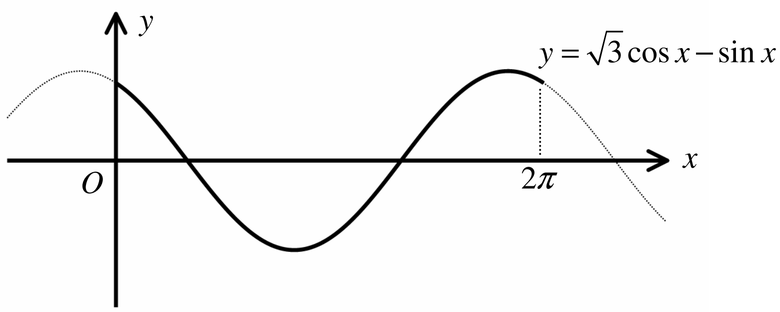
**Question 10**




The graph of $y = \sqrt{3}\cos x - \sin x$ for $0 \le x \le 2\pi$ is shown in the figure above.

**a)** Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$. **(4)**

The function $f$ is defined as

$$f(x) = \sqrt{3}\cos x - \sin x, \ x \in \mathbb{R}, \ 0 \le x \le 2\pi.$$

**b)** State the range of $f(x)$. **(1)**

**c)** Explain why $f(x)$ does not have an inverse. **(1)**

The function $g$ is defined as

$$g(x) = \sqrt{3}\cos x - \sin x, \ x \in \mathbb{R}, \ 0 < x_1 \le x \le x_2 < 2\pi.$$

It is further given that $f$ and $g$ have identical ranges, and the inverse function $g^{-1}(x)$ exists.

**d)** Find …

**i.** …the value of $x_1$ and the value of $x_2$. **(2)**

**ii.** … an expression for $g^{-1}(x)$. **(3)**