IYGB GCE

Core Mathematics C3

Advanced

Practice Paper J

Difficulty Rating: 3.6333/1.6901

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

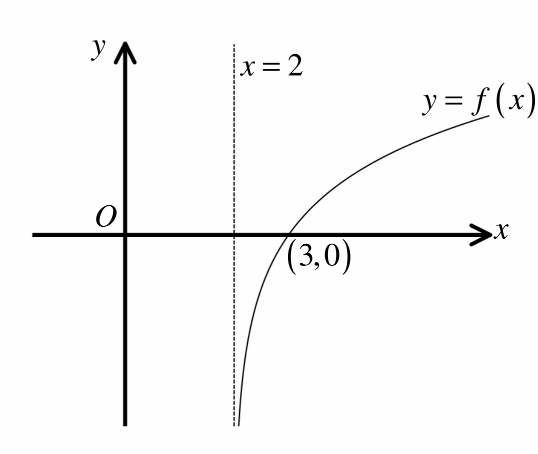
Question 1

The curve C has equation

$$y = \frac{x}{1 + \ln x}, \ x > 0, \ x \neq e^{-1}.$$

Show that C has a single stationary point and find its coordinates. **(6)**

Question 2



The figure above shows the graph of the curve with equation

$$y = f(x), \ x \in \mathbb{R}, \ x > 2.$$

The curve meets the x axis at the point with coordinates $(3,0)$, and the straight line with equation $x = 2$ is an asymptote to the curve.

Sketch, on separate diagrams, each of the following graphs.

a) $y = f^{-1}(x)$. **(3)**

b) $y = f(|x|)$. **(2)**

c) $y = f(2x-1)$. **(2)**

Each sketch must include the coordinates of any points where the graph meets the coordinate axes and the equations of any asymptotes.

Question 3

$$x^3 - 1 - \frac{1}{x} = 0, \quad x \neq 0.$$

a) By sketching two suitable graphs in the same diagram, show that the above equation has one positive root α and one negative root β.

The sketch must include the coordinates of the points where the curves meet the coordinate axes. **(3)**

b) Explain why $\alpha > 1$. **(1)**

To find α the following iterative formula is used

$$x_{n+1} = \sqrt[3]{\frac{1}{x_n} + 1}, \quad x_0 = 1.5.$$

c) Find, to 2 decimal places, the value of x_1, x_2 and x_3. **(2)**

d) By considering the sign of an appropriate function $f(x)$ in a suitable interval, show clearly that $\alpha = 1.221$, correct to 3 decimal places. **(2)**

Question 4

The point P, where $x = \pi$, lies on the curve with equation

$$f(x) = e^x \sin 2x, \quad 0 \leq x < 2\pi.$$

Show that an equation of the normal to the curve at P, is given by

$$x + 2ye^{\pi} = \pi.$$ **(7)**

Question 5

The function f is defined as

$$f : x \mapsto \frac{2x-1}{x^2-x-2} - \frac{1}{x-2}, \ x \in \mathbb{R}, \ x > 4.$$

a) Show clearly that

$$f : x \mapsto \frac{1}{x+1}, \ x \in \mathbb{R}, \ x > 4.$$ **(4)**

b) Find the range of f. **(1)**

c) Determine an expression for the inverse function, $f^{-1}(x)$. **(3)**

d) State the domain and range of $f^{-1}(x)$. **(2)**

The function g is given by

$$g : x \mapsto 3x^2 - 2, \ x \in \mathbb{R}.$$

e) Solve the equation

$$fg(x) = \frac{1}{11}.$$ **(5)**

Question 6

In this question it is given that the exact value of $\tan 20° = t$.

a) Express $\tan 25°$ in terms of t. **(3)**

b) By using the result of part (a) show that

$$\tan 25° \tan 65° = 1.$$ **(2)**

c) Show further that if

$$2\cos(\theta° + 20°) = 5\sin(\theta° - 20°),$$

then

$$\tan\theta = \frac{2+5t}{5+2t}.$$ **(5)**

Question 7

It is given that

$$\cos(A+B) + \cos(A-B) \equiv 2\cos A \cos B.$$

a) Prove the validity of the above trigonometric identity. **(1)**

b) Hence, or otherwise, solve the trigonometric equation

$$2\cos\left(x + \frac{\pi}{6}\right) = \sec\left(x + \frac{\pi}{2}\right), \quad 0 \le x \le \pi,$$

giving the answers in terms of π. **(7)**

Question 8

The temperature, θ °C, of an oven t minutes after it was switched on is given by

$$\theta = 300 - 280e^{-0.05t}, \ t \geq 0.$$

a) State the initial temperature of the oven. **(1)**

b) Find the value of t when the temperature of the oven …

i. … reaches 160 °C. **(4)**

ii. … is increasing at the rate of 4 °C per minute. **(3)**

c) Determine, with justification, the maximum temperature this oven can reach. **(1)**

The temperature θ °C of a **different** oven t minutes after it was switched on is modelled by a similar equation

$$\theta = 250 - 230e^{-0.1t}, \ t \geq 0.$$

d) Assuming that both ovens are switched on at the same time find the time when both ovens will have the same temperature since they were switched on. **(5)**