1. $1+\frac{x-8}{x^2+2x-8}-\frac{2}{x+4} = 1+\frac{x-8}{(x+4)(x-2)}-\frac{2}{x+4}$

$= \frac{1(x+4)(x-2)+(x-8)-2(x-2)}{(x+4)(x-2)} = \frac{x^2+\cancel{2x}-8+x-8-\cancel{2x}+4}{(x+4)(x-2)}$

$= \frac{x^2+x-12}{(x+4)(x-2)} = \frac{\cancel{(x+4)}(x-3)}{\cancel{(x+4)}(x-2)} = \frac{x-3}{x-2}$ // i.e. $p=3$, $q=2$

2. a) LET $y = 4-\ln(2x-1)$

$\ln(2x-1) = 4-y$

$2x-1 = e^{4-y}$

$2x = 1+e^{4-y}$

$x = \frac{1}{2}(1+e^{4-y})$

$\therefore f^{-1}(x) = \frac{1}{2}(1+e^{4-x})$ //

b) $f(f(1)) = f(4-\cancel{\ln 1})$

$= f(4)$

$= 4-\ln 7$ //

c) $f(x) = f(f(1))$

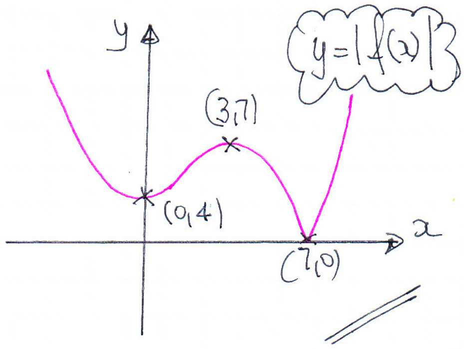
$\cancel{4}-\ln(2x-1) = \cancel{4}-\ln 7$

$\ln(2x-1) = \ln 7$

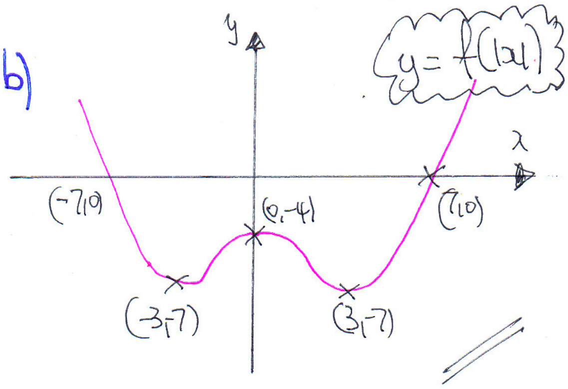
$2x-1 = 7$

$x = 4$ //

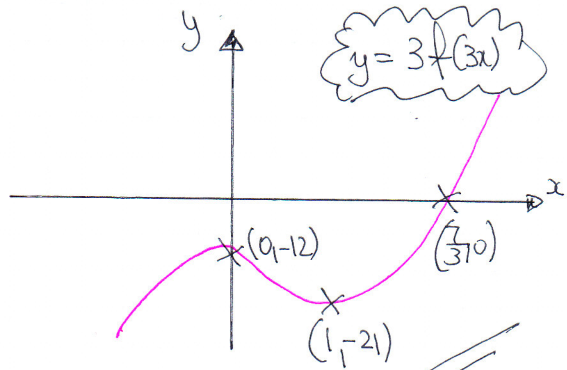
3. a)



b)

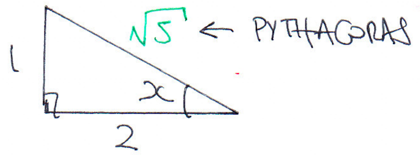


c)



THIS IS A HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{3}$ AND A VERTICAL STRETCH BY SCALE FACTOR 3 (ANY ORDER)

4. a) $\tan x = \frac{1}{2}$



$\therefore \sin x = \frac{1}{\sqrt{5}}$

$\therefore \operatorname{cosec} x = \sqrt{5}$

b) $\tan(x+y) = 2$

$\Rightarrow \frac{\tan x + \tan y}{1 - \tan x \tan y} = 2$

$\Rightarrow \frac{\frac{1}{2} + \tan y}{1 - \frac{1}{2}\tan y} = 2$

$\Rightarrow \frac{1}{2} + \tan y = 2\left(1 - \frac{1}{2}\tan y\right)$

$\Rightarrow \frac{1}{2} + \tan y = 2 - \tan y$

$\Rightarrow 2\tan y = \frac{3}{2}$

$\Rightarrow \tan y = \frac{3}{4}$

5. a) $V = 10 + 8e^{-\frac{1}{12}t}$

when $t=0$

$V = 10 + 8e^{0}$

$V = 18$

b) $14 = 10 + 8e^{-\frac{1}{12}t}$

$\Rightarrow 4 = 8e^{-\frac{1}{12}t}$

$\Rightarrow \frac{1}{2} = e^{-\frac{1}{12}t}$

$\Rightarrow 2 = e^{\frac{1}{12}t}$

$\Rightarrow \ln 2 = \frac{1}{12}t$

$\Rightarrow t = 12\ln 2 \simeq 8.3177\ldots$

$\therefore$ 08:19 ← $0.3177\ldots \times 60$

c) $V = 10 + 8e^{-\frac{1}{12}t}$

$\frac{dV}{dt} = -\frac{8}{12}e^{-\frac{1}{12}t}$

$\frac{dV}{dt} = -\frac{2}{3}e^{-\frac{1}{12}t}$

$\left.\frac{dV}{dt}\right|_{t=12} = -\frac{2}{3}e^{-\frac{1}{12}\times 12}$

$= -\frac{2}{3}\times e^{-1}$

$\simeq -0.245$

(MINUS IMPLIES DECREASE)

d) AS $t \to \infty$ $\quad e^{-\frac{1}{12}t} \to 0$

$\therefore V \to 10 + 8\times 0$

$\therefore$ LIMITING VALUE IS 10

6. a) $y = e^{2x}(x^2 - 4x - 2)$

$$\frac{dy}{dx} = 2e^{2x}(x^2 - 4x - 2) + e^{2x}(2x - 4)$$
$$= 2x^2e^{2x} - 8xe^{2x} - 4e^{2x} + 2xe^{2x} - 4e^{2x}$$
$$= 2x^2e^{2x} - 6xe^{2x} - 8e^{2x}$$
$$= 2e^{2x}(x^2 - 3x - 4)$$

AS REQUIRED

b) $\frac{dy}{dx} = 0 \qquad 2e^{2x}(x^2 - 3x - 4) = 0$

$$x^2 - 3x - 4 = 0 \qquad e^{2x} \neq 0$$
$$(x+1)(x-4) = 0$$
$$x = \begin{cases} -1 \\ 4 \end{cases} \qquad y = \begin{cases} e^{-2}((-1)^2 - 4(-1) - 2) = 3e^{-2} \\ e^{8}(4^2 - 4\times 4 - 2) = -2e^{8} \end{cases}$$

$\therefore (-1, 3e^{-2})$ & $(4, -2e^{8})$

7. a) $y = (e^{2x} - 2x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(e^{2x} - 2x)^{-\frac{1}{2}}(2e^{2x} - 2) = (e^{2x} - 1)(e^{2x} - 2x)^{-\frac{1}{2}} = \frac{e^{2x} - 1}{\sqrt{e^{2x} - 2x}}$$

$$\left.\frac{dy}{dx}\right|_{x=p} = \frac{e^{2p} - 1}{\sqrt{e^{2p} - 2p}} \quad \leftarrow \text{tangent gradient}$$

NOW AT P $x = p \implies P(p, (e^{2p} - 2p)^{\frac{1}{2}})$

$$\therefore y - (e^{2p} - 2p)^{\frac{1}{2}} = \frac{e^{2p} - 1}{(e^{2p} - 2p)^{\frac{1}{2}}}(x - p)$$

PASSES THROUGH $O(0,0) \implies -(e^{2p} - 2p)^{\frac{1}{2}} = \frac{-p(e^{2p} - 1)}{(e^{2p} - 2p)^{\frac{1}{2}}}$

$$\implies -(e^{2p} - 2p)^{1} = -p(e^{2p} - 1)$$
$$\implies -e^{2p} + 2p = -pe^{2p} + p$$

$$\Rightarrow pe^{2p} - e^{2p} = -p$$
$$\Rightarrow e^{2p}(p-1) = -p$$
$$\Rightarrow e^{2p}(1-p) = p$$

AS REQUIRED SINCE $x=p$

b) $(1-x)e^{2x} = x$

$(1-x)e^{2x} - x = 0$

- LET $f(x) = (1-x)e^{2x} - x$

$f(0.8) = 0.1906\ldots$
$f(1) = -1$

AS $f(x)$ IS CONTINUOUS & CHANGES SIGN IN THE INTERVAL, THERE IS AT LEAST ONE ROOT IN THE INTERVAL $(0.8, 1)$

c) $x_0 = 0.8$

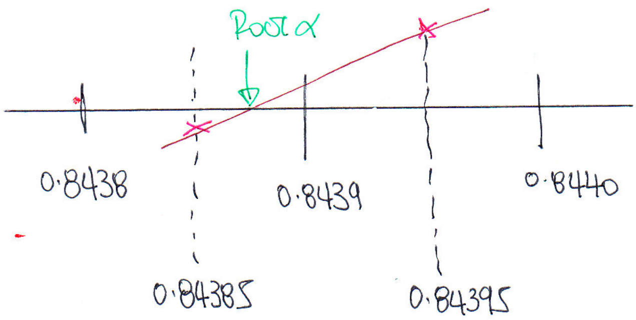
$x_1 = 0.838$

$x_2 = 0.843$

$x_3 = 0.844$

$x_4 = 0.844$

d)



$f(0.84385) = 0.000460$
$f(0.84395) = -0.000014$ $\Rightarrow$

$0.84385 < \alpha < 0.84395$

$\alpha = 0.8439$ 4. d.p

8. a) $\sqrt{3}\cos x - \sin x \equiv R\cos(x+\alpha)$

$\equiv R\cos x\cos\alpha - R\sin x\sin\alpha$

$\equiv (R\cos\alpha)\cos x - (R\sin\alpha)\sin x$

THUS $\left.\begin{array}{l} R\cos\alpha = \sqrt{3} \\ R\sin\alpha = 1 \end{array}\right\}$ SQUARE & ADD $R = \sqrt{(\sqrt{3})^2 + 1^2} = 2$

DIVIDE EQUATIONS $\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \frac{1}{\sqrt{3}}$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\therefore f(x) = 2\cos\left(x + \frac{\pi}{6}\right)$

b) $f_{MAX}(x) = 2$

IT OCCURS WHEN $\cos\left(x + \frac{\pi}{6}\right) = +1$

$x + \frac{\pi}{6} = 0$

$x = -\frac{\pi}{6}$) Add 2π

$x = \frac{11\pi}{6}$

c) $D = 13 + \sqrt{3}\cos\left(\frac{\pi t}{6}\right) - \sin\left(\frac{\pi t}{6}\right)$ THIS IS PART OF (a) WITH $x = \frac{\pi t}{6}$

$D = 13 + 2\cos\left(\frac{\pi t}{6} + \frac{\pi}{6}\right)$

$D_{MAX} = 13 + 2 = 15$

THIS MAX OCCUR WHEN $x = \frac{11\pi}{6}$

$\frac{\pi t}{6} = \frac{11\pi}{6}$

$t = 11$ OR 11:00

NOTE IT ALSO HAPPENS IN CONTEXT AT 23:00

d) $D = 12$

HENCE

$$\Rightarrow 12 = 13 + 2\cos\left(\frac{\pi t}{6} + \frac{\pi}{6}\right)$$

$$\Rightarrow -1 = 2\cos\left(\frac{\pi t}{6} + \frac{\pi}{6}\right)$$

$$\Rightarrow -\frac{1}{2} = \cos\left(\frac{\pi t}{6} + \frac{\pi}{6}\right)$$

$$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \begin{cases} \frac{\pi t}{6} + \frac{\pi}{6} = \frac{2\pi}{3} \pm 2n\pi \\ \frac{\pi t}{6} + \frac{\pi}{6} = \frac{4\pi}{3} \pm 2n\pi \end{cases}$$

$n = 0, 1, 2, 3, \ldots$

(DIVIDE THROUGH BY π)

$$\Rightarrow \begin{cases} \frac{t}{6} + \frac{1}{6} = \frac{2}{3} \pm 2n \\ \frac{t}{6} + \frac{1}{6} = \frac{4}{3} \pm 2n \end{cases}$$

(MULTIPLY THROUGH BY 6)

$$\Rightarrow \begin{cases} t + 1 = 4 \pm 12n \\ t + 1 = 8 \pm 12n \end{cases}$$

$$\Rightarrow \begin{cases} t = 3 \pm 12n \\ t = 7 \pm 12n \end{cases}$$

$$\therefore t = 3, 15, 7, 19$$

OR 03:00
07:00
15:00
19:00